URSI GASS 2020, Rome, Italy, 29 August - 5 September 2020

Computation of Antenna Covariance Matrix for mm-Wave Massive MIMO — A Statistical Treatment

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Abstract

This paper presents a derivation of the antenna covariance matrix of a 3-D Rayleigh fading multi-input multi-output (MIMO) channel. The receive antenna covariance is calculated by considering the impinged electric field as random process to capture the true statistical behavior of the fading channel. Furthermore, this paper attempts to bridge the gap between antenna engineering and wireless communication by considering several practical aspects of the antenna and the physical channel like the relative size of the aperture of the whole antenna array and the relative size of the scatterers, active pattern of the array elements, etc. The contemporary millimeter wave massive MIMO communication system is considered. It is shown that the expression of the correlation coefficient, obtained in this work, reduces to the well known expression of the correlation coefficient of Clark's model in a special case.

1 Introduction

Massive multi-input multi-output (MIMO) technology has become one of the main enabling technologies of the 5G and forth coming standards of wireless communication [1, 2]. MIMO channel is generally modeled using four approaches: (*i*) Antenna Correlation Approach (*ACA*) (*ii*) Geometry Based Stochastic Approach (*GBSA*) (*iii*) Correlation Based Stochastic Approach (*CBSA*) and (*iv*) Ray-Tracing Approach (*RTA*) [3]. Here we attempt to bring *ACA*, generally used by antenna engineers [4] and *CBSA*, commonly used by experts of wireless communication together [5]. In the previous works, the electric field ($\vec{\mathscr{E}}$), impinged on a receive antenna is considered as a random variable having probability density function (*pdf*) $p_{\vec{\mathscr{E}}}(\theta,\phi)$. In contrary, a through statistical analysis of this topic is carried out in this paper, where $\vec{\mathscr{E}}$ is duly treated as a *random process*. Further, the active radiation pattern of the antennas is considered to take into account the mutual coupling. Modeling a wireless channel presently has got a special importance [2], because it is not only computationally expensive to estimate the channel state information (CSI), but knowledge of imperfect CSI compels the communication engineers to use far more complicated algorithms as well [6].

2 Model of Signal Received in Multipath Fading Channel

Let us consider communication over a Rayleigh fading MIMO 3-D wireless channel with a static transmitter (`Tx`) and a dynamic receiver (`Rx`), moving with velocity $\vec{v}_{\mathtt{Rx}}(\theta,\phi)$. We also assume that the interacting objects (IO) are static. The `Tx` has N_t antennas and the `Rx` has N_r antennas. Let $\mathbf{x} \in \mathbb{C}^{N_t\times 1}$ be the transmitted signal vector and $\mathbf{y} \in \mathbb{C}^{N_r\times 1}$ be the received signal vector. In a MIMO channel, the received signal is related to the transmitted signal in the following way

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}, \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{N_r\times N_t}$ is the complex channel matrix and $\mathbf{w} \in \mathbb{C}^{N_r\times 1}$ is the additive white Gaussian noise (AWGN) vector, added at each receiving antenna and the corresponding analog circuitry. Here, $h_{i,j} = \mathbf{H}(i,j)$ is the channel gain of the communication link between the $j-$th `Tx` and the $i-$th `Rx` antennas. However, in the case of down link of massive MIMO channel, there appears a $N_t \times N_r$ channel precoder matrix $\mathbf{D}$ due to digital precoding and we write $\mathbf{x} = \mathbf{D}\mathbf{s}$, where $\mathbf{s}$ is the message signal, to be transmitted to a single user, and $\mathbf{x}$ is the transmitted signal [1]. The general case of multi-user communication is bit more complicated and that is kept outside the scope of this summary paper. This channel matrix can be written in the form [2, 3]

$$\mathbf{H} = \frac{1}{\sqrt{tr\left(\mathbf{R}_{\mathtt{Rx}}\right)}}\mathbf{R}_{\mathtt{Rx}}^{\frac{1}{2}}\mathbf{H}_u\mathbf{R}_{\mathtt{Tx}}^{\frac{1}{2}}. \tag{2}$$

In (2), $\mathbf{R}_{\mathtt{Rx}}^{\frac{1}{2}} = \frac{1}{N_r}\mathbb{E}\left[\mathbf{H}\mathbf{H}^H\right] \in \mathbb{C}^{N_r\times N_r}$ is the *receive antenna covariance matrix* and $\mathbf{R}_{\mathtt{Tx}}^{\frac{1}{2}} = \frac{1}{N_t}\mathbb{E}\left[\mathbf{H}^H\mathbf{H}\right] \in \mathbb{C}^{N_t\times N_t}$ is the *transmit antenna covariance matrix*. Here, $\mathbf{H}_u$ is the $N_r \times N_t$ uncorrelated channel matrix and its elements are identically and independently distributed (i.i.d.).

Let $\vec{G}_{Ai}(\theta,\phi) = G_\theta^{Ai}(\theta,\phi)\hat{\theta} + G_\phi^{Ai}(\theta,\phi)\hat{\phi}$ be the far-zone radiated electric field of A_i, when A_k(for $k = 1,2,...,i-1,i+1,...,N$) are terminated at matched load. For an incident electric field $\vec{\mathscr{E}}_{Ai}(\theta,\phi)$, the open-circuit voltage induced at the terminal of A_i, is given by [4, 7]

$$v_i = \int_\phi\int_\theta \vec{G}_{Ai}(\theta,\phi)\cdot\vec{\mathscr{E}}_{Ai}(\theta,\phi)\, d\theta d\phi. \tag{3}$$

In modern I/Q communication, generally the transmitted signal takes the following form $s(t) = Re\{u(t)e^{j\omega_c t}\}$, where

$u(t)$ is the **low-pass equivalent** of the transmitted signal and ω_c is the carrier frequency. ***We assume that the channel does not change during a symbol duration.*** Therefore, without loss of generality, we drop $u(t)$ from the forthcoming expressions. In the case of Rayleigh fading channel, both the θ and ϕ polarizations of the electric field, incident on an antenna 'A' in an angle (θ,ϕ), can be expressed as [8]

$$\mathscr{E}^{A}_{\theta}(\theta,\phi) = I^{A}_{\theta,\phi}(\theta,\phi)\cos\omega_c t - Q^{A}_{\theta,\phi}(\theta,\phi)\sin\omega_c t, \quad (4a)$$

$$\text{with } I^{A}_{\theta,\phi}(\theta,\phi) = A_{\theta,\phi}(\theta,\phi)\cos(\omega_D t + \psi_{\theta,\phi}(\theta,\phi)), \quad (4b)$$

$$\text{and } Q^{A}_{\theta,\phi}(\theta,\phi) = A_{\theta,\phi}(\theta,\phi)\sin(\omega_D t + \psi_{\theta,\phi}(\theta,\phi)). \quad (4c)$$

However, in this paper, we deal with the in-phase component only to comply with the space constraint. So, the electric field incident on the $i-$th antenna is

$$\mathscr{E}^{Ai}_{\theta,\phi}(\theta,\phi) = A_{\theta,\phi}(\theta,\phi)\cos(\omega_D t + \psi_{d_i} + \psi_{\theta,\phi}(\theta,\phi))\cos\omega_c t, \quad (5)$$

Here $A_\theta(\theta,\phi)$ and $A_\phi(\theta,\phi)$ are the random amplitudes of unknown probability distribution function (PDF) i.e. for any given $\{\theta,\phi\}$, A_θ and A_ϕ are two random variables. $A_\theta(\theta,\phi)$ and $A_\phi(\theta,\phi)$ depend upon path-loss and shadowing. $\omega_D(\theta,\phi)$ is the doppler frequency shift experienced by the receiving antenna moving with velocity $\vec{v}_{Rx}$ and $\omega_D(\theta,\phi)$ can be expressed in terms of the carrier frequency and velocity of the receiver

$$\omega_D(\theta,\phi) = |\vec{v}_{Rx}(\theta,\phi)|\omega_c\cos\xi(\theta,\phi)/v_p, \quad (6)$$

where v_p is the phase velocity of the propagating wave and $\xi(\theta,\phi)$ is the angle subtend between direction of propagation of the incident wave and the direction of $\vec{v}_{Rx}$. In (4b), (4c) and (5), $\psi_\theta(\theta,\phi)$ and $\psi_\phi(\theta,\phi)$ are the random phase shifts i.e. for any given $\{\theta,\phi\}$, ψ_θ and ψ_ϕ are two random variables. The random phase shift depend upon path length and phase shift due to reflection on lossy scatterers. As the phase shift changes much faster than amplitude, the standard practice is to assume uniform PDF of $\psi_\theta(\theta,\phi)$ and $\psi_\phi(\theta,\phi)$ [8]. $A_{\theta,\phi}(\theta,\phi)$ and $\psi_{\theta,\phi}(\theta,\phi)$ have distribution other than normal distribution, but we consider infinite number of incoming rays in (3) due to presence of micro-scatterers. Consequently, the central limit theorem applies and the envelope of I/Q modulated v_i becomes Rayleigh faded.

For sub-6 GHz Massive MIMO antenna, the amplitude of the electric field, incident on different antennas may vary because of large physical aperture of the whole array. To avoid this complication, for the time being, we assume that the amplitude of field incident on all the antenna elements are same. This approximation remains valid as long as the distance between antennas remain small with respect to the size of the scatterers. *So, we consider a mm-wave massive MIMO communication system.*

The phase of the incident field, however, is not same for all the array elements, because of the phase shift ψ_{d_i} due to spatial separation of the antennas (see (5)). In massive MIMO, 2-D and even 3-D antenna arrays have been attempted, but we consider 1-D array in this summary paper to keep the treatment simple. Without loss of generality, we assume that the antennas are positioned along $X-$axis with a separation d_i. In the case of mm-wave Massive MIMO communication system, we consider the angle of arrival (AoA) to be same for all the array elements. Under these assumptions, the incoming waves received at A_i experience an extra phase shift of amount ψ_{d_i} relative to phase center; and ψ_{d_i} is given by

$$\psi_{d_i} = k_{d_i}\sin\theta\cos\phi, \quad (7)$$

where k is the propagation constant and (θ,ϕ) is the AoA.

We know that the amplitude of an incident wave depends upon the path-loss and shadowing, whereas its phase mainly depends upon the path length. Therefore, we assume the amplitude and the phase of the incident waves are statistically uncorrelated.

$Assumption-I$:

$$\overline{A_\theta\cdot\psi_\theta} = \overline{A_\theta}\cdot\overline{\psi_\theta}, \text{ and} \quad (8a)$$

$$\overline{A_\phi\cdot\psi_\phi} = \overline{A_\phi}\cdot\overline{\psi_\phi} \quad (8b)$$

As two polarizations propagate independently, we assume ψ_θ and ψ_ϕ are uncorrelated i.e.

$Assumption-II$:

$$\overline{\psi_\theta\ \psi_\phi} = \overline{\psi_\theta}\ \overline{\psi_\phi} \quad (9)$$

We also assume a "Kronecker Delta" channel model [8], where the electric fields incident at different angle of AoA are statistically uncorrelated.

$Assumption-III$:
For $(\theta_1 \neq \theta_2)$ and/or $(\phi_1 \neq \phi_2)$,

$$\overline{\mathscr{E}_\theta(\theta_1,\phi_1)\cdot\mathscr{E}_\theta(\theta_2,\phi_2)} = \overline{\mathscr{E}_\theta(\theta_1,\phi_1)}\cdot\overline{\mathscr{E}_\theta(\theta_2,\phi_2)}, \text{ and} \quad (10a)$$

$$\overline{\mathscr{E}_\phi(\theta_1,\phi_1)\cdot\mathscr{E}_\phi(\theta_2,\phi_2)} = \overline{\mathscr{E}_\phi(\theta_1,\phi_1)}\cdot\overline{\mathscr{E}_\phi(\theta_2,\phi_2)}. \quad (10b)$$

3 Derivation of Correlation Coefficient

The combined cross correlation coefficient between v_i and v_j is given by:

$$\rho_{v_iv_j} = \frac{\overline{(v_i-\overline{v_i})\cdot(v_j-\overline{v_j})^*}}{\left[\overline{(v_i-\overline{v_i})^2}\cdot\overline{(v_j-\overline{v_j})^2}\right]^{\frac{1}{2}}}. \quad (11)$$

In order to evaluate (11), we compute $\overline{v_i}$, $\overline{v_j}$, $\overline{v_i.v_j{}^*}$, $\overline{|v_i|^2}$ and $\overline{|v_j|^2}$. Using $Assumption-I$ and utilizing the fact that $\overline{\cos(\omega_D t+\psi_{d_{i,j}}+\psi_\theta(\theta,\phi))}=0$, we get $\overline{\vec{\mathscr{E}}_{Ai}(\theta,\phi)}=0$ and $\overline{\vec{\mathscr{E}}_{Aj}(\theta,\phi)}=0$. Therefore, from (3) we get $\overline{v_i}=\overline{v_j}=0$. Consequently, (11) becomes

$$\rho_{v_iv_j} = \frac{\overline{v_i\cdot v_j{}^*}}{\left[\overline{|v_i|^2}\cdot\overline{|v_j|^2}\right]^{\frac{1}{2}}}. \quad (12)$$

$$
\begin{aligned}
\overline{v_i \cdot v_j{}^*} &= \overline{\left[\int_\Omega \vec{G}_{Ai}(\Omega)\cdot\vec{\mathscr{E}}_{Ai}(\Omega)\, d\Omega\right]\cdot\left[\int_\Omega \vec{G}_{Aj}(\Omega)\cdot\vec{\mathscr{E}}_{Aj}(\Omega)\, d\Omega\right]^*} \\
&= \overline{\left[\int_\Omega G_\theta^{Ai}(\Omega)\mathscr{E}_\theta^{Ai}(\Omega)\, d\Omega\right]\left[\int_\Omega G_\theta^{Aj}(\Omega)\mathscr{E}_\theta^{Aj}(\Omega)\, d\Omega\right]^*} + \overline{\left[\int_\Omega G_\phi^{Ai}(\Omega)\mathscr{E}_\phi^{Ai}(\Omega)\, d\Omega\right]\left[\int_\Omega G_\phi^{Aj}(\Omega)\mathscr{E}_\phi^{Aj}(\Omega)\, d\Omega\right]^*} \\
&+ \overline{\left[\int_\Omega G_\theta^{Ai}(\Omega)\mathscr{E}_\theta^{Ai}(\Omega)\, d\Omega\right]\left[\int_\Omega G_\phi^{Aj}(\Omega)\mathscr{E}_\phi^{Aj}(\Omega)\, d\Omega\right]^*} + \overline{\left[\int_\Omega G_\phi^{Ai}(\Omega)\mathscr{E}_\phi^{Ai}(\Omega)\, d\Omega\right]\left[\int_\Omega G_\theta^{Aj}(\Omega)\mathscr{E}_\theta^{Aj}(\Omega)\, d\Omega\right]^*} \qquad (13)
\end{aligned}
$$

3.1 Derivation of $\overline{v_i \cdot v_j}^*$

Using (1) and (2) we get $\mathbb{E}[v_i \cdot v_j^*]$, which is given in (13) at the top of the next page. Let us name the four terms of (21) as E_1, E_2, E_3 and E_4 respectively. Using *Assumption* $-$ *III* we can write

$$
\begin{aligned}
E_1 &= \int_{\Omega_j}\int_{\Omega_i} G_\theta^{Ai}(\Omega_i)G_\theta^{Aj^*}(\Omega_j)\overline{[\mathscr{E}_\theta^{Ai}(\Omega_i)\mathscr{E}_\theta^{Aj^*}(\Omega_j)]}\, d\Omega_i d\Omega_j \\
&= \iint_{\theta_j,\phi_j}[\iint_{\theta_i,\phi_i} G_\theta^{Ai}(\theta_i,\phi_i)G_\theta^{Aj^*}(\theta_j,\phi_j)\sin\theta_i\sin\theta_j\cdot \\
&\overline{[\mathscr{E}_\theta^{Ai}(\theta_i,\phi_i)\mathscr{E}_\theta^{Aj^*}(\theta_j,\phi_j)]}\ \delta(\theta_i-\theta_j)\delta(\phi_i-\phi_j)d\theta_i d\phi_i]d\theta_j d\phi_j
\end{aligned} \qquad (14)
$$

Therefore,

$$E_1 = \int_\Omega G_\theta^{Ai}(\Omega)G_\theta^{Aj^*}(\Omega)\ \mu_{\theta\theta}\ d\Omega, \qquad (15a)$$

Similarly,

$$E_2 = \int_\Omega G_\phi^{Ai}(\Omega)G_\phi^{Aj^*}(\Omega)\ \mu_{\phi\phi}\ d\Omega, \qquad (15b)$$

$$E_3 = \int_\Omega G_\theta^{Ai}(\Omega)G_\phi^{Aj^*}(\Omega)\ \mu_{\theta\phi}\ d\Omega, \qquad (15c)$$

$$E_4 = \int_\Omega G_\phi^{Ai}(\Omega)G_\theta^{Aj^*}(\Omega)\ \mu_{\phi\theta}\ d\Omega, \qquad (15d)$$

where $\mu_{\theta\theta} = \overline{[\mathscr{E}_\theta^{Ai}\mathscr{E}_\theta^{Aj}]}$, $\mu_{\phi\phi} = \overline{[\mathscr{E}_\phi^{Ai}\mathscr{E}_\phi^{Aj}]}$, $\mu_{\theta\phi} = \overline{[\mathscr{E}_\theta^{Ai}\mathscr{E}_\phi^{Aj}]}$, $\mu_{\phi\theta} = \overline{[\mathscr{E}_\phi^{Ai}\mathscr{E}_\theta^{Aj}]}$. Using (5) we get

$$
\begin{aligned}
\mu_{\theta\theta} =& \overline{[\mathscr{E}_\theta^{Ai}\mathscr{E}_\theta^{Aj}]} \\
=& \cos^2\omega_c t\cdot\overline{A_\theta^2(\Omega)}\cdot\overline{\cos(\omega_D t+\psi_{d_i}+\psi_\theta(\Omega))} \\
& \overline{\cos(\omega_D t+\psi_{d_j}+\psi_\theta(\Omega))}
\end{aligned} \qquad (16)
$$

Using basic trigonometry and probability theory we get

$$\mu_{\theta\theta} = \overline{P_\theta(\Omega)}\cdot\cos(\psi_{d_i}-\psi_{d_j})\cos^2\omega_c t, \qquad (17)$$

where $P_\theta(\Omega) = \frac{1}{2}A_\theta^2(\Omega)$ is the power carried by the $\theta-$polarization of the incoming wave.
Similarly,

$$\mu_{\phi\phi} = \overline{P_\phi(\Omega)}\cos(\psi_{d_i}-\psi_{d_j})\cos^2\omega_c t, \qquad (18)$$

where $P_\phi(\Omega) = \frac{1}{2}A_\phi^2(\Omega)$ is the power carried by the $\phi-$polarization of the incoming wave.

For cross-polarized components we use *Assumption II* (9) to get

$$\mu_{\theta\phi} = \mu_{\phi\theta} = 0 \qquad (19)$$

Therefore,

$$
\begin{aligned}
\overline{v_i \cdot v_j{}^*} = &\left[\int_\Omega G_\theta^{Ai}(\Omega)G_\theta^{Aj^*}(\Omega)\ \overline{P_\theta(\Omega)}\cos(\psi_{d_i})\ d\Omega\right. \\
&\left.\int_\Omega G_\phi^{Ai}(\Omega)G_\phi^{Aj^*}(\Omega)\ \overline{P_\phi(\Omega)}\cos(\psi_{d_j})\ d\Omega\right] cos^2\omega_c t
\end{aligned} \qquad (20)
$$

3.2 Derivation of $\overline{|v_i|}^2$ and $\overline{|v_j|}^2$

Using (1) we get (21). Let us name the four terms of (27) as E_5, E_6, E_7 and E_8 respectively. Using *Assumption* $-$ *III* we can write

$$E_5 = \int_{\Omega_j}\int_{\Omega_i} G_\theta^{Ai}(\Omega_i)G_\theta^{Ai^*}(\Omega_j)\overline{[\mathscr{E}_\theta^{Ai}(\Omega_i)\mathscr{E}_\theta^{Ai^*}(\Omega_j)]}\ d\Omega_i d\Omega_j \qquad (22)$$

Therefore,

$$E_5 = \int_\Omega \left|G_\theta^{Ai}(\Omega)\right|^2\overline{P_\theta(\Omega)}\cos^2\omega_c t\ \ d\Omega$$

Similarly,

$$E_6 = \int_\Omega \left|G_\phi^{Ai}(\Omega)\right|^2\overline{P_\phi(\Omega)}\cos^2\omega_c t\ \ d\Omega, \qquad (23a)$$

And $E_7 = E_8 = 0$ due to *Assumption* $-$ *II*. Therefore,

$$
\begin{aligned}
\overline{|(v_{i,j})|}^2 = &\int_\Omega \left|G_\theta^{Ai,j}(\Omega)\right|^2\ \overline{P_\theta(\Omega)}\cos(\psi_{d_{i,j}}).\cos^2\omega_c t\ d\Omega \\
+&\int_\Omega \left|G_\phi^{Ai,j}(\Omega)\right|^2\ \overline{P_\phi(\Omega)}\cos(\psi_{d_{i,j}}).\cos^2\omega_c t\ d\Omega
\end{aligned} \qquad (24)
$$

Therefore, correlation between v_i and v_j is given by $\rho_{v_i v_j}$ as shown in (25) in the next page.

4 Derivation of Correlation Coefficients for Omnidirectional Antennas with Uniform Exposure in the Azimuthal Plane

For omnidirectional antennas $G_\theta^{Ai}(\theta,\phi) = G_\theta^{Aj}(\theta,\phi) = G_\theta$ and $G_\phi^{Ai}(\theta,\phi) = G_\phi^{Ai}(\theta,\phi) = G_\phi$. Similarly, for uniform exposure $A_\theta(\theta,\phi) = A_\theta$ and $A_\phi(\theta,\phi) = A_\phi$. In the Clark's model, incident waves comes uniformly in the azimuthal plane. Therefore, from (27) we get

$$
\begin{aligned}
\left[\overline{v_i v_j{}^*}\right]_{Omni}^{Clark} &= \frac{1}{2}\mathscr{A}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\cos(\psi_d)\sin\ \delta(\theta-\pi/2)d\theta d\phi \\
&= \frac{1}{2}\mathscr{A}\int_{\phi=0}^{2\pi}\cos(kd\cos\phi)\ d\phi \\
&= \pi\mathscr{A}(-1)^n\int_{\phi=0}^{\pi/2}\cos(kd\cos\phi)\cos(2n\phi)\ d\phi\ [\text{for } n=0]
\end{aligned}
$$

Therefore, $\left[\overline{v_1 v_2{}^*}\right]_{Omni}^{Clark} = \pi\mathscr{A}J_o(kd), \qquad (26)$

$$
\begin{aligned}
\overline{|(v_i)|^2} =& \overline{[\int_\Omega \vec{G}_{Ai}(\Omega)\cdot\vec{\mathscr{E}}_{Ai}(\Omega)\, d\Omega]\, [\int_\Omega \vec{G}_{Ai}(\Omega)\cdot\vec{\mathscr{E}}_{Ai}(\Omega)\, d\Omega]^*} \\
=& \overline{[\int_\Omega G_\theta^{Ai}(\Omega)\mathscr{E}_\theta^{Ai}(\Omega)\, d\Omega]\, [\int_\Omega G_\theta^{Ai}(\Omega)\mathscr{E}_\theta^{Ai}(\Omega)\, d\Omega]^*} + \overline{[\int_\Omega G_\phi^{Ai}(\Omega)\mathscr{E}_\phi^{Ai}(\Omega)\, d\Omega]\, [\int_\Omega G_\phi^{Ai}(\Omega)\mathscr{E}_\phi^{Ai}(\Omega)\, d\Omega]^*} \\
&+ \overline{[\int_\Omega G_\theta^{Ai}(\Omega)\mathscr{E}_\theta^{Ai}(\Omega)\, d\Omega]\, [\int_\Omega G_\phi^{Ai}(\Omega)\mathscr{E}_\phi^{Ai}(\Omega)\, d\Omega]^*} + \overline{[\int_\Omega G_\phi^{Ai}(\Omega)\mathscr{E}_\phi^{Ai}(\Omega)\, d\Omega]\, [\int_\Omega G_\theta^{Ai}(\Omega)\mathscr{E}_\theta^{Ai}(\Omega)\, d\Omega]^*}
\end{aligned} \tag{21}
$$

$$
\begin{aligned}
\rho_{v_i v_j} =& \left[\int_\Omega G_\theta^{Ai}(\Omega) G_\theta^{Aj^*}(\Omega)\, \overline{P_\theta(\Omega)}\cos(\psi_{d_i}-\psi_{d_j})\, d\Omega + \int_\Omega G_\phi^{Ai}(\Omega) G_\phi^{Aj^*}(\Omega)\, \overline{P_\phi(\Omega)}\cos(\psi_{d_i}-\psi_{d_j})\, d\Omega\right] \\
&\cdot \left[\left\{\int_\Omega \left|G_\theta^{Ai}(\Omega)\right|^2 \overline{P_\theta(\Omega)}\cos(\psi_{d_i})\, d\Omega + \int_\Omega \left|G_\phi^{Ai}(\Omega)\right|^2 \overline{P_\phi(\Omega)}\cos(\psi_{d_i})\, d\Omega\right\}\right. \\
&\left.\left\{\int_\Omega \left|G_\theta^{Aj}(\Omega)\right|^2 \overline{P_\theta(\Omega)^2}\cos(\psi_{d_j})\, d\Omega \int_\Omega \left|G_\phi^{Aj}(\Omega)\right|^2 \overline{P_\phi(\Omega)}\cos(\psi_{d_j})\, d\Omega\right\}\right]^{-\frac{1}{2}}
\end{aligned} \tag{25}
$$

where, J_o is the zero-th order Bessel's function of first kind and $\mathscr{A} = \left(|G_\theta|^2 \overline{P_\theta} + |G_\phi|^2 \overline{P_\phi}\right)$. Substituting $d = 0$ in (13) we get

$$
\left[\overline{v_1 v_1^*}\right]_{Omni}^{Clark} = \pi\mathscr{A} \quad \text{and} \quad \left[\overline{v_2 v_2^*}\right]_{Omni}^{Clark} = \pi\mathscr{A}. \tag{27}
$$

Therefore, from (16)

$$
\rho_{v_1 v_2}^{Clark} = J_o(kd). \tag{28}
$$

5 Conclusion

A correlation coefficient has been computed considering the impinged electric field as a random process and the final result has been verified against the special case of Clark's model for which we know the closed form expression of correlation coefficient. Notably, the expression of the correlation coefficient, obtained in this paper can be computed using the concept of cross-correlation Green's function (CCGF) [9] without determining the farfield radiation pattern of the antennas. However, it is worth mentioning that indoor channels have depolarization effects and shows that $Assumption-II$ is not valid inside a common factory hall [10]. In such cases, an antenna-statistics combined channel modeling becomes challenging.

6 Acknowledgments

This work is supported by Young Faculty Research Fellowship (YFRF) reference no: DIC/MUM/GA/10(37)D of MeitY, Govt. of India.

Further, the authors are thankful to Mr. Abhishek Kumar, research scholar of the department of E.C.E, NIT Patna for his support in this work.

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