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Statistical model for MIMO propagation channel in cavities and random media

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Abstract

We present a physics-based channel matrix that establishes the connection between electromagnetism and information theory. The approach is based on impedance matrices and related to the scattering matrix formulation. Cavities and random media support a wave chaotic behaviour that yields an exponential proliferation of multiple paths due to the presence of multiple reflection or scattering. It is shown that fading can be included using the random coupling model (RCM). This model leads to theoretical and numerical predictions of the probability density function of the mutual information in large and complex electromagnetic environments. The effect of losses and modal density on channel capacity can thus be explored and channel hardening bounds quantified.

1 Introduction

Indoor wireless communications experience severe fading conditions due to multi-path propagation and interference. Multiple-input multiple-output (MIMO) systems have been proposed to develop multiple orthogonal channels from multiple interfering propagation paths. Thanks to MIMO technologies, the channel capacity of wireless links can be increased resulting in high digital data rates. Although the channel status can be probed regularly (typically in the ms scale) through pilot tone techniques, the design of MIMO systems, e.g., in terms of number of radiating elements in the antenna array, benefits from assuming that the information transfer takes place through a rich fading electromagnetic environment (EME). This is motivated by the generic EME complexity, causing a high sensitivity to perturbations and a strong dependence on frequency, which suggests that the information theoretic channel matrix shows a random/stochastic behaviour [1]. Therefore, it is important to link the channel matrix randomness to the physics of propagation underpinning the EME propagation. Statistical approaches to create a physical model of the channel matrix are very appealing as they lead to asymptotic results for large dimensional systems predicted by random matrix theory (RMT) [2]. Formulations based on ergodicity assumptions and universality, i.e. RMT, can be tested in the laboratory through electromagnetic reverberation chambers and planar microwave billiards [3]. These can be used to emulate the multi-path fading in a wireless channel through wall reflection [4]. To account for non-universal features such as direct lines of sight, more sophisticated technologies can be implemented. This allows to exploit the chaotic background and perform the transmission by coupling through individual eigenstates that experience higher signal-to-noise ratio [5]. An important question arises on what are the performances of MIMO wireless systems when they operate inside confined environments which support a large number of multi-paths while having moderate losses due to absorption of the electromagnetic energy. Information theoretic measures are needed to establish the maximum data rate achievable within cavities and random media. Starting from an information theoretic end-to-end voltage channel matrix given in the impedance matrix formalism, we include fading through the random coupling model (RCM) [6], thus making an analogy between the electromagnetic environment and a chaotic cavity coupled to antennas. Using the generalised expression of mutual information for multi-antenna systems, we derive the probability density function (PDF) of the mutual information that are instrumental to evaluate and maximise the channel capacity of real MIMO systems operating in complex environments and media. This work aims at contributing to establish a solid link between the electromagnetics and the wireless communication communities.

2 Random Coupling Model

The impedance matrix of an antenna array radiating inside a resonant environment of large dimension and complex geometry can be predicted through a well-established statistical model, the RCM. In this model, cavity impedance Z and radiation (free-space) impedance matrices Z^{rad} are connected by a clear relation, which is

$$Z_{TT} = i\Im\left\{Z_{TT}^{rad}\right\} + \Re\left\{Z_{TT}^{rad}\right\}^{1/2} \xi_\alpha \Re\left\{Z_{TT}^{rad}\right\}^{1/2}, \quad (1)$$

$$Z_{RR} = i\Im\left\{Z_{RR}^{rad}\right\} + \Re\left\{Z_{RR}^{rad}\right\}^{1/2} \xi_\alpha \Re\left\{Z_{RR}^{rad}\right\}^{1/2}, \quad (2)$$

for the self-impedance, and

$$Z_{TR} = \Re\left\{Z_{TT}^{rad}\right\}^{1/2} \xi_\alpha \Re\left\{Z_{RR}^{rad}\right\}^{1/2}, \quad (3)$$

$$Z_{RT} = \Re\left\{Z_{RR}^{rad}\right\}^{1/2} \xi_\alpha \Re\left\{Z_{TT}^{rad}\right\}^{1/2}, \quad (4)$$

for the trans-impedance between transmitting and receiving antenna array, respectively. The random matrix ξ_α has the physical meaning of a normalised impedance, whose elements $\xi_{\alpha,pp'}$ are given by the sum of resonant terms, viz.,

$$\xi_{\alpha,pp'} = -\frac{i}{\pi}\sum_n \frac{\left(\Phi_n \Phi_n^T\right)_{pp'}}{\mathscr{E}_0 - \mathscr{E}_n + i\alpha}, \quad (5)$$

where $\mathscr{E}_{(\cdot)} = k_{(\cdot)}^2/\Delta k^2$, Δk^2 is the mean mode spacing, k_n are cavity mode wave-numbers, and α is an (average) loss parameter defined as

$$\alpha = \frac{k_0^2}{Q\Delta k^2}, \quad (6)$$

where Q is the (average) cavity quality factor, and Δk^2 is the (average) mode-to-mode spacing. In (5), Φ_n is a vector of uncorrelated, zero mean, unit width Gaussian random variables, and $\mathscr{E}_n^2$ are the eigenvalues of a matrix selected from the Gaussian orthogonal ensemble (GOE). The matrix (5) is the only statistical part in (1) - (4), capturing the chaotic multi-path scattering throughout the cavity. This is a universal random matrix ξ_α that incorporates homogeneous cavity losses. The form in (5) is valid also for the admittance matrix of apertures radiating inside a cavity [7].

3 Channel matrix and mutual information

The physics of the wireless link between transmitting and receiving antenna arrays is captured by the impedance matrix Z, which brings together self- and mutual-impedance in a compact form

$$Z = \begin{pmatrix} Z_{TT} & Z_{TR} \\ Z_{RT} & Z_{RR} \end{pmatrix}. \quad (7)$$

From this, an information theoretic channel matrix $\mathbf{H}$ can be obtained. It contains the essential physics of a loss-less multi-antenna MIMO system under noise matching at the receiver and power matching at the transmitter. Furthermore it models isotropic background noise being received by the antennas [8, Eq. (105)] in addition to amplifier noise. The expression for the channel matrix reads

$$\mathbf{H} = \frac{e^{-j\phi}}{R_r}\mathbf{C}_T^{-1/2} Z_{TR} \mathbf{C}_R^{-1/2}, \quad (8)$$

with antenna array correlation matrices

$$\mathbf{C}_T = \frac{\Re\{Z_{TT}\}}{R_r}, \qquad \mathbf{C}_R = \frac{\Re\{Z_{RR}\}}{R_r}, \quad (9)$$

which involve elements of the open port impedance matrix (7). In (8), the phase factor is defined as $\phi = \text{angle}(R + Z_{opt})$, R is the generator impedance, Z_{opt} is the impedance seen at the receiver when an impedance matching network is employed to decouple the antenna in the receiving antenna array. The antennas are assumed to be identical and their radiation resistance is $R_r = \Re\{[Z_{RR}]_{ii}\}$. It is worth noticing that the phase factor ϕ in (8) does not influence the Gramian $\mathbf{HH}^\dagger$ matrix nor the mutual information.

Upon application of the RCM, the channel matrix becomes a random matrix whose elements have statistical moments depending on the average environment loss parameter α. Therefore, the channel matrix of a MIMO system operating in a cavity is readily obtained by application of (1) - (4) in (8). If we assume $\alpha > 1$, i.e. an overmoded environment where losses are moderate and many resonances contribute to the coherence bandwidth of the environment, i.e., $\alpha > 1$, we can approximate the array self-impedances in (9) as

$$Z_{TT} \approx Z_{TT}^{rad}, \qquad Z_{RR} \approx Z_{RR}^{rad}. \quad (10)$$

Interestingly, in the moderate loss regime, the mathematical structure of both RCM and array correlation matrices leads to a universal form of the Gramian matrix that is independent on the antenna correlation matrices. More precisely, we have

$$\mathbf{HH}^\dagger \approx \xi_\alpha \xi_\alpha^\dagger \quad (11)$$

if the decoupling and matching networks (DMNs) employed to optimise the MIMO system are expected to have a weak average effect on improving the channel capacity. In presence of arbitrary (low) losses, i.e. a highly resonant environment supporting a large number of paths, the full expression (8) should be used with (9) and (1)-(2). This is important to account for the coupling radiating elements within the antenna arrays as they correlate the wireless channels at both transmitter and receiver [9]. However, correlation effects do not play an important role in the moderate loss regime, as suggested by (11). Given that ξ_α is a $N_T \times N_R$ matrix with complex valued entries whose columns are N_T-variate normal distributions

$$\Re\{\xi_{\alpha,(c)}\}, \Im\{\xi_{\alpha,(c)}\} \sim \mathscr{N}_{N_T}\left(\mathbf{0}, \frac{1}{\sqrt{2\pi\alpha}}\mathbf{I}\right), \quad (12)$$

the statistics of the singular values of the channel matrix can be derived observing that (11) has a complex Wishart matrix with probability distribution [2]

$$\mathbf{HH}^\dagger \sim \mathscr{CW}_{N_T}\left(N_R, \sqrt{\frac{2}{\pi\alpha}}\mathbf{I}\right), \quad (13)$$

where $\mathbf{I}$ indicates the identity matrix of appropriate dimension. Importantly, in moderate loss regime, the impedances in (8) can be replaced by their scattering matrix expressions, using the effective Hamiltonian model [10] or other progressive scattering theories [11] applicaple for chaotic cavities and media. Since the impedance matrix has an isomorphism with the Wigner K-matrix in nuclear reaction theory, recent achievements in this area can be inherited to give the PDF expression of the off-diagonal elements in closed form [12]. The eigenvalue distribution of (13) is known in closed form and predicted by the Marchenko-Pastur (MP) law [1]. In the context of (11) and $\alpha > 1$ the empirical distribution of the eigenvalues converges almost surely to

$$f_\beta(\lambda) = \left(1 - \frac{\alpha}{\beta}\right)^+ \delta(\lambda) + \frac{\sqrt{(\lambda - a)^+}\sqrt{(b - \lambda)^+}}{2\pi\lambda\left(\frac{\beta}{\alpha}\right)}, \tag{14}$$

with $\beta = \frac{N_T}{N_R}$, $(z)^+ = \max(0,z)$, $a = (1 - \sqrt{\frac{\beta}{\alpha}})^2$, and $b = (1 + \sqrt{\frac{\beta}{\alpha}})^2$. Therefore, from (14) we can estimate the diversity gain of the MIMO wireless link operating in chaotic cavities with moderate losses

$$\langle \lambda \rangle \sim \frac{\beta}{\alpha}, \qquad \alpha > 1. \tag{15}$$

Furthermore, the mutual information follows from the Gramian matrix. The mutual information was extended and maximised for multi-antenna systems to obtain a Shannon channel capacity [13]. Under the hypothesis of additive Gaussian noise, knowledge of the channel transfer matrix at the receiver, uncorrelated noise and uncorrelated input signals (consistent with the optimised channel matrix (8)), the mutual information reads

$$C = \log_2 \left(\det \left(\mathbf{I} + \frac{\rho}{N_T} \mathbf{H}\mathbf{H}^\dagger \right) \right), \tag{16}$$

with signal-to-noise ratio ρ. The random Gram matrix makes C a random variable whose statistics can be calculated through (14), exploiting the properties of determinants and logarithms, viz.,

$$C = \sum_{n=1}^{N_C} \log_2 \left(1 + \frac{\rho}{N_T} \lambda_n \right), \tag{17}$$

where N_C is the number of non-zero eigenvalues of the channel matrix.

4 Numerical results

We use the Monte-Carlo method to generate the distribution functions of the Gram matrix (11) as well as of the mutual information (16). Details on how to produce statistical eigenvalue and eigenfunction distribution in (5) that are universal and capture the symmetries of the physical system at hand, e.g., time-reversal symmetry. We consider an antenna array of N_T transmitting elements in communication with an antenna array of N_R receiving elements through a wave chaotic cavity. This represents the end-to-end model of a generic MIMO system operating in presence of rich fading. We remind that the wave chaotic cavity has distributed losses that can be captured through the average parameter α that is defined in (6) and can be estimated by separate scattering measurements in real-life wireless environments. For two selected MIMO configurations with $N_T = 3$ and $N_R = 9$, as well as $N_T = 5$ and $N_R = 5$, both operating in a chaotic cavity with $\alpha = 5$, the probability distribution of the singular values of the Gram matrix are represented in

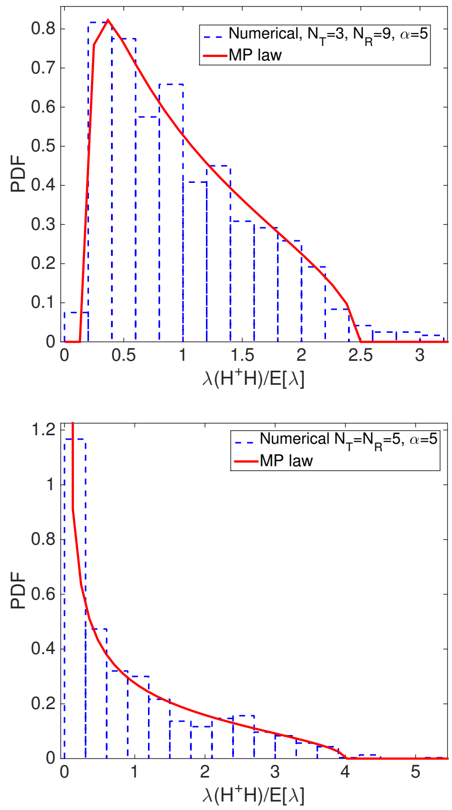

Figure 1. Probability density function of the normalised eigenvalues compared with the Marchenko-Pastur distribution, for $N_T = 3$, $N_R = 9$, $\alpha = 5.0$ (upper) and $N_T = 5$, $N_R = 5$, $\alpha = 5.0$ (lower). Note the different scale on the abscissa.

Fig. 1 (upper) for $N_T = 3$ and $N_R = 9$ and in Fig. 1 (lower) for $N_T = 5$ and $N_R = 5$. Note that the eigenvalues are normalised to the mean value for comparison with the theoretical prediction from (14). Furthermore, the numerical computation of the PDF of the channel capacity (16) for a 2×2 MIMO wireless link is performed and the results are reported in Fig. 2 for selected values of the loss factor. In particular, it is possible to appreciate the reduction of the average mutual information with the increase of the loss factor α. This is related to the ergodic channel capacity through maximisation of the input signal correlation function constrained to the total transmitted power. While the magnitude of fluctuation is reduced as the average losses increase, the coefficient of variation is not expected to do so. Therefore, the stability of the wireless link does not improve. This can be achieved by increasing the number of elements in the receiving antenna arrays, a phenomenon called channel hardening [14].

5 Conclusion

We have presented a model for the channel matrix of MIMO systems operating in chaotic environments and random media with multiple scattering. The model develops a physics-based, information theoretic channel matrix from

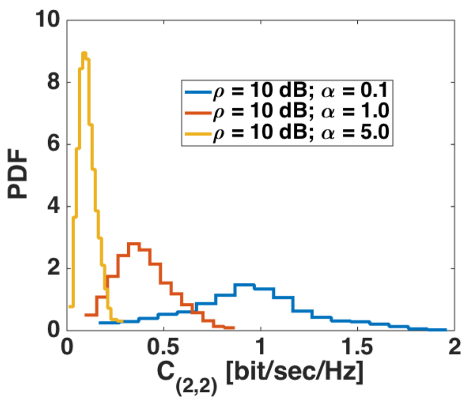

Figure 2. Probability density function of the mutual information, for $N_T = 2$, $N_R = 2$, and α increasing from low to high losses.

the impedance matrix of the antenna arrays. In environments with rich multi-path fading, the radiation resistance of the antenna becomes a random matrix on account of the uniform coupling with the cavity eigenmodes. Thanks to an established link between circuit and information theory, the normalised version of the random resistance matrix becomes the information theoretic channel matrix. The channel matrix is universal and depends only on the loss factor. The probability density function of the mutual information can be computed from the statistics of the Gramian matrix, which belongs to the complex Wishart ensemble. Numerical results of the eigenvalue and the mutual information distribution functions are shown for antenna arrays with a selected number of number of radiating elements. Results are useful to estimate the outage probability expected in resonant electromagnetic environments with distributed losses.

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