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# Power maximization for a multiple–input and multiple–output wireless power transfer system described by the admittance matrix

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## Abstract

This paper analyses a multiport wireless power transfer system described by its admittance matrix. The general case of a link using a multiple–input and multiple–output configuration is solved by determining the optimal loads for maximizing the total power delivered to the loads. As a specific application of interest of the proposed theory, a wireless link based on capacitive coupling is analysed.

## 1 Introduction

By using wireless power transfer (WPT), energy can be transferred without any conducting path between transmitter and receiver. Although most WPT devices on the market consist of one transmitter and one receiver, it can be expected that the need will arise to charge multiple receivers by multiple transmitters simultaneously (multiple–input and multiple–output configuration).

In this work, the optimal loads to maximize the power transfer for a WPT system with *any* number of transmitters and receivers are determined. This was already done for WPT systems where the impedance matrix is given [1], but for certain applications, an admittance matrix approach is much more straightforward. For example, a *capacitive* WPT multiport can be easier described by its admittance matrix [2], contrary to an *inductive* WPT multiport that is more manageable if characterized by its impedance matrix [1].

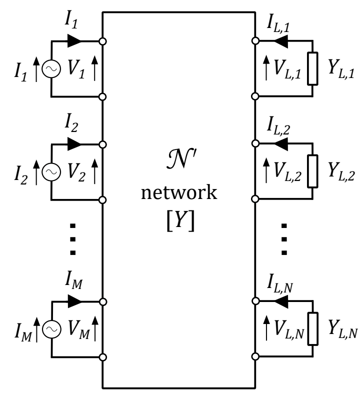


**Figure 1.** At a multiport network $\mathcal{N}'$, characterized by its admittance matrix $\boldsymbol{Y}$, are $M$ current sources (left side) and $N$ loads (right side) connected.

An easy procedure to calculate the optimal loads for power maximization is proposed, based on the methodology of [1]. The method is valid for *any* multiple–input and multiple–output WPT system. The procedure is demonstrated on a capacitive WPT system with 2 transmitters and 3 receivers as a representative example and validated by circuital simulation.

## 2 General circuit with *M* transmitters and *N* receivers

### 2.1 Description multiport

A multiport network $\mathcal{N}'$ with $M$ transmitters and $N$ receivers is considered. The $M$ input ports of the network are connected to $M$ current sources (Figure 1, left side). At the $N$ output ports, loads admittances $Y_{L,i} = G_{L,i} + j.B_{L,i}$ are present ($i = 1,\ldots,N$), with $G_{L,i}$ and $B_{L,i}$ the load conductance and load susceptance, respectively (Figure 1, right side). The peak current and voltage phasors at the $M$ input ports and $N$ output ports, respectively, are defined in Figure 1. The following matrices are introduced:

$$\boldsymbol{i_M} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_M \end{bmatrix}, \boldsymbol{v_M} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ \vdots \\ V_M \end{bmatrix}, \boldsymbol{i_N} = \begin{bmatrix} I_{L,1} \\ I_{L,2} \\ I_{L,3} \\ \vdots \\ I_{L,N} \end{bmatrix}, \boldsymbol{v_N} = \begin{bmatrix} V_{L,1} \\ V_{L,2} \\ V_{L,3} \\ \vdots \\ V_{L,N} \end{bmatrix} \quad (1)$$

The relation between the voltages and the currents of the multiport can be described by:

$$\begin{bmatrix} \boldsymbol{i_M} \\ \boldsymbol{i_N} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Y}_{MM} \boldsymbol{Y}_{MN} \\ \boldsymbol{Y}_{NM} \boldsymbol{Y}_{NN} \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{v_M} \\ \boldsymbol{v_N} \end{bmatrix} \quad (2)$$

where the admittance matrix of the network $\mathcal{N}'$ has been partitioned into four submatrices $\boldsymbol{Y}_{MM}$, $\boldsymbol{Y}_{MN}$, $\boldsymbol{Y}_{NM}$ and $\boldsymbol{Y}_{NN}$. The subscript of the submatrices indicate their dimension.

## 2.2 Norton equivalent circuit

By applying the Norton's theorem, for the multiport network it is possible to derive the equivalent circuit illustrated in Figure 2, where the currents $I_i^{(no)}$ are the Norton currents ($i = 1,\ldots,N$). Notice that the $M$ input ports are replaced by open circuits. The $N$ loads of the receiver are represented by the network $\mathcal{N}_L$, described by the admittance matrix $\boldsymbol{Y_L}$.

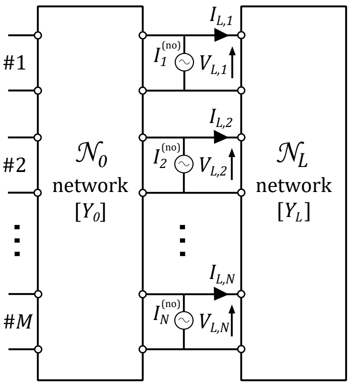


**Figure 2.** Norton equivalent circuit with load network $\mathcal{N}_L$.

The Norton currents can be derived by short-circuiting the output ports ($\boldsymbol{v}_N = \mathbf{0}$). Equation (2) changes into

$$\boldsymbol{i_M} = \boldsymbol{Y}_{MM}.\boldsymbol{v_M} \tag{3}$$

$$\boldsymbol{i_N} = \boldsymbol{Y}_{NM}.\boldsymbol{v_M} \tag{4}$$

From equation (3), the following relation is obtained

$$\boldsymbol{v_M} = \boldsymbol{Y}_{MM}^{-1}.\boldsymbol{i_M} \tag{5}$$

Substituting the above expression into equation (4) results in the values of the Norton currents:

$$\boldsymbol{i_N} = \boldsymbol{Y}_{NM}\boldsymbol{Y}_{MM}^{-1}.\boldsymbol{i_M} \equiv \boldsymbol{i}_{\boldsymbol{N}}^{(no)} = \begin{bmatrix} I_1^{(no)} \\ I_2^{(no)} \\ I_3^{(no)} \\ \vdots \\ I_N^{(no)} \end{bmatrix} \tag{6}$$

The Norton admittance matrix $\boldsymbol{Y_0}$, which characterizes network $\mathcal{N}_0$, is defined by

$$\boldsymbol{i_N} = \boldsymbol{Y_0}.\boldsymbol{v_N} \tag{7}$$

under the assumption that the original current sources are open circuited ($\boldsymbol{i_M}$=$\mathbf{0}$). Under these conditions, equation (2) changes into

$$\mathbf{0} = \boldsymbol{Y}_{MM}.\boldsymbol{v_M} + \boldsymbol{Y}_{MN}.\boldsymbol{v_N} \tag{8}$$

$$\boldsymbol{i_N} = \boldsymbol{Y}_{NM}.\boldsymbol{v_M} + \boldsymbol{Y}_{NN}.\boldsymbol{v_N} \tag{9}$$

From equation (8), it is possible to obtain

$$\boldsymbol{v_M} = -\boldsymbol{Y}_{MM}^{-1}.\boldsymbol{Y}_{MN}.\boldsymbol{v_N} \tag{10}$$

Substituting the above expression into equation (9) results in:

$$\boldsymbol{i_N} = -\boldsymbol{Y}_{NM}.\boldsymbol{Y}_{MM}^{-1}.\boldsymbol{Y}_{MN}.\boldsymbol{v_N} + \boldsymbol{Y}_{NN}.\boldsymbol{v_N} \tag{11}$$

Combining this relation with equation (7) leads to the Norton admittance matrix $\boldsymbol{Y_0}$:

$$\boldsymbol{Y_0} = \boldsymbol{Y}_{NN} - \boldsymbol{Y}_{NM}.\boldsymbol{Y}_{MM}^{-1}.\boldsymbol{Y}_{MN} \tag{12}$$

# 3 Optimal loads for power maximization

The goal of this work is to determine the loads that realize maximum power transfer from the $M$ transmitters to the $N$ receivers, i.e. that maximize the total output power $P_{out}$ defined as

$$P_{out} = \sum_{i=1}^{N} P_i \tag{13}$$

with $P_i$ the output power delivered to load $Y_{L,i}$. By applying Norton's theorem, the original circuit of Figure 1 was replaced by the equivalent circuit of Figure 2 with Norton admittance matrix $\boldsymbol{Y_0}$ and current sources $I_i^{(no)}$.

Baudrand [3] determined a generalized condition for the voltage-current relations at the ports of a multiport network. Applying the duality principle on this generalized condition results in the following expression for achieving maximum power transfer to passive loads:

$$\boldsymbol{v_N} = (\boldsymbol{Y_0} + \boldsymbol{Y_0^+})^{-1}.\boldsymbol{i}^{(no)} \tag{14}$$

with $\boldsymbol{Y_0^+}$ the conjugate transpose of $\boldsymbol{Y_0}$. For a reciprocal network, as is the case for WPT, the conjugate transpose coincides with the conjugate. Since $\boldsymbol{Y_0}$ and $\boldsymbol{i}^{(no)}$ are known from the previous section, the optimal voltages $\boldsymbol{v_N}$ at the $N$ output ports that realize maximum power transfer are known.

For the network of Figure 2, the equations are:

$$\boldsymbol{i_N} = \boldsymbol{i}^{(no)} - \boldsymbol{Y_0}.\boldsymbol{v_N} \tag{15}$$

$$\boldsymbol{i_N} = \boldsymbol{Y_L}.\boldsymbol{v_N} \tag{16}$$

Combining both expressions results into

$$\boldsymbol{Y_L}.\boldsymbol{v_N} = \boldsymbol{i}^{(no)} - \boldsymbol{Y_0}.\boldsymbol{v_N} \tag{17}$$

Substituting equation (14) into the right hand side results in:

$$\boldsymbol{Y_L}.\boldsymbol{v_N} = \boldsymbol{i}^{(no)} - \boldsymbol{Y_0}.(\boldsymbol{Y_0} + \boldsymbol{Y_0^+})^{-1}.\boldsymbol{i}^{(no)} \tag{18}$$

Note that all parameters are known, except $\boldsymbol{Y_L}$.

For a WPT system, the loads of the receivers should be uncoupled. This implies that the admittance matrix $\boldsymbol{Y_L}$ is a diagonal matrix, with as diagonal elements the $N$ loads of the receivers. From equation (16), the expressions of the optimal currents can be obtained:

$$\boldsymbol{i_N} = \boldsymbol{i}^{(no)} - \boldsymbol{Y_0}.(\boldsymbol{Y_0} + \boldsymbol{Y_0^+})^{-1}.\boldsymbol{i}^{(no)} \tag{19}$$

Equations (14) and (19) determine the voltages and currents at the loads for the maximum power configuration. The load admittances that realize maximum power transfer are therefore given by ($i = 1, \dots, N$):

$$Y_{L,i} = G_{L,i} + jB_{L,i} = \frac{I_{L,i}}{V_{L,i}} \tag{20}$$

with $V_{L,i}$ and $I_{L,i}$ the elements of $\boldsymbol{v_N}$ and $\boldsymbol{i_N}$, respectively, as described by equations (14) and (19).

The procedure to find the loads for power maximization for a WPT system with any number of transmitters and receivers can be summarized as follows.

1. Establish (e.g., by measurement) the admittance matrices $\boldsymbol{Y}_{MM}$, $\boldsymbol{Y}_{MN}$, $\boldsymbol{Y}_{NM}$ and $\boldsymbol{Y}_{NN}$ of the multiport network given by equation (2).
2. Determine the Norton current sources $\boldsymbol{I}_{\boldsymbol{N}}^{(no)}$ from equation (6).
3. Set up the Norton admittance matrix $\boldsymbol{Y_0}$ using equation (12).
4. Calculate the voltages $\boldsymbol{v_N}$ (equation (14)) and currents $\boldsymbol{i_N}$ (equation (19)) for the loads at the maximum power configuration.
5. Determine the optimal loads $Y_{L,i}$ from equation (20).

# 4 Application to a capacitive WPT link with 2 transmitters and 3 receivers

## 4.1 Optimal loads for power maximization

The theory presented in 3 is valid for any multiport network, regardless of the total number of $(M+N)$ ports and its specific implementation; in order to be applied, the presented methodology just requires the experimental or analytical derivation of the admittance matrix of the network. In this section an example of application of the presented theory is provided for the specific case of a capacitive WPT system with two transmitters ($M$=2) and three receivers ($N$=3). The approximated equivalent circuit of the analysed link is illustrated in Figure 3 [4].

At the two input ports (left side of Figure 3), two current sources $I_1$ and $I_2$ power the system. At the three output port (right side of Figure 3) load admittances $Y_{L,1}$, $Y_{L,2}$ and $Y_{L,3}$ are connected. The voltages $V_j$ and currents $I_j$ at the ports are defined in the figure ($j = 1, \dots, 5$).

The shunt conductances $g_{jj}$ describe the losses in the circuit. The mutual capacitances $C_{13}$, $C_{14}$, $C_{15}$, $C_{23}$, $C_{24}$,

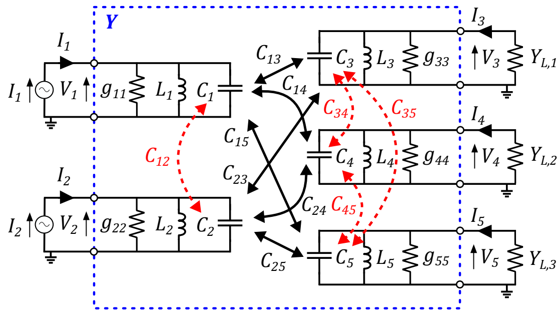


**Figure 3.** Equivalent circuit of a capacitive WPT system with two transmitters and three receivers.

and $C_{25}$ represent the desired electric coupling between the transmitter capacitances $C_1$, $C_2$, and the receiver capacitances $C_3$, $C_4$, $C_5$. They realize the wireless link between transmitters and receivers [5]. Undesired electric coupling is present between both transmitters, indicated by the mutual capacitance $C_{12}$. Also between the receivers, an undesired coupling is present: $C_{34}$, $C_{35}$ and $C_{45}$.

In order to obtain a resonant scheme, the inductors $L_j$ are added:

$$L_j = \frac{1}{\omega_0^2 C_j} \tag{21}$$

with $\omega_0$ the operating angular frequency of the current sources $I_1$ and $I_2$. The coupling factor $k_{ij}$ is defined as ($i, j = 1, \dots, 5$):

$$k_{ij} = \frac{C_{ij}}{\sqrt{C_i C_j}} \tag{22}$$

The entire multiport system (indicated by the dashed rectangle in Figure 3) is considered fixed and fully determined by the admittance matrix $\boldsymbol{Y}$ which is, at the resonance angular frequency $\omega_0$, equal to:

$$\boldsymbol{Y} = \begin{bmatrix} \boldsymbol{Y}_{MM} \boldsymbol{Y}_{MN} \\ \boldsymbol{Y}_{NM} \boldsymbol{Y}_{NN} \end{bmatrix} = \\ = \begin{bmatrix} g_{11} & -jb_{12} & -jb_{13} & -jb_{14} & -jb_{15} \\ -jb_{12} & g_{22} & -jb_{23} & -jb_{24} & -jb_{25} \\ -jb_{13} & -jb_{23} & g_{33} & -jb_{34} & -jb_{35} \\ -jb_{14} & -jb_{24} & -jb_{34} & g_{44} & -jb_{45} \\ -jb_{15} & -jb_{25} & -jb_{35} & -jb_{45} & g_{55} \end{bmatrix} \tag{23}$$

with $b_{ij} = \omega_0 C_{ij}$.

## 4.2 Numerical verification

As an example, consider the system with numerical values indicated in Table 1 and operating at $f_0$=10 MHz. The coupling factors between the transmitters and receivers are indicated in Table 2.

By using (20), the values reported in Table 3 can be obtained for the optimal terminating admittances. The optimal load susceptances are negative, i.e. they correspond to shunt inductors whose values are reported in the table.

**Table 1.** Given circuit parameters of the example capacitive WPT system.

| Quantity | Value | Quantity | Value |
|---|---|---|---|
| $g_{11}$ | 1.00 mS | $C_1$ | 350 pF |
| $g_{22}$ | 1.25 mS | $C_2$ | 300 pF |
| $g_{33}$ | 1.50 mS | $C_3$ | 250 pF |
| $g_{44}$ | 1.75 mS | $C_4$ | 225 pF |
| $g_{55}$ | 2.00 mS | $C_5$ | 200 pF |
| $I_1$ | 100 mA | $f_0$ | 10 MHz |
| $I_2$ | 200 mA | | |

**Table 2.** Coupling factors of the analysed numerical example

| Desired coupling | Value | Undesired coupling | Value |
|---|---|---|---|
| $k_{13}$ | 30 % | $k_{12}$ | 10 % |
| $k_{14}$ | 25 % | $k_{34}$ | 5 % |
| $k_{15}$ | 20 % | $k_{35}$ | 2 % |
| $k_{23}$ | 25 % | $k_{45}$ | 5 % |
| $k_{24}$ | 20 % | | |
| $k_{25}$ | 15 % | | |

In order to verify the analytical results summarized in Table 3 circuital simulations have been performed in SPICE. First of all, a simulation with the network terminated on the optimal admittances given in Table 3 has been performed; SPICE returns an output power of 4.64 W. Next, six simulations were performed by varying among the six parameters of interest one parameter at a time (either one of the three conductances $G_{Li}$ or one of the load inductors $L_{L,i}$), while keeping all the others constant at their optimal value shown in the Table 3. The achieved results are reported in Figures 4 and 5. They confirm the data provided by the theory for this example: a maximum power output is achieved when the loads are the ones calculated according to (20).

## 5 Conclusion

A procedure to easily find the optimal load values in order to maximize the power transfer for a multiple–input and multiple–output wireless power transfer system has been presented. The proposed methodology is valid for *any* number of transmitters and receivers, even when non-negligible coupling is present between the transmitters, or between the receivers. The calculation of the optimal loads just requires the knowledge of the admittance matrix of the system, which can either analytically calculated or measured. The reported formulas are validated through circuital simulations performed for a numerical example referring to a capacitive link using three transmitters and two loads.

**Table 3.** Optimal terminating admittances for the analysed numerical example

| $G_{L,1}$ (mS) | $L_{L1}$ (μH) | $G_{L,2}$ (mS) | $L_{L2}$ (μH) | $G_{L,3}$ (mS) | $L_{L3}$ (μH) |
|---|---|---|---|---|---|
| 21.4 | 524.0 | 23.0 | 443.0 | 22.6 | 370 |

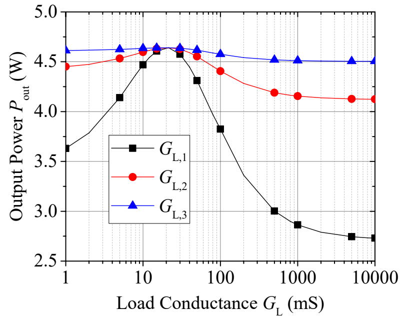


**Figure 4.** Simulated output power $P_{out}$ as function of varying load conductance for the analysed numerical example. One of the three load conductances is varied, while keeping the other two fixed at their optimal value.

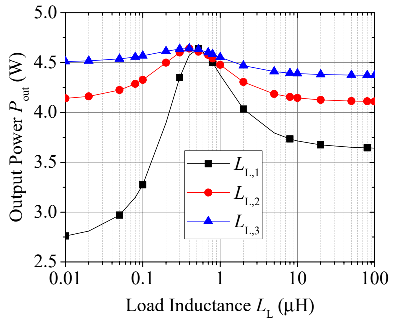


**Figure 5.** Simulated output power $P_{out}$ as function of varying load inductance for the analysed numerical example. One of the three load inductances is varied, while keeping the other two fixed at their optimal value.

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