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Particle Swarm Optimization of Layered Media Cloaking Performance

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Abstract

Electromagnetic scattering problems are considered concerning the excitation of a spherically-layered medium by an external dipole. The purpose of this work is to determine suitable parameters of the dielectric layers covering a perfectly conducting core so that the generated scattered far-field is significantly reduced for all observation angles. A Particle Swarm Optimization (PSO) algorithm is developed and applied to the associated optimization problem. Some preliminary numerical results exhibiting reduced values of the total scattering cross section are reported.

1 Introduction

Optimizations of the characteristic input parameters of devices, utilized in several applications of Electromagnetics and Photonics, are of primary importance for achieving required operational characteristics with respect to prescribed field variations. In this context, obtaining exact or semi-analytic solutions for the involved electromagnetic scattering and radiation problems is significant for achieving fast and efficient optimization schemes. Such solutions can be exploited as suitable objective functions in optimization problems concerning the determination of the physical and geometrical parameters of the configuration under examination. Considering, in particular, the efficient design of layered media exhibiting desired far-field patterns, relevant optimizations involve the determination of the layers widths as well as their permittivities and permeabilities. For achieving cloaking behavior of spherically layered media, aspects of related optimization problems have been investigated in [1]-[4], and are mainly focusing on plane incident waves. An overview of optimization techniques for the design of various meta-devices is presented in [5].

In this work, we present certain initial numerical optimization results for the reduction in the total scattering cross section of a spherical medium containing a perfect electric conducting (PEC) core covered by a suitable number of dielectric layers. The primary excitation is due to an external magnetic dipole. As the distance between the dipole and the boundary of the medium becomes sufficiently large, approximations of the scattering performance due a plane incident wave are obtained. The optimization variables are the radii, the permittivities and the permeabilities of the spherical layers. The core's radius is kept constant in the presented numerical experiments. An evolutionary algorithm based on the Particle Swarm Optimization (PSO) is developed and subsequently utilized for the determination of suitable values of the optimization variables yielding significantly reduced scattered far-field contributions. The exact solution in the form of a Mie series of the considered scattering problem is crucial for the fast and efficient implementation of the PSO algorithm in the present setting.

2 The Scattering Problem

A layered spherical medium V with radius a_1 is excited by an external magnetic dipole, with position vector $\mathbf{r}_0$ on the z-axis and with dipole moment along the direction $\hat{\mathbf{y}}$; the case of an arbitrary dipole can also be considered and treated by similar techniques with those presented below. The interior of V is divided by $P-1$ concentric spherical interfaces $r = a_p$ $(p = 2,\ldots,P)$ into $P-1$ homogeneous dielectric layers V_p $(p = 1,\ldots,P-1)$, consisting of materials with dielectric permittivities ε_p and magnetic permeabilities μ_p, and surrounding a PEC core (layer V_P). The exterior V_0 of V is an unbounded homogeneous medium with permittivity ε_0, permeability μ_0, and wavenumber k_0.

The exact solution of the scattering problem is determined analytically by applying a combined Sommerfeld T-matrix methodology, which is developed and analyzed in [6] and [7]. More precisely, the electric fields in each region of the problem are decomposed into primary and secondary components, which are then expressed as series of the spherical vector wave functions [8]. The unknown coefficients in the expansions of the secondary fields are determined analytically by imposing the transmission boundary conditions on the interfaces of the spherical shells and applying a T-matrix method. By this methodology, the following expressions are obtained for the bistatic (differential) scattering cross section and the total scattering cross section, respectively,

$$\sigma(\theta,\phi;\mathbf{r}_0) = \frac{4\pi}{k_0^2}\left[|S_\theta(\theta;\mathbf{r}_0)|^2\cos^2\phi + |S_\phi(\theta;\mathbf{r}_0)|^2\sin^2\phi\right], \tag{1}$$

$$\begin{aligned}\sigma^{\mathrm{t}}(\mathbf{r}_0) &= \frac{1}{4\pi}\int_{S^2}\sigma(\theta,\phi;\mathbf{r}_0)\mathrm{d}s(\hat{\mathbf{r}}) \\ &= \frac{2\pi}{k_0^2}\sum_{n=1}^{\infty}(2n+1)\left[|\gamma_n|^2+|\delta_n|^2\right],\end{aligned} \tag{2}$$

where functions S_θ and S_ϕ and coefficients γ_n and δ_n are defined in [6], while S^2 denotes the unit sphere in $\mathbb{R}^3$.

3 Particle Swarm Optimization (PSO) Algorithm

PSO belongs to the class of evolutionary optimization algorithms, which are based on biological behaviors and phenomena that can be observed in real life. Such biological mechanisms usually have to do with how a population of a specific species (like e.g. a swarm, herd or a pride) acts in unison, changes in time and solves problems. The related algorithms are required to handle and organize the mentioned populations, using meta-heuristic methods (i.e. self-training methods) in order to simulate a specific biological evolutionary mechanism.

In PSO, the population evolving in time is a group of particles (points with no mass), which move in the space $\mathbb{R}^n$ (n being the number of variables of the problem's objective function) searching for solutions representing a maximum (or minimum). Biologically, this is interpreted as the particles seeking for the greatest path towards "food". Each particle represents a possible set of variables that serves as a potential solution to the optimization problem.

The two main characteristics of each particle are its position $\mathbf{p}$ and its velocity $\mathbf{u}$. These describe the current state and the short-term evolution of the particle. The vector $\mathbf{p} \in \mathbb{R}^n$ includes the variables of the objective function, which are allowed to have continuous variations in specific chosen intervals. The components of the vector $\mathbf{u} \in \mathbb{R}^n$ describe the movement of the particle. Each particle holds a memory "spot" for its best attained position $\mathbf{p}_{\text{best}}$ alongside with the obtained value $\mathbf{g}_{\text{best}}$ at this position. Additionally, the swarm as a whole keeps track of the best position $\mathbf{P}_{\text{best}}$ per iteration with its corresponding value $\mathbf{G}_{\text{best}}$. These two types of memory allude to the fact that swarm particles have both *cognitive* and *social* learning taking place while the algorithm runs. It is worth to note that the swarm has three specific fundamental characteristics

1. Cohesion (the particles move as a swarm and not completely independently)

2. Separation (the particles do not merge with other particles, or hinder them)

3. Alignment (the particles have common cause and judgement)

Moreover, there is an inertia mechanism added to the particles movement; see e.g. [9] and [10]. Inertia may improve the results by "slowing down" particles that move too fast and, hence, might miss an optimum. The developed algorithm is presented below in a pseudo-code form, where l and u are the lower and upper limits of the positions domain, $\theta_{\min} = 0.4$ and $\theta_{\max} = 0.9$, and c_1 and c_2 are the cognitive and social learning rates (both set to 0.5 here).

Algorithm 1: Particle Swarm Optimization (PSO)

Input: N, l, u, c_1, c_2, $it_{\max}$

Output: A swarm S of size N with each current position(s)

Initialize S with random values for the position of each particle w.r.t. the domain of the objective function;
Initialize all velocities $\mathbf{u}$ to zero;
Initialize best positions (and respective values) both for individual particles and S;
Choose randomly two values in $[0,1]$ for r_1 and r_2;
Iteration $it = 0$;
Initialize $\theta_{\min}, \theta_{\max}$;
while $it < it_{\max}$ **do**
  calculate inertia as: $\theta = \theta_{\max} - \frac{\theta_{\max}-\theta_{\min}}{it_{\max}}\, it$;
  For each particle in S:

  1. Update velocity: $\mathbf{u}(i) = \theta\,\mathbf{u}(i-1) + c_1\, r_1\,(\mathbf{p}_{\text{best}} - \mathbf{p}(i-1)) + c_2\, r_2\,(\mathbf{G}_{\text{best}} - \mathbf{p}(i-1))$;
  2. Update position: $\mathbf{p}(i) = \mathbf{p}(i-1) + \mathbf{u}(i)$

  Validate particles (find corresponding values);
  Upgrade $\mathbf{g}_{\text{best}}, \mathbf{G}_{\text{best}}$ and their corresponding values;
  Upgrade iteration: $it = it + 1$;
  (Optional) Check for convergence.;
end
return S;

4 Implementation of PSO to the Scattering Problem and Numerical Results

The objective function we consider in the optimization schemes is the *normalized total scattering cross section* $\sigma^{\mathrm{t}}(\mathbf{r}_0)/(\pi a_1^2)$ (instead of the backscattering cross section, as in a previous work [11]). Achieving small values of this objective function provides efficient designs in terms of significant reductions in the scattered far-field contributions.

The above described PSO algorithm was implemented in MATLAB®. The swarm was a MATLAB struct for which we followed the steps in Algorithm 1. The chosen N for the experiments was 20, and the iterations were 800 in most cases. The components of the position vector consisted of the optimization variables a_p of the radii, ε_p of the permittivities, and μ_p of the permeabilities of the first $P-1$ dielectric layers. The radius a_P of the PEC core was chosen constant at $k_0 a_P = 2\pi$ (one free-space wavelength). Thus, for scattering by a medium with P layers in total, the number of optimization variables for the particles position is $3(P-1)$. The differences $a_{p+1} - a_p$ between two consecutive layers radii were considered in the range $[\frac{\pi}{10}, \pi]$. The values of ε_p and μ_p were allowed to vary in the range [0.5,10]. We also executed experiments for the range [0.5,3]. The distance r_0 of the dipole from the scatterer was taken $r_0 = 10a_1$ for which case the obtained far-field results are close to the ones due to plane-wave incidence; see also [6].

By applying the developed PSO algorithm, optimized val-

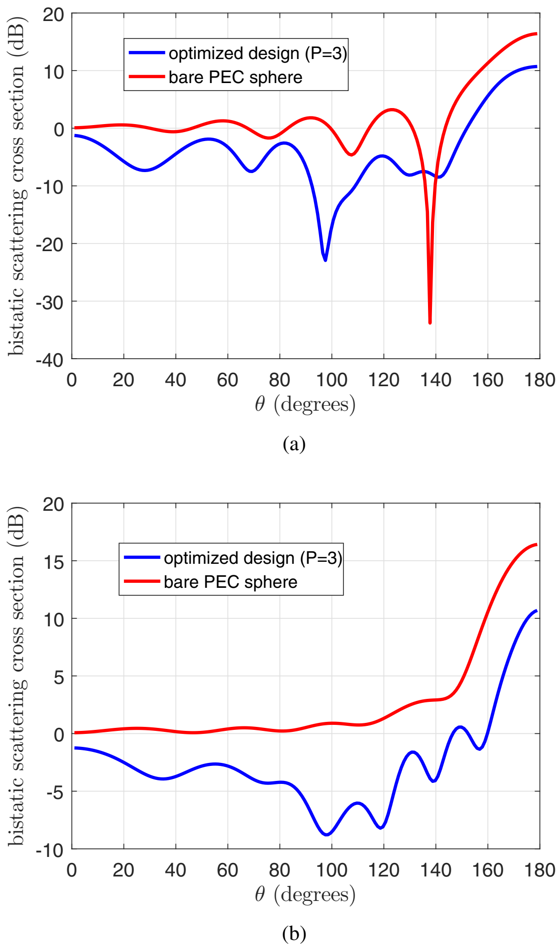

Figure 1. Normalized bistatic scattering cross section versus the observation angle θ for $P = 3$ optimized layers with parameters computed by the PSO algorithm; (a) and (b) refer to xOz and the yOz planes, respectively.

ues -2.66, -5.63, and -3.86 dB of the normalized total cross section were obtained for a spherical medium with P =3, 4, and 5 total number of layers, respectively. To demonstrate the actual reduction in the far-field with respect to the angles of observation, we depict in Fig. 1 a representative plot of the normalized bistatic scattering cross section $\sigma(\theta, \phi; \mathbf{r}_0)/(\pi a_1^2)$ as function of the angle θ in the xOz and yOz planes. The curves in Fig. 1 correspond to the optimized design obtained for $P = 3$, and it is evident that they exhibit significantly reduced far-field contributions with respect to the bare PEC sphere for all observation angles (with the exception of a resonant angle on the xOz plane).

Optimizations for dipoles lying close to the scattering medium have also been considered. Extensions to radially inhomogeneous media [12] are also feasible.

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