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Physical and Statistical Model for Dual-Wave Weather Radar Observations

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Abstract

The development of the method for dual-wave calibration (MDC) of weather dual-wave radar precipitation measurements is presented. The results of the study of the MDC algorithm characteristics obtained by the Monte Carlo statistical test method and the results of applying the MDC method for interpretation of the data from real dual-wave measurements by the AKSOPRI radar system installed in Moscow are under consideration.

1 Introduction

Today the weather radar coverage area almost completely covers the territory of industrially developed countries in the world and so allows to make continuous observations of atmospheric cloud in the interests of weather service, transport and other industries. One of the main goal of weather radar is precipitation measurements. It is generally assumed that the main factor limiting the accuracy of the radar method (RM-radar methods) for precipitation measurements is the impossibility of taking into account the natural variations in the rainfall microstructure (DSD - drop size distribution) for the classic single-wave RM. To overcome this problem, a transition to multi-parameter sensing methods is desired in particular polarization and dual-wave RM. The method for dual-wave calibration (MDC) of Russian weather dual-wave radar MRL-5 rainfall measurements was proposed in [1]. The method provides measurement of reflectivity (Z) under two wavelengths: attenuated in rainfall λ1 = 3 cm and non-attenuated ("almost") λ2 = 10 cm. The proposed method is used for interpretation of observation data obtained by the dual-wave the weather radar in Moscow. The presented report discusses the mathematical formulation of MDC.

2 Model development and calculation experiments

Physical and statistical model for dual-wave radar sensing of the cloud atmosphere was formulated by the authors. The model is based on the one-dimensional radar equation for distributed target [2] taking into account the attenuation of microwave radiation in rainfall. For closure of the model a "rain parameter diagram" was used [2], which made it possible for the first time to obtain self-consistent on rainfall microstructure model for dual-wave radar measurements. A feature of this formulation is the selection of the attenuation effect in the form of a dependence on a single parameter, the A_C coefficient, in the Z-R ratio in the approximation of the two-parameter function of rainfall particle distribution on size (RFD). In MDC, to take into account variations in the rainfall microstructure it is necessary to identify the A_C parameter by measurements of reflectivity profiles at two wavelengths and to make its further application in calculating of rainfall intensity on the Z-R ratio. The A_C parameter is estimated using the least square method [3]. The stability analysis revealed a strong dependence of the A_C coefficient on random fluctuations by radial profiles Z and ZAtt, by which the radiophysical measurements noise are simulated.

To study the performance characteristics of the MDC algorithm (offset value and evaluation dispersion, minimum required sample length) the statistical test method (Monte Carlo) was used in the Gaussian approximation of noise signal statistics. In the report, the scheme of computational experiments is considered and the results of studying for the performance characteristics of the MDC algorithm are presented.

Radar sensing is an azimuthal scan of the upper hemisphere at several elevation angles ϕ and obtaining in result of measurements the distribution of radar reflectivity Z (r, α, ϕ) in the form of a set of conical sections. Here r is the radial coordinate, α is the azimuth coordinate.

The radial distribution of reflectivity on the attenuated (ZAtt) and non- attenuated (Z) wavelengths at a distance r from the radar are related by the following way (1) according to the main radar equation [2]:

$$Z^{Att}(r) = Z(r) \cdot exp\left[-2\int_{r_0}^{r} k\, ds\right], \qquad (1)$$

here κ – is a specific coefficient of microwave radiation attenuation in rainfall, r0 is the coordinate of the leading cloud front.

Precipitation characteristics such as radar reflectivity Z (mm^6/m^3), intensity R (mm / h), attenuation coefficient of microwave radiation in precipitation K (dB/km) are the different order moments of the distribution function for precipitation particles on size N (D):

$$Z_e = \int_0^\infty D^6 N(D)\, dD, \quad R = \int_0^\infty D^3 N(D)\, \omega_t(D)\, dD, \quad K = \int_0^\infty N(D)\, \sigma_t(D)\, dD, \qquad (2)$$

Here, ω_t is the steady-state drop rate, σ_t is the attenuation cross section in precipitation, and D is the diameter of the drops.

The relationship between the parameters R, Z and K is looking for in the form of power dependencies.

$$Z = A\,R^{b},\;\; K = \alpha\,R^{\gamma} \qquad (3)$$

the coefficients {A, b, α, γ} of which are not independent, but are interconnected through the distribution function N (D), which describes the microstructure of precipitation in a pulsed radar volume.

To close the system of equations (1-3), we used the “rain parameter diagram” (RPD), which is based on the principle of the sufficiency of the measurements for any two characteristics for a complete description of a two-parameter RPD, the coefficients of which were calculated for several wavelengths λ and drop temperature t [2]. After some transformations, the system of equations (1-3) is reduced to

$$Z^{Att}(r) = Z(r) - \Theta(A)\int_{r_0}^{r} exp\big(\varphi \cdot Z(s)\big)\,ds, \qquad (4)$$

where ϕ and Θ are expressed in terms of the coefficients of relations (3) and the parameters of the RPD.

If we consider (4) as the equation for determining Z by ZAtt, then an analytical solution of this nonlinear integral equation with an exponential core was obtained by Hitschfeld and Bordan [4]. They also demonstrated that the problem formulated by this way “task of correction for the attenuation effect in precipitation” is incorrect: there is no uniformly continuous dependence for the solution from the input values, namely, from the value of the absolute calibration error ZAtt. In other words: correction is possible, firstly, in the absence of a systematic error in the ZAt data, and, secondly, with known coefficients in relations (3). Since these requirements are never fulfilled in practice, the described approach has purely theoretical significance.

To exclude stability problems, we proposed in the MDC to consider (4) as equation for estimating the parameter A in the Z-R relation (3) using two-wave measurements of Z and ZAtt by the AKSOPRI radar system as input data.

The AC parameter estimation (calibration) is executed by using the least squares method, by minimization of the residual functional between the calculated and measured radial reflectivity profiles at the wavelength attenuated in the precipitation. The calculation of the radial profile ZAtt (r) is carried out using the measured profile on non-attenuated wavelength Z (r) according to equation (4).

The advantage of MDC is the ability to make calibration at an arbitrary interval of radial profile. In this case, it is assumed that the ZAtt (r) data contains an unknown systematic errors, and an additional boundary condition ZAtt (r0) = Z (r0) is formulated. In this case, the obtained estimation of the parameter A_C will be weighted average over the interval [r0, r1].

Figure 1 shows the results of four calculations of the A_C coefficient with different noise levels during input data: 0.5, 1.0, 1.5 and 2.0 dB. The initial reflectivity profile was set in the form of a sin wave, and the initial value of the coefficient A0 = 200 corresponded to the classical Marshall – Palmer value [2]. Every point on the graph represents the result of calculating the A_C coefficient according to equation (5) with a random realization of the noise component of the measured signal. Along axis of ordinate the values of the coefficient A (left axis) are plotted, along the x-axis the sample length of calibration values are pointed.

σ=0,5 dB

σ=1,0 dB

σ=1,5 dB

σ=2,0 dB

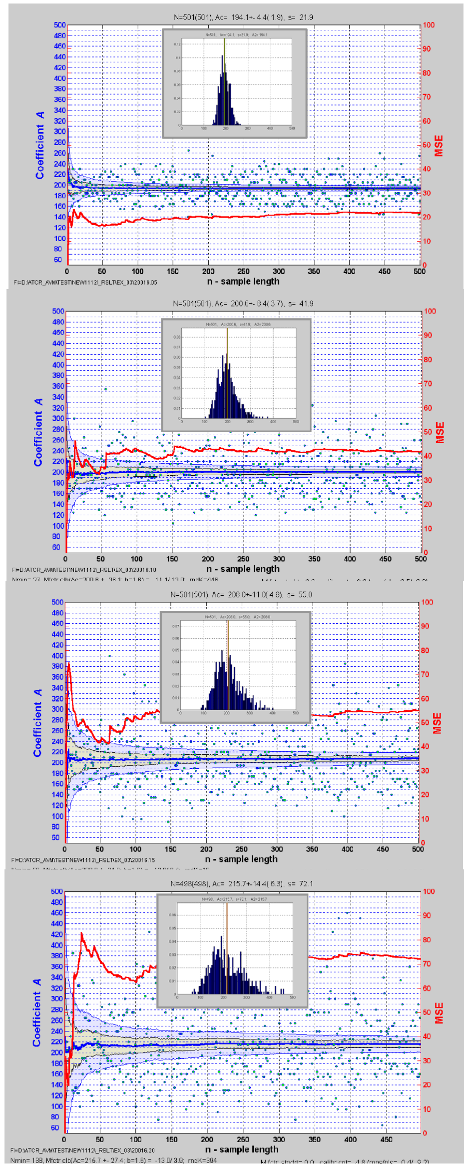

Figure 1. Diagramms of the calibration A_C values scattering due to random fluctuations of the profiles ZAtt (r) and Z (r) at various noise amplitudes σ = 0.5, 1, 1.5, and 2 dB.

The initial values of the coefficient A0 = 200. The model noise has a Gaussian spectrum with zero mean and a given dispersion σ2. The ordinate is the A_C coefficient, the abscissa is the sample length N. Blue shadow shows the profile of the sample mean, red line shows the sample variance (left scale and sample dispersion, right scale).

At the insert for every graph the frequency histogram of the A_C coefficient estimates is presented.
The solution of the variational problem with respect to the coefficient A_C has the following form:

$$A_C(r_0,r_1)=2\varepsilon_1\left\{\frac{(r_1-r_0)\int_{r_0}^{r_1}f^2(s)\,ds+\left(\int_{r_0}^{r_1}f(s)ds\right)^2}{(r-r_0)\int_{r_0}^{r_1}\left\{\left[Z(s)-Z^{Att}(s)\right]f(s)\right\}ds+\int_{r_0}^{r_1}\left[Z(s)-Z^{Att}(s)\right]ds\times\int_{r_0}^{r_1}f(s)ds}\right\}^{b_C/\varepsilon_3},$$

$$f(s)\equiv\int_{r_0}^{S}\exp\left[\xi\cdot Z(t)\right]dt, \qquad (5)$$

where the coefficients εi, ξ, bc are expressed through the known coefficients (3) and RPD.
Validation of the MDC algorithm was carried out on a combination of direct and inverse models for dual-wave radar sensing [5].

3 Conclusion

In the absence of noise ($\sigma = 0$), the calculation according to equation (5) gives the exact initial value $A_C = A_0$. As the noise amplitude increases, the scattering of the calibration A_C values increases. To obtain a stable estimate, averaging of several calibration A_C values obtained in independent measurements is required, for which the MDC uses calculations in several adjacent azimuthal directions.
The sample average $\langle A_C \rangle$ has the displacement relative to the initial value A_0, moreover, displacement value depends on the noise amplitude σ. To reduce the effect of displacement, special input data filtering procedures (Z, ZAtt) and averaging of calibration values $(A_C)i$ were applied.
As a result of computational experiments [6], it was found that the calibration accuracy of $\mid A_0 - A_C \mid$ depends on several factors: the amount of accumulated attenuation in the interval PIA $\equiv$ (Z-ZAtt) $\mid$ r1 – (Z-ZAtt) $\mid$ r0, noise amplitude σ, averaging sample length.
The study of the MDC algorithm characteristics obtained by the Monte Carlo statistical test method was made as well as the application of the MDC method for interpretation of the data from real dual-wave measurements by the AKSOPRI radar system installed in Moscow.

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