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Josephson Effect based Superconducting Electronics

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Abstract

Josephson parametric amplifiers (JPAs) operated in the coherent quantum regime exhibit ultimate sensitivity. We review physical principles, properties and applications of JPAs in quantum information processing.

1 Introduction

Josephson effect based devices allow generation, detection, mixing and parametric amplification of high frequency signals up into the THz region and exhibit high sensitivity, low energy consumption and small size. Nanotechnological fabrication techniques and the availability of novel materials give a strong impact on the development of novel Josephson-effect-based devices and systems [1]. According to the theory of Bardeen, Cooper and Schrieffer (BCS), superconductivity is a microscopic effect caused by a Bose-Einstein condensation of electrons into Cooper-pairs [2]. The superconducting ground state can be described by a coherent macroscopic matter wave function. The condensation of the electrons in a quasi-coherent state leads to special high-frequency properties and low electronic noise.

In 1962 B.D. Josephson presented the theory of superconductive tunneling through superconductor–isolator–superconductor (SIS) junctions based on the microscopic BCS theory [3,4]. A Josephson junction is an arrangement of two superconductors weakly coupled across a tunnel barrier with a thickness of a few nanometers. Josephson junctions are suitable for quantum information processing.

2 General Energy Relations

A voltage $v(t)$ applied to the Josephson junction determines the time variation of the quantum phase difference φ of the macroscopic matter waves describing the superconducting state. Current $i(t)$ and voltage $v(t)$ are related via

$$i(t) = I_J \sin\varphi(t), \quad \text{(1a)} \qquad v(t) = \frac{\hbar}{2e_0}\frac{\mathrm{d}\varphi(t)}{dt}, \quad \text{(1b)}$$

where $\hbar = h/2\pi$ and h is Planck's constant, e_0 is the magnitude of the electron charge, and I_J is the maximum Josephson current [1]. Applying a DC voltage V_0 yields an AC Josephson current with frequency $f_0 = 2e_0V_0/h = 483.6\ V_0$ (MHz/μV) and amplitude I_J. The energy $w_J(\varphi)$ stored in the Josephson junction is

$$w_J(\varphi) = \frac{\hbar I_J}{2e_0}[1-\cos\varphi] = \frac{\hbar I_J}{2e_0}[1-\cos(2\pi\Phi(t)/\Phi_0)]\,, \tag{2}$$

where we have introduced a magnetic flux Φ, defined via $d\Phi/dt = v$ and the flux quantum $\Phi_0 = h/2e_0 \approx 2.06783461\cdot 10^{-15}$ Vs. The Josephson junction acts as a nonlinear lossless inductor and can be applied for mixing and parametric amplification in the microwave region. The general energy relations for the Josephson junction [5,6] are similar to the *Manley-Rowe* equations [7], however, they exhibit an additional term for the DC power.

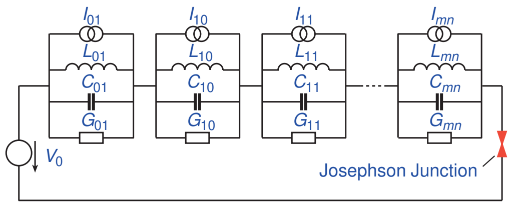

Figure 1. Frequency conversion with Josephson junctions.

In the circuit depicted in Fig. 1, power exchange with the Josephson junction is only possible at DC and at the combination frequencies mf_1+nf_2, where m and n are integers. Applying a voltage with a DC component V_0 and AC components with the frequencies f_1 and f_2 yields a Josephson current with frequency components $f_0+mf_1+nf_2$, where m,n are integers. For the case $f_0 = kf_1+lf_2$ the following general energy relations were derived in [6]:

$$\sum_{m=1}^{\infty}\sum_{n=-\infty}^{\infty}\frac{mP_{mn}}{mf_1+nf_2} = -\frac{kP_0}{kf_1+lf_2}\,, \tag{3a}$$

$$\sum_{n=-\infty}^{\infty}\sum_{n=1}^{\infty}\frac{mP_{mn}}{mf_1+nf_2} = -\frac{lP_0}{kf_1+lf_2}\,, \tag{3b}$$

where P_{mn} is the active power flowing into the Josephson junction at the combination frequency mf_1+nf_2, and P_0 is the DC power flowing into the Josephson junction. If the Josephson junction is connected only to two resonant circuits with frequencies f_1 and f_2 and the junction is DC voltage biased to generate a Josephson oscillation at $f_0 = f_1+f_2$, we obtain $P_1/f_1 = P_2/f_2 = -P_0/(f_1+f_2)$.

3 Josephson Parametric Amplifiers

The first experimental realization of an AC pumped Josephson parametric amplifier (ACPJPA) was reported in 1967 by H. Zimmer [8]. This ACJPA consisted of a thin film Josephson junction evaporated on a rutile resonator and was operated at 9646 MHz in the doubly degenerate mode.

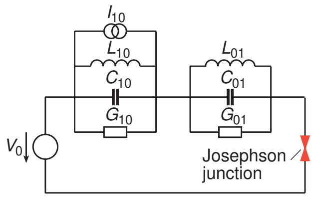

Figure 2. Josephson parametric amplifier [5].

In 1969 P. Russer proposed a DC pumped Josephson parametric amplifier (DCPJPA) [5]. Figure 2 shows the equivalent circuit of the DCPJPA exhibiting a signal circuit consisting of the inductor L_{10}, the capacitor C_{10}, the conductor G_{10} and the impressed signal source I_{10} and an idler circuit consisting of L_{01}, C_{01} and G_{01}. Terminating the Josephson junction at the idler frequency ω_2 with the admittance Y_{01}, it exhibits at the signal frequency ω_1 the impedance

$$Z_{J10} = -\frac{\omega_1 \omega_2 \hbar^2}{e_0^2 I_J^2} Y_{01}^* . \quad (4)$$

Since the negative real part of Z_{J10} is related to the positive real part of Y_{01}, the gain at ω_1 is related to losses in the idler circuit.

H. Kanter has realized DCPJPAs for signal frequencies of 30 MHz [9] and 9 GHz [10–12], as up-converters from 115 MHz to 9 GHz [11, 13], and for 89 GHz [14]. M.L. Yu proposed a DCJPA where the Josephson oscillation synchronizes with the input signal [15]. With an ACPJPA operating at 9 GHz, a power gain of 16 dB in a 4-MHz 3-dB bandwidth was achieved by J. Mygind et al. [16,17]. A JPA for 36 GHz with a point contact Josephson junction with 11 dB gain was realized by Y. Taur and P. Richards [18,19]. Further work on JPAs is presented in [20–24]. Low-noise JPAs with power gain greater than 20 dB and bandwidths from 1 MHz to 10 MHz have been realized [25–29].

4 Traveling Wave Parametric Amplifiers

A Josephson traveling wave parametric amplifier (JTWPA) achieves unilateral and higher gain, together with improved stability and bandwidth, than a discrete JPA with a single Josephson junction [1, 30–38]. The JTWPA can be realized either by connecting Joephson junctions in parallel to a transmission line or by insertion of Josephson elements in series between transmission line segments.

Fig. 3 shows a JTWPA structure with the Josephson junction in parallel to the transmission line as described in [30, 31]. The JTWPA is based on a Josephson transmission line (JTL) formed by two superconductors of length l connected via a distributed Josephson junction exhibiting an insulating tunneling layer of thickness d. A transverse magnetic field B_y flows through a layer of thickness $d_m = d + 2\lambda_L$, where λ_L is the London penetration depth of the magnetic field, into the superconductor [39, p. 61]. The magnetic field yields a spatial variation of the quantum phase difference φ [40]. For a uniform transverse magnetic field B_{y0}

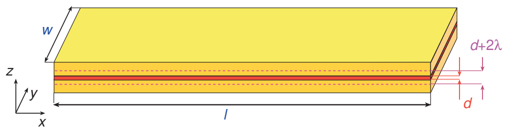

Figure 3. Josephson transmission line.

the magnetic flux increases in the x-direction proportional to $x d_m B_{y0}$. Applying a DC voltage V_0 and a static transverse magnetic field B_{y0}, we excite a pump wave with angular frequency $\omega_0 = 2e_0 V_0/\hbar$ and wave number $k_0 = 2e_0 B_{y0} d_m/\hbar$. For the applied idler frequencies $\omega_{i\pm}$, both frequency and phase conditions $\omega_{i\pm} = \omega_0 \pm \omega_s$ and $k_{i\pm} = k_0 \pm k_s$ must be fulfilled. Small signal analysis [1, 30] yields a power gain at ω_s given by

$$G_s(x) = \frac{1}{2}\left[\cosh(2\kappa l) + 1\right] \quad (5)$$

with

$$\kappa = \frac{4\pi\alpha d_m J_J}{c_0 e_0}, \quad (6)$$

where J_J is the maximum Josephson current density and α is the fine structure constant $\alpha = e_0^2 Z_{F0}/4\pi\hbar \approx 1/137$.

The JTWPA with Josephson junctions in parallel to the transmission line has the advantages that it can be realized as a continuous structure, it can be DC pumped and the phase velocity of the pump wave can be tuned by a DC magnetic field. Difficulties for realization arise from the low impedance of Josephson junctions.

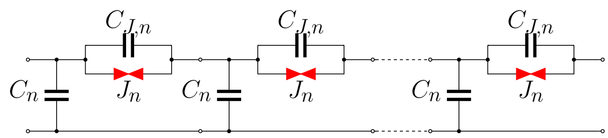

Figure 4. JTWPA with series connected junctions.

In 1985 M. Sweeny and R. Mahler proposed a JTWPA consisting of a superconducting transmission line interrupted in series by a large number of junctions. Similar structures with series connected Josephson junctions were investigated in [33–38]. Figure 4 shows the circuit of a Josephson traveling wave parametric amplifier consisting of a long chain of capacitively shunted Josephson junctions along a transmission line.

5 Circuit Quantum Electrodynamics of JPAs

If signal amplitudes are small enough, an understanding and a correct description of the phenomena taking into account

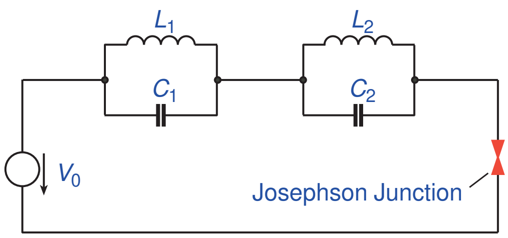

Figure 5. Josephson Junction circuit.

the quantum statistical properties of the circuits requires a treatment on the basis of circuit quantum electrodynamics (CQED) allowing the investigation of lumped element or distributed linear lossless circuits on the basis of a Hamiltonian description of Foster equivalent circuits [41–44].

Figure 5 shows a Josephson junction embedded in a circuit consisting of two lossless resonant circuits L_1, C_1 and L_2, C_2 and a DC source V_0. Introducing the destruction operators a_i and the creation operators $a_i^\dagger$ by

$$a_i = \sqrt{\frac{1}{2\hbar\omega_i L_i}}\Phi_i + j\sqrt{\frac{\omega_i L_i}{2\hbar}}Q_i\,, \tag{7a}$$

$$a_i^\dagger = \sqrt{\frac{1}{2\hbar\omega_i L_i}}\Phi_i - j\sqrt{\frac{\omega_i L_i}{2\hbar}}Q_i\,, \tag{7b}$$

where the operators Q_i and Φ_i represent charge and flux in the capacitors C_i and the inductors L_i, respectively, † denotes the Hermitian conjugate, and $\omega_i = \sqrt{1/L_i C_i}$, we can write the components of the Hamiltonian in the form

$$H_0 = \frac{1}{2}\hbar\omega\left(a_1^\dagger a_1 + a_1 a_1^\dagger + a_2^\dagger a_2 + a_2 a_2^\dagger\right), \tag{8}$$

$$H_1 = W_J\left[1 - \cos\left[\omega_0 t + \kappa_1(a_1 + a_1^\dagger) + \kappa_2(a_2 + a_2^\dagger)\right]\right]$$

with

$$\omega_0 = \frac{2e_0 V_0}{\hbar}, \quad \text{(9a)} \qquad W_J = \frac{\Phi_0 I_J}{2\pi}, \quad \text{(10a)}$$

$$\Phi_0 = \frac{\pi\hbar}{e_0}, \quad \text{(9b)} \qquad \kappa_i = \sqrt{\frac{2\alpha Z_i}{\pi Z_{F0}}}, \quad \text{(10b)}$$

where $Z_{F0} \approx 377\ \Omega$ is the free space wave impedance and $Z_i = \sqrt{L_i/C_i}$.

The time dependence of the expectation value of the photon energy has been calculated in [1,44] as

$$\langle W(t)\rangle = \hbar\omega|w|^2\cosh^2\gamma_{12}t + \frac{\hbar\omega_1}{2}\coth\frac{\hbar\omega_1}{k_B T_1}\cosh^2\gamma_{12}t + \frac{\hbar\omega_1}{2}\coth\frac{\hbar\omega_2}{k_B T_2}\sinh^2\gamma_{12}t\,, \tag{11}$$

with

$$\gamma_{ij} = \frac{e_0 I_J}{\pi h}\sqrt{Z_i Z_j} = \frac{\alpha I_J}{\pi e_0}\frac{\sqrt{Z_i Z_j}}{Z_{F0}} = \frac{I_J}{2\pi\Phi_0}\sqrt{Z_i Z_j}\,. \tag{12}$$

The first term on the right-hand side represents the amplified signal, the second term is the amplified noise of the signal circuit L_1, C_1 and the third term is the amplified noise down-converted from the idler circuit L_2, C_2 to the signal circuit. In [45], the thermal losses in the resonator are described by a heat bath and treated by the Langevin approach. A quantum mechanical treatment of the JTWPA was given in [46,47]. Dissipation and fluctuation in DCPJPAs were investigated in [48–50] on the basis of the quantum Langevin equations.

The JTWPA as given in [35] using a Hamiltonian describing a discrete chain of unit cells [46], and extended by a circuit [47] for matching the phases of the idler and signal waves, was investigated in [37]. In addition to the approximations used in [35,51], the flux operator was expanded into a set of modes and a strong classical pump field was assumed [47].

6 Squeezed States and Entangled States

Squeezed states have less uncertainty in one quadrature than coherent states [52–54]. Squeezed states of the radiation field are eigenstates of the operator

$$b = \mu a + \nu a^\dagger \quad \text{with} \quad |\mu|^2 - |\nu|^2 = 1\,, \tag{13}$$

where $a^\dagger$ and a are photon creation and destruction operators and μ and ν are complex numbers. Splitting a into self-adjoint operators $a = a_c + ja_q$, representing cophasal- and quadrature components, with $a_c = a_c^\dagger$ and $a_q = a_q^\dagger$ yields

$$\langle\Delta a_c^2\rangle = \tfrac{1}{4}|\mu - \nu|^2\,, \qquad \langle\Delta a_q^2\rangle = \tfrac{1}{4}|\mu + \nu|^2\,. \tag{14}$$

Squeezed-states can be generated by degenerate parametric amplification [55]. Squeezed-state generation using a Josephson parametric amplifier has been discussed in [56–59]. The achievable degree of squeezing in a degenerated DCPJPA with identical signal and idler frequencies $f_1 = f_2 = f_0/2$ has been calculated in [58,59]. In this case (8) can be approximated by

$$H_1^{DCPJA} = \kappa_1\left[a^{\dagger 2}e^{-j\omega_0 t} + a^2\, e^{j\omega_0 t}\right]. \tag{15}$$

According to Yuen, this Hamiltonian can produce squeezed states [52–54]. Squeezed states allow to transfer entanglement to a pair of quantum bits [60]. Squeezing of photons with JPAs was investigated in [47,61,62].

Classical computers operate with *bits* whereas quantum computers use quantum bits. While a classical bit can only assume either the state 0 or 1 a qubit can be in a weighted superposition of states $|0\rangle$ and $|1\rangle$. A two-qubit state which is not decomposable into two one-bit states is called entangled. An example for an entangled state is $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

A photonic NOON state is a many-photon entangled state representing a superposition of a state with N photons in a first mode and zero photons the second mode with a state with zero photons in the first mode and N photons in the second mode [63,64] and has the form

$$|\psi\rangle_{NOON} = \frac{1}{\sqrt{2}}\left(|N,0\rangle + e^{j\phi}|0,N\rangle\right). \tag{16}$$

NOON states allow to make precision phase measurements in optical interferometry. Schemes for generating NOON photon states with circuits based on superconducting resonators and Josephson junctions are reported in [65, 66].

7 Applications

Superconducting quantum information processing devices will operate at microwave and millimeterwave frequencies and will be based on future RF nanoelectronics. Josephson junctions will find interesting applications in quantum state engineering. In [67–69] the basic concepts and quantum computing applications of Josephson junctions are discussed. Quantum superposition and entanglement has the potential of high parallelism and efficiency in CAD, but there are still fundamental problems to be solved [70].

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