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Simplicial Homology Global Optimisation in the Problem of Point-to-Point Ionospheric Ray Tracing

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Abstract

This work discusses the application of simplicial homology global optimization in the shooting procedure of point-to-point ionospheric HF ray tracing. The presented examples demonstrate the capability of suggested method to resolve various ray configurations which occur in 3D inhomogeneous anisotropic ionosphere, including multipath propagation in the vicinity of maximum usable frequency, as well as to produce accurate simulated oblique ionograms.

1 Introduction

High-frequency (HF) radio waves currently have a wide range of applications including long-range communications, as well as ionospheric remote sensing. HF wave propagation in the ionosphere can be described in terms of geometrical optics approximation using the well-established mathematical basis that has been developed in the early 50's [1].

In ray tracing, one can outline two common types of problems. The first are initial value problems, where ray trajectories of HF radio waves are determined from given parameters of the transmitter and medium. The second are boundary value problems, where solution is defined as a set of all possible ray trajectories linking predefined transmitter and receiver positions through a given medium. The former can be directly solved by integrating a characteristic system for eikonal equation provided in [1]. Solving the latter is more difficult, and various techniques have been developed for that purpose. A direct variational approach based on Fermat's principle [2] is often used to provide a reliable way for finding ray solutions, in particular, a recent development of a generalized force approach [3] should be noted.

Techniques based on Fermat's principle can be applied in an efficient manner in many cases, but issues are encountered when geomagnetic field-induced effects have to be taken into account [2]. In those cases, shooting method remains the often-used alternative, despite the vast amount of necessary calculations due to the trial-and-error nature of the method. The major issue when considering point-to-point ray tracing is the sheer number of possible ray trajectories: for each transmission frequency in the range between critical frequency and maximum usable frequency (MUF), at least two solutions are possible for each ionospheric layer (commonly dubbed as *high* and *low* rays). To find all of them (especially near the MUF of a layer), a significant number of required computations is necessary.

There are multiple approaches to solving point-to-point ray tracing with shooting method which can be roughly split into two groups. The first is based on multidimensional search [4], while the second employs penalty function optimization, where penalties are assigned according to distance between ray target and the desired target. The latter are generally more efficient due to lower dimensionality of the problem [5].

In particular, it has been shown [5, 6] that Nelder-Mead simplex optimization method [7] can be successfully applied to find both high and low rays. A significant advantage of simplex optimization is that it is derivative-free, removing some constraints on the penalty function, and also that it requires less penalty function evaluations than other methods (e.g. Powell's method and its successors). The downside is that as with all local optimization techniques, the solution found using this method is only a local minimum and is highly dependable on the initial guess. To find multiple solutions, an extensive search in the initial guess space is required, without guarantee that all, if any, correct solutions would be found. To perform this search, a manual intervention or preconditioning in a form of global optimization is required.

There are numerous global optimization techniques, yet only a few of them are derivative-free. In particular, simplicial homology global optimization technique (SHGO) [8] has received significant attention in recent years. It had been successfully applied to various problems, demonstrating its efficiency and convergence speed. This article is concerned with applying SHGO to the problem of ionospheric point-to-point ray tracing.

2 Methods

In this work, ray tracing is performed by integrating Haselgrove's equations [1] in the way described in [9]. For description of the medium, NeQuick2 model [10] is used to provide a sensible estimation of electron density in the ionosphere, and IGRF 12 model [11] describes Earth's magnetic field. To account for absorption, electron collision frequency is estimated using neutral particle concentrations provided by NRLMSISE-00 model [12].

The cost function for optimization is defined as a following function of initial ray azimuth α and elevation ε:

$$C(\alpha,\varepsilon) = |\theta_{ray}(\alpha,\varepsilon) - \theta_{rec}|^2 + |\phi_{ray}(\alpha,\varepsilon) - \phi_{rec}|^2 + P_{space}$$
$$P_{space} = \begin{cases} \varepsilon^2, & \text{if } h_{ray}(\alpha,\varepsilon) > h_{rec} \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

Where $\theta_{ray,rec}$ and $\phi_{ray,rec}$ are the geographic coordinates of the final ray point and the intended receiver, respectively. Penalty P_{space} is introduced so that if a local optimization iteration ends up with a ray leaving the ionosphere, the penalty gradient becomes directed to lower elevation, leading the algorithm back into the viable search space. Correct solutions that satisfy the boundary conditions are therefore found at points where $C(\alpha,\varepsilon) = 0$. The resulting cost function becomes rather complex to compute, rendering most optimization methods inefficient.

The cost function is then subjected to SHGO algorithm as a preconditioning measure. SHGO constructs a simplicial complex (in our case, a triangulated mesh) out of a set of chosen vertices, giving a rough approximation of the surface of the cost function. A set of locally-convex areas is then discovered, and each of these is used as an initial guess for local optimization. To speed up the search, Sobol' sequence [13] vertex sampling method is used, providing a evenly-spaced pseudo-random coverage of the cost function domain. All candidate points discovered by SHGO are provided as initial guesses for Nelder-Mead simplex optimization [7] or any other suitable local optimizer.

When point-to-point ray tracing is considered, SHGO offers the following advantages when compared with other global optimization techniques: (a) it's derivative-free, requiring less cost function evaluations, and hence less traced rays; (b) it can determine multiple local minima with ease, especially useful when high and low rays are considered; (c) with increasing function sampling density, the number of candidate points for local optimization converges to a fixed value, avoiding numerous pointless local minima computations; and (d) most of its routines can be easily parallelized.

3 Results

The approach above was tested on a simulated HF communication link between Lovozero (68.006N, 35.015E) and Dixon (73.508N, 80.529E) for the test day of May 5th, 14:05 UT. $F10.7$ = 74.8 s.f.u was used to initialize NeQuick2 model.

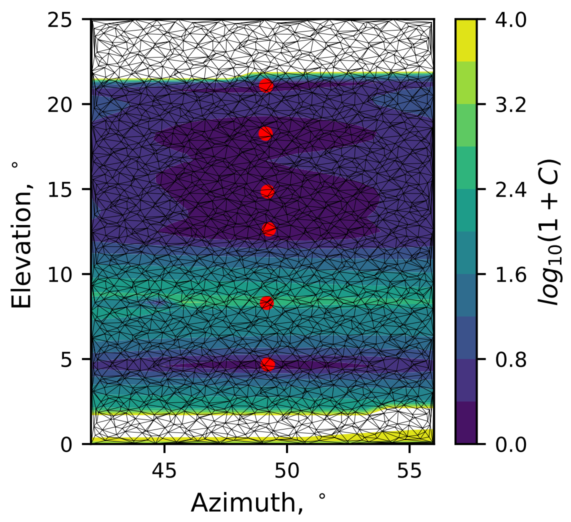

Figure 1. Cost function for frequency 10.48 MHz. Triangulated mesh is the constructed simplicial complex, red dots are discovered local minima.

Fig. 1 represents cost function (1) in (azimuth, elevation) domain for the frequency 10.48MHz (O-mode). The triangulated mesh is a simplicial complex constructed by SHGO from a Sobol' sequence, which is then used to locate initial guesses for Nelder-Mead simplex optimization. Six red dots are obtained local minima that satisfy $C(\alpha,\varepsilon) = 0$ with tolerance of 10^{-3}.

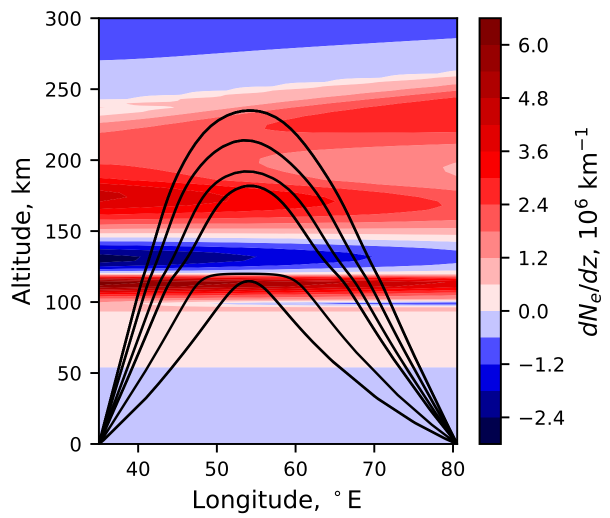

Figure 2. Ray trajectories corresponding to local minima of the cost function, presented in Fig. 1

Corresponding ray trajectories are shown in Fig. 2 along with vertical gradient of electron density according to NeQuick2 model. It can be seen that for the chosen frequency propagation is possible via both high and low rays for each E, F1 and F2 ionospheric layer. It's worth noting that SHGO is capable for resolving local minima located in very narrow and sharp valleys of the cost function with sufficient sampling density, as can be seen with the second lowest minimum at ∼8° elevation (E layer high ray), and also in regions where rays tend to escape the ionosphere, e.g. upper minimum at ∼22° elevation (F2 layer high ray).

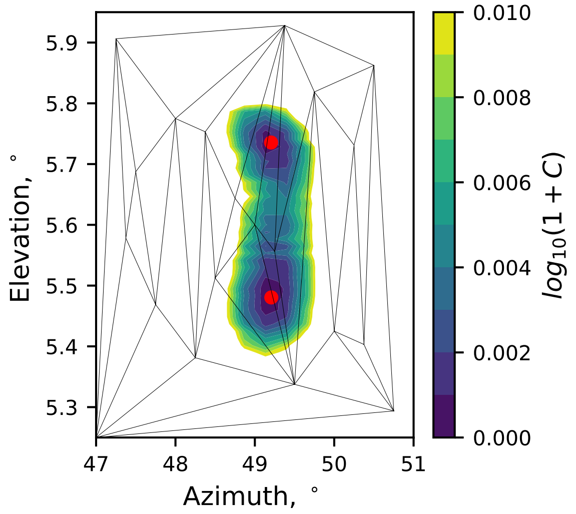

Figure 3. Cost function for frequency 11.46 MHz close to the MUF of E layer. Triangulated mesh is the constructed simplicial complex, red dots are discovered local minima.

As the operational frequency approaches MUF of a layer, local minima corresponding to high and low rays become closer in the cost function space, making it harder for the optimizer to resolve them. Nevertheless, SHGO is capable of doing so even for very closely located minima. Similar to Fig. 1, Fig. 3 shows the cost function for 11.46 MHz frequency, which is close to the MUF for E layer. Note that suggested approach effectively resolves two observed local minima, corresponding to high and low rays separated by less than 0.3° in initial elevation.

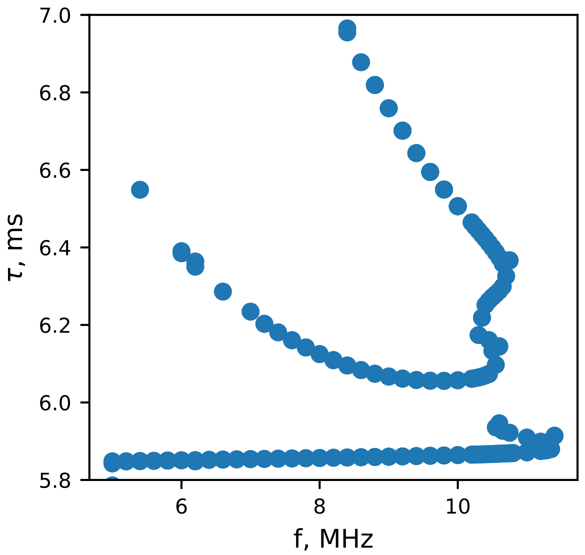

Figure 4. Simulated oblique ionogram for Lovozero - Dixon, May 5, 14:05 UT, $F10.7 = 74.8$ s.f.u

Fig. 4 presents a simulated oblique ionogram using SHGO for the considered HF link in a broad range of operational frequencies (3-15 MHz). Note that while the figure presents only O-mode single-hop traces, SHGO can as well solve both X-mode and multi-hop propagation. Propagation via all three layers presented in the model is observed with both high and low rays present. It should be noted that the simulated ionogram agrees well with real experimental data for this HF link [14].

4 Conclusions

In this paper we have presented an improved version of ionospheric point-to-point ray tracing in a realistic 3D inhomogeneous anisotropic ionosphere constructed using NeQuick2, IGRF12 and NRLMSISE-00 models, developed by introducing simplicial homology global optimization in the shooting procedure. This approach allows to resolve various ray configurations, including multipath propagation via close high and low rays in the vicinity of MUF and does not require manual intervention or preconditioning, as many other techniques based on local optimization or multidimensional search do. Therefore, the proposed method can become an useful tool in various applications, including ionospheric remote sensing and HF communications.

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