



## A Statistical Estimation of 5G Massive MIMO's Exposure using Stochastic Geometry

Maarouf Al Hajj<sup>(1)</sup>, Shanshan Wang<sup>(1)</sup>, Philippe de Doncker<sup>(2)</sup>, Claude Oestges<sup>(3)</sup>, and Joe Wiant<sup>(1)</sup>

(1) Télécom Paris, Institut Polytechnique de Paris, Palaiseau, France

(2) Université libre de Bruxelles, Brussels, Belgium

(3) Université catholique de Louvain, Brussels, Belgium

### Abstract

This paper aims to estimate the exposure in 5G massive MIMO networks using a stochastic geometric approach. The paper investigates the effect of beamforming, and the effect of multi-user massive MIMO on the exposure of a 5G network. The massive MIMO antenna pattern distribution is obtained by fitting the radiation pattern obtained by running a large amount of channel simulations on NYUSIM. The distribution is then implemented into an analytical framework based on stochastic geometry, so we can obtain a close form expression of the downlink exposure.

### 1 Introduction

5G systems are being designed to be extremely flexible in terms of both the antennas used, the resource allocation, and the cellular system architecture. New Radio will have major differences from previous technologies in handling data transmission and reception. And while the exact architecture and models are still not yet complete, and each constructor/operator can develop vastly different technologies of antennas and RAN architecture due to the huge flexibility of the 5G technology, the characterization of the emission can be hard, and it may depend on lots of variables[1].

The exposure of 5G antennas proved to be a big problem for regulatory bodies since the configuration and implementation can vary largely between networks, thus making isolated antenna measurements more difficult to use as prediction. And since massive MIMO antennas are not yet deployed, simulating the behavior of the 5G network becomes the sole way to model the exposure.

Stochastic geometry studies the random spatial patterns, such as point processes which are often considered the main subject of stochastic geometry[2], and it can be used in wireless and cellular networks due to the uncertainty in the deployment of wireless nodes, especially in 5G networks.

The antenna pattern is produced using the NYUSIM channel simulator[3], which is based on measurements in mmWave in the city of New York. The simulator spatially divides the paths of transmission into lobes depending on the angle-of-departures of each path.

### 2 System Description

We consider a 5G cellular network where the base stations are distributed in a 2D plane following a Poisson point process  $\Phi_{BS}$  with intensity  $\lambda$ . The users are assumed to be uniformly distributed in the cell. The system is assumed to be in full buffer mode, which is an acceptable conservative assumption for exposure estimation, and users are being served over the whole transmission period. Therefore, the power allocated to each user is the same due to the lack of downlink power control.

As a simplistic approach, we assume a free-space path loss model,  $L = r^{-\alpha}$ . Where,  $\alpha$  is the path loss exponent and is assumed as  $\alpha = 3$  and  $r$  is the distance between the user and the BS. We also assume that there is no shadowing, and no background noise. The antenna is simulated in lign-of-sight mode to insure maximum directed beamforming gain, which will produce the maximum exposure.

### 3 Distribution of Antenna Pattern

We run a large number of channel simulations using the NYUSIM simulator with the characteristics summarized in Table 1.

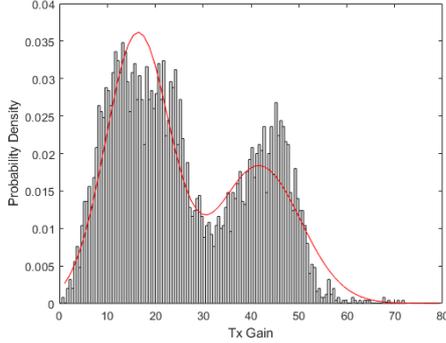
**Table 1.** List of channel parameters used in NYUSIM

Notation	Value	Meaning
$f$	28 GHz	Carrier Frequency
$N_{TX}$	64	Number of transmit antenna elements
Scenario	UMi	BS deployment scenario
Environment	LOS	Link type between the Tx and Rx
Antenna spacing	0.5	Antenna element spacing per wavelength
Polarization	Co-Pol	Polarization of the Tx

A generalized gain distribution can also be determined from the simulations Figure 1 and for this example, the gain has been fitted into a multimodal normal distribution where the pdf can be written as in (1)

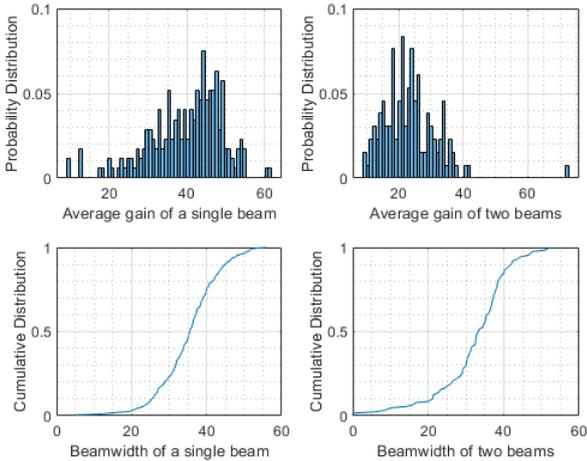
$$y = p \frac{1}{\sigma_1 \sqrt{2\pi}} e^{-0.5 \left(\frac{x-\mu_1}{\sigma_1}\right)^2} + (1 - p) \frac{1}{\sigma_2 \sqrt{2\pi}} e^{-0.5 \left(\frac{x-\mu_2}{\sigma_2}\right)^2} \quad (1)$$

Where  $p, \sigma_1, \sigma_2, \mu_1, \mu_2$  are the fitted parameters and have the respective values [0.6, 16.35, 41.67, 6.68, 8.63]. The two observed peaks in the gain distribution can be attributed to the cases where multiple beams are formed (Multi-user MIMO) versus the single user case.



**Figure 1.** Fitting of the gain distribution obtained from NYUSIM

To simplify the analytical formulation and to better allow the comparison of cases, we fit separate distribution for each case of single or multi-user usage and we approximate the distribution to fit a scaled uniform distribution with the peak as the average gain in the beam's angle, and the width as the average HPBW of the beam. The gain and HPBW distributions for single user MIMO and 2-beam configuration are presented in Figure 2.



**Figure 2.** Gain and HPBW distributions for one and two-beam cases

The gain can then be expressed by as the product of the average gain per path and half-power beamwidth as expressed in (2).

$$G = G_{avg} \times HPBW_{avg} \quad (2)$$

## 4 Analytical Formulation Using Stochastic Geometry

We assume that the base stations are distributed following a Poisson point process  $\Phi_{BS}$ , a simple path loss model  $r^{-\alpha}$  where  $\alpha$  is the path loss exponent, and  $r$  is the distance from the BS to the user.

The exposure can be calculated by summing the received power at a certain point in space from the serving base station and from all other interferers.

Where  $\phi(t) = E\{\exp(jtX)\}$  is the characteristic function of the exposure. It can also be written as in (3).

$$\begin{aligned} \phi(t) &= E \left\{ \exp \left( jt \sum_{r_i \in \Phi_{BS}} P_{tx} |h|^2 G_{Tx}(\theta) r^{-\alpha} \right) \right\} \\ &= E_r \left\{ E_{|h|^2} \left\{ \prod_{x \in \Phi_{BS}} \int \exp(j t P_{tx} |h|^2 G(\theta) r^{-\alpha} d\theta) \right\} \right\} \end{aligned} \quad (3)$$

The close-form equation can then be expressed by (4):

$$\begin{aligned} \phi(t) &= \exp \left( -2\pi\lambda BW G P_G \frac{jt P_{tx} G \bar{\gamma} m^{2-\alpha}}{\alpha - 1} {}_2F_1 \left( 1, \frac{\alpha - 2}{\alpha}, 2 \right. \right. \\ &\quad \left. \left. - \frac{2}{\alpha}, jt P_{tx} G BW P_G \bar{\gamma} m^{-\alpha} \right) \right) \end{aligned} \quad (4)$$

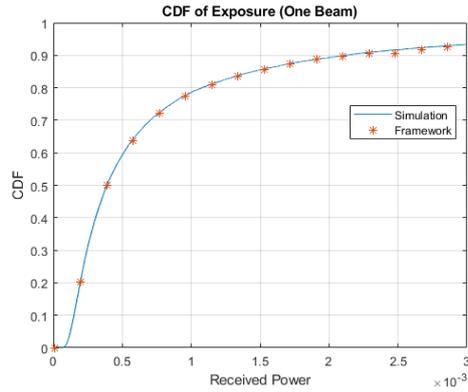
Where,  $\alpha$  is the path loss exponent,  $BW$  is the beamwidth,  $\lambda$  is the PPP intensity,  $G$  is the average gain,  $P_{tx}$  the transmit power, and  $m$  the minimum separation distance between the BS and the user to avoid infinite values at the BS location.

## 5 Results

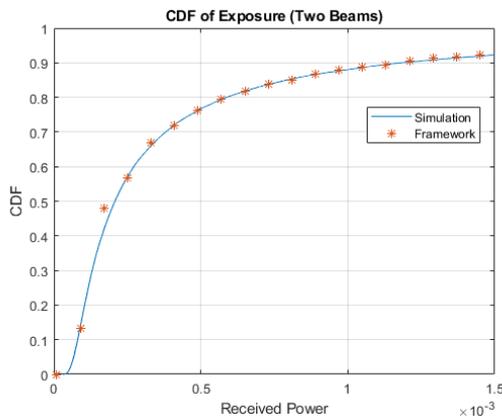
The CDF of the exposure can be determined using the Gil-Peleaz theorem [4] represented by (5).

$$F(x) = \frac{1}{2} - \frac{1}{\pi} \int_0^\infty \frac{Im[e^{-jtx} \phi(t)]}{t} dt \quad (5)$$

The CDF produced from the stochastic-geometric framework is then compared with a Monte-Carlo simulation for both the case of a single emitted beam, and the case of two emitted beams, as shown in Figure 3 and Figure 4.



**Figure 3.** CDF of the exposure (Simulation vs Framework) for one beam case



**Figure 4.** CDF of the exposure (Simulation vs Framework) for two beam case

These two figures clearly show the difference between operating the antenna with a single beam and multiple beams. The difference in the exposure is attributed to the division of the antenna elements into two parts to form two different beams, which reduces the gain of the lobe serving a certain user.

## 6 Conclusion

In this paper, we have presented an accurate framework to analytically estimate the exposure in a massive MIMO network using stochastic geometry.

We have also made the comparison between two simple implementation cases of a massive MIMO antenna which can have significant effect on the exposure.

Future work will focus on producing more accurate and representative models in order to better study the variability of massive MIMO networks, and the effects the changes of the characteristics and the network usage on the exposure in addition to the spatio-temporal behavior of antenna.

## 7 References

- [1] P. Baracca, A. Weber, T. Wild, and C. Grangeat, "A Statistical Approach for RF Exposure Compliance Boundary Assessment in Massive MIMO Systems," 2018.
- [2] M. Haenggi, *Stochastic geometry for wireless networks*, vol. 9781107014. 2009.
- [3] S. Sun, G. R. Maccartney, and T. S. Rappaport, "A novel millimeter-wave channel simulator and applications for 5G wireless communications," *IEEE Int. Conf. Commun.*, vol. 10, no. 1, pp. 1–7, 2017.
- [4] J. GIL-PELAEZ, "Note on the inversion theorem," *Biometrika*, vol. 38, no. 3–4, pp. 481–482, 1951.