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Chaos-based anytime reliable coded communications over fading channels with and without information aging

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Abstract

We show that two chaos-coded modulation schemes, that are proved to be anytime reliable on the AWGN channel, can also work in time-varying fading channels. In particular, we devise a transmission scheme that takes advantage of transmitter channel-state information and show by simulation that, whenever such CSI undergoes information aging, the schemes can lose their anytime reliability property.

1 Introduction

In the context of automatic control, it may happen that the measurement sensor and the controller are not physically co-located, and communicate through a wireless channel. In such a scenario channel encoding may be required in order to defend against communication errors. In order for the control to be successful, the encoding-decoding scheme must satisfy the property of *anytime reliability* (AR) [1]. Loosely speaking, an encoding-decoding scheme is said to be AR if its bit error probability decreases exponentially with the decoding delay d, i.e., it goes down as $\mathrm{e}^{-\beta d}$, where $\beta > 0$ is the *anytime exponent* of the scheme.

The basic structure of an AR coding scheme is well represented by an infinite-memory, ever-growing convolutional trellis. This fact, together with the continuous character of the information to be transmitted (the measurements), allows thinking that possible solutions for such a scenario could spring from the realm of chaotic systems. In [2], two schemes based on chaos-coded modulations [3] have been proposed. The first, called adaptive-size chaos-based coded modulation (AS-CCM), has fixed spectral efficiency and adaptive instantaneous power, while the second, adaptive-bandwidth CCM (AB-CCM), has adaptive spectral efficiency and instantaneous power. Provided that some conditions on the scheme parameters are met, the two schemes are shown in [2] to be AR on the AWGN channel, and lower bounds to their anytime exponents are obtained.

In this paper, we consider the two schemes of [2] on time-varying Rice-fading channels. By simulations, we show that using channel-state information (CSI) at the receiver, under a sufficiently high SNR and Rice factor, is enough for maintaining the AR property for both schemes. However, for small Rice factors, the schemes become unreliable, with AS-CCM especially prone to errors. In order to improve the performance, we consider a scheme with CSI feedback to the transmitter, and a transmission strategy that can be provably anytime reliable with only a constant increase of transmitted power. Finally, we show that, if CSI is fed back only once in a while, and the channel changes fast enough between two CSI updates, information aging becomes harmful and spoils the good property of the proposed CCM schemes.

The structure of the paper is as follows. In Section 2, we briefly describe the two CCM schemes proposed in [2], i.e., AS-CCM and AB-CCM. In Section 3, we provide simulation results under different scenarios of CSI usage, with and without information aging. Finally, Section 4 draws some conclusions.

2 Anytime reliable CCM schemes

We consider in this paper a discrete-time chaotic map, defined as a nonlinear map $f : [0,1] \to [0,1]$. We define the *invariant cdf* F_f of the map as the one which is preserved by the map itself, i.e., if ξ is a R.V. satisfying $\mathbb{P}\{\xi \leq x\} = F_f(x)$, then also $\mathbb{P}\{f(\xi) \leq x\} = F_f(x)$.

2.1 Adaptive-size CCM

For a given map f, AS-CCM is a CCM scheme able to encode a semiinfinite binary information sequence $\mathbf{b} = (b_1, b_2, \dots)$. Let us define a mapper $\mathscr{M}_f$ that maps semiinfinite binary sequences into $[0,1]$. For a map f with nonuniform invariant density, we choose $\mathscr{M}_f(\mathbf{b}) = F_f^{-1}\left(\mathscr{M}_{f_u}(\mathbf{b})\right)$, where f_u is the map with uniform invariant density that is topologically conjugate with f. Examples of mappers for several maps are given later.

Ideally, given $\mathbf{b}$, the transmitted symbols are obtained by quantization of chaotic samples $z_1, z_2, \dots$ as follows. First,

$$z_n = \begin{cases} \mathscr{M}_f(\mathbf{b}), & n = 1 \\ f^{(\delta_n)}(z_{n-1}), & n > 1 \end{cases}, \tag{1}$$

where δ_n is a nonnegative integer and $f^{(\delta_n)}$ means that the chaotic map is applied δ_n times. Then the quantized chaotic samples are obtained as

$$z_n^Q = F_f\left(\mathscr{Q}_U^{(q_n)}\left(F_f^{-1}(z_n)\right)\right), \tag{2}$$

where $\mathscr{Q}_U^{(q_n)}$ is a uniform quantizer on $[0,1]$ with 2^{q_n} levels. In practice, by a suitable choice of the mapper, the n-th quantized sample can be written as

$$z_n^Q = \mathscr{M}_f\left([\mathbf{b}_{\varepsilon_n}^n, 0, 0, \ldots]\right), \tag{3}$$

where ε_n is the oldest bit to be encoded at time n ($\varepsilon_1 = 1$) and $q_n = n - \varepsilon_n + 1$, $\delta_n = \varepsilon_n - \varepsilon_{n-1}$. See [2] for more details.

For the Bernoulli shift map (BSM), natural mapping, i.e., $\mathscr{M}_{\text{BSM}}(\mathbf{b}) = \sum_{n=1}^{\infty} b_n 2^{-n}$ is suitable for the scheme. For the tent map, Gray mapping is needed, as per [2, Eqn. (13)].

The symbol to be transmitted is denoted s_n, which is a scaled and zero-mean version of z_n^Q. Let $\mathbf{r}_1^n$ be the received sequence of samples. We suppose that the receiver implements optimal ML detection of $\mathbf{b}$. We also make the assumption that the receiver is able to discriminate the bits which are reliably decoded from the bits whose decoding is still unreliable and that it conveys to the transmitter, through a noise-free dedicated feedback channel, the index ε_n of the oldest information bit which is not yet reliably decoded.

It is shown in [2] that, if the instantaneous power of the transmitted sample increases exponentially with q_n and the inverse invariant density F_f^{-1} has a derivative bounded away from zero, AS-CCM is anytime reliable on the AWGN channel for a sufficiently low channel noise variance. AR for the logistic map is verified by simulations, since such map does not match the condition on F_f^{-1}.

2.2 Adaptive-bandwidth CCM

In AB-CCM, given a map f, we first choose a pair of initial conditions $z^{(0)}, z^{(1)} \in [0,1]$. Then, at step n, we transmit a vector symbol

$$\mathbf{s}_n = \left(s_n^1, \cdots, s_n^{q_n}\right) \tag{4}$$

through q_n parallel orthogonal channels. The i-th subsymbol at time n is given by

$$s_n^i = \begin{cases} f\left(s_{n-1}^{i-1}\right), & i > 1 \\ f\left(z^{(b_n)}\right), & i = 1 \end{cases}, \tag{5}$$

where $q_n = n - \varepsilon_n + 1$, ε_n being as before the oldest bit to be encoded at time n. Once again, the ML receiver feeds back the index ε_n, corresponding to the oldest bit that has not yet been reliably decoded. It is shown in [2] that, if the initial conditions $z^{(0)}, z^{(1)}$ are chosen properly depending on the map, AB-CCM is anytime reliable on the AWGN channel, irrespective of the channel noise variance.

3 CCM schemes on the fading channel

If the above CCM schemes are used on a fading channel, then for AS-CCM the received sample at time n, $n = 1, 2, \ldots$, will be given by

$$r_n = \alpha_n s_n + w_n, \tag{6}$$

where α_n is the fading coefficient and w_n is the AWGN sample with power σ^2. For AB-CCM, instead,

$$r_n^{(i)} = \alpha_n^{(i)} s_n^{(i)} + w_n^{(i)}, \ i = 1, \ldots, q_n. \tag{7}$$

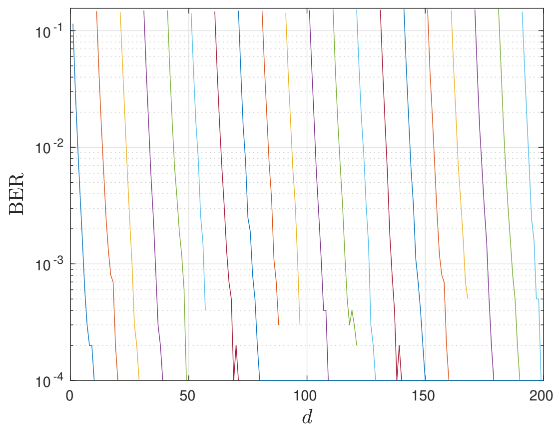

Figure 1. BER as a function of the decoding delay with perfect CSI at the receiver: AS-CCM case. Channel parameters: $\sigma^2 = 0.5$, $K = 7$ dB, and $f_{max,d} = 10^{-2}$ Hz.

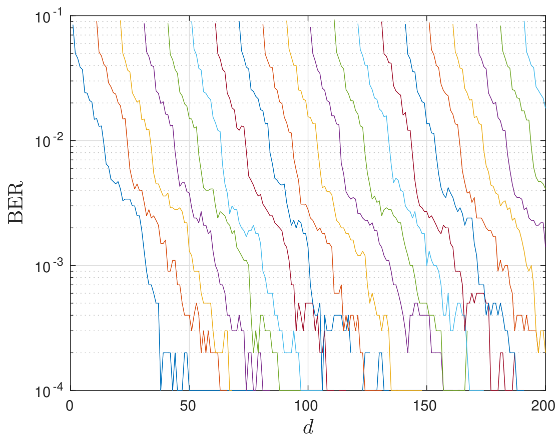

Figure 2. BER as a function of the decoding delay with perfect CSI at the receiver: AB-CCM case. Channel parameters: $\sigma^2 = 0.25$, $K = 7$ dB, and $f_{max,d} = 10^{-2}$ Hz.

We assume in this paper time-varying Rician fading, parameterized by the factor K (ratio between the power in the LoS path and the power in the scattered paths) and by the maximum Doppler shift $f_{max,d}$. Notice that, for AB-CCM, we will always assume in the following flat fading, i.e., $\alpha_n^{(i)} = \alpha_n$, for $i = 1, \ldots, q_n$.

In the following subsections, we consider first the case in which there is perfect channel state information (CSI) only at the receiver. Then, we pass to the case in which such CSI is available also at the transmitter, possibly because it is communicated by the receiver through the feedback channel. Finally, we consider the case where the receiver only feeds back CSI only every B channel uses, to see the effect of information aging on the performance of the CCM schemes. Without loss of generality, all the tests will be made with the BSM [2] for illustrative purposes.

3.1 Perfect receiver CSI

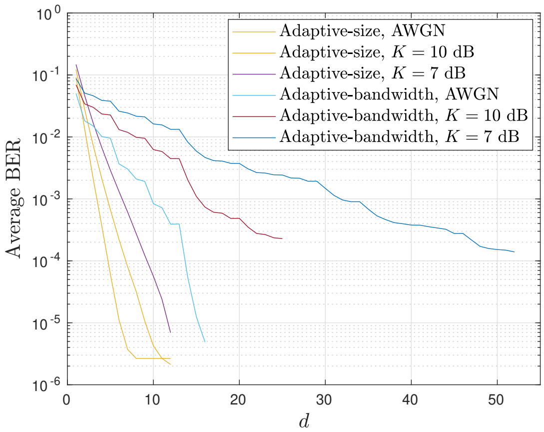

Figure 3. Measured average BER when there is perfect CSI at the receiver for the adaptive-size ($\sigma^2 = 0.5$) and adaptive-bandwidth ($\sigma^2 = 0.25$) schemes, for different K values and $f_{max,d} = 10^{-2}$ Hz. AWGN results are also given.

In this section, we consider the case in which the receiver has perfect CSI and implements an optimal Viterbi-based decoding scheme that takes into account such CSI. In particular, Figure 1 shows the performance of AS-CCM in the case of a Rician channel with parameter $K = 7$ dB, which varies due to relative motion between transmitter and the receiver. Precisely, the Doppler frequency shift is set to $f_{max,d} = 10^{-2}$ Hz. Finally, the noise variance is $\sigma^2 = 0.5$. The figure shows the behavior of the bit error probability of some bits within the simulated block as a function of the decoding time step. All curves show a rapid decay with the increasing delay, certifying that the AR property is preserved.

Figure 2 shows a similar plot for AB-CCM. The noise variance is now $\sigma^2 = 0.25$, while all the other parameters are the same. Again, the decrease with the decoding delay shows that AB-CCM is AR in this scenario. In Figure 3, we show the BER as a function of the decoding delay, for both schemes, with different Rice parameters, averaged with respect to the bit position in the block. Although the slope decreases with decreasing K, the exponential decrease of the average BER certifies the AR property.

3.2 Perfect transmitter CSI

While AR is achieved with just receiver CSI only for relatively large Rice factors, for smaller values, we can imagine that the receiver feeds back the CSI to the transmitter. In the case of perfect transmitter CSI, we can devise the following transmit strategy: given a fixed threshold value α_{th}, if $\alpha_n \geq \alpha_{\text{th}}$, the channel is inverted at the transmitter, i.e., the transmitted symbol is $\alpha_n^{-1} s_n$, while if $\alpha_n < \alpha_{\text{th}}$, the transmitted symbol is $\alpha_{\text{th}}^{-1} s_n$. It can be seen that the average transmitted power will be raised by a factor of at most α_{th}^{-2}.

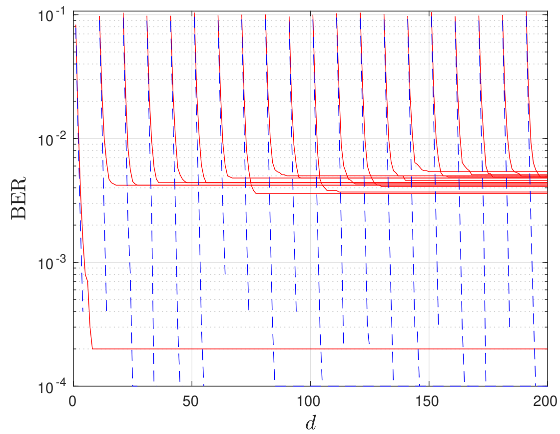

Figure 4. BER as a function of the decoding delay with perfect CSI at the trasmitter, when $\alpha_{th} = 0.1$ (red continuous line) and $\alpha_{th} = 10^{-8}$ (blue dashed line): AS-CCM. Channel parameters: $\sigma^2 = 0.5$, $K = 3$ dB, and $f_{max,d} = 10^{-3}$ Hz.

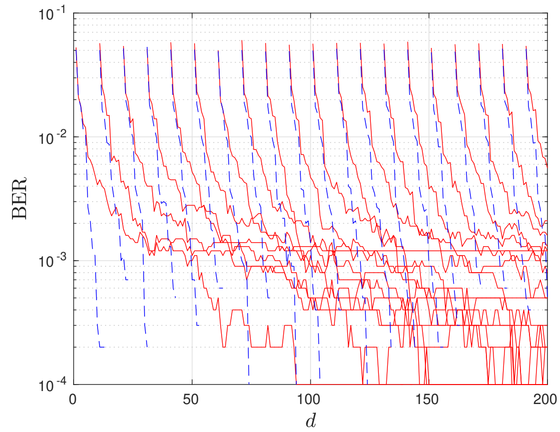

Figure 5. BER as a function of the decoding delay with perfect CSI at the trasmitter, when $\alpha_{th} = 0.1$ (red continuous line) and $\alpha_{th} = 10^{-8}$ (blue dashed line): AB-CCM. Channel parameters: $\sigma^2 = 0.25$, $K = 3$ dB, and $f_{max,d} = 10^{-3}$ Hz.

A simple analysis extending the proof in [2] shows that, with this strategy, AB-CCM is still AR. It is possible to show that AR still holds also for AS-CCM *provided that the received SNR is large enough*. Let $\pi = \Pr\{\alpha_n < \alpha_{\text{th}}\}$ and let γ be the anytime exponent of the CCM scheme on the AWGN channel. Then, for both schemes, the anytime exponent on the fading channel with transmit CSI and the strategy described above can be bounded as follows

$$\gamma' \geq -\log\left((1-\pi)e^{-\gamma}+\pi\right). \qquad (8)$$

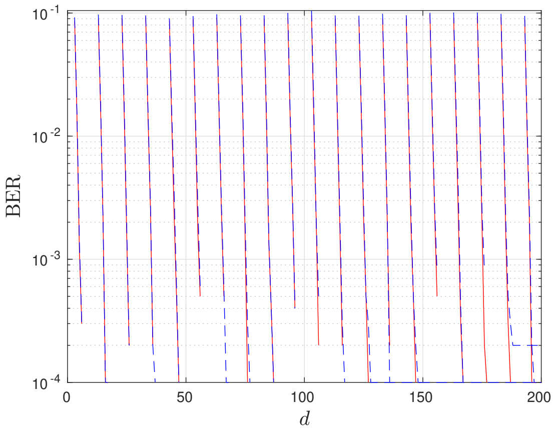

Figure 6. BER as a function of the decoding delay when there is CSI with information aging at the trasmitter with $\alpha_{th} = 0.1$: AS-CCM. Channel parameters: $\sigma^2 = 0.5$, $K =$ 10 dB, and $f_{max,d} = 10^{-2}$ Hz. Red continuous line: $B = 1$ (no mismatch). Blue dashed line: $B = 9$.

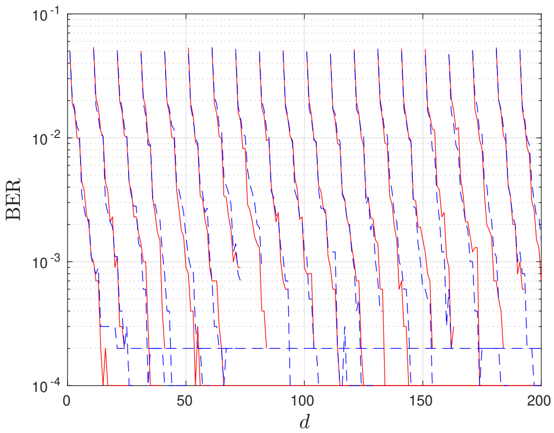

Figure 7. BER as a function of the decoding delay when there is CSI with information aging at the trasmitter with $\alpha_{th} = 0.1$: AB-CCM. Channel parameters: $\sigma^2 = 0.25$, $K =$ 7 dB, and $f_{max,d} = 10^{-2}$ Hz. Red continuous line: $B = 1$ (no mismatch). Blue dashed line: $B = 4$.

In Figures 4 and 5, we can see some results for both schemes. It is to be noted that the results with $\alpha_{th} = 0.1$ ascertain that the systems are no longer anytime reliable, while, when $\alpha_{th} = 10^{-8}$, the results are practically the same as under AWGN. This examples illustrate the importance of choosing an appropriate value for α_{th}: it cannot be too low to keep the power bounded, but it cannot be too high with respect to the typical faded values if we want to keep the AR property. Though equation (8) points towards the idea that the AR property should hold, the fact is that, when the channel is in deep fade, for the parameters K and $f_{max,d}$ chosen, the channel gains keep very low values during long bursts, and therefore the effective SNR is so low (and the effective noise power is so high) that the AR property cannot be met.

3.3 Perfect transmitter CSI with information aging

In this section, we consider the case in which CSI at the transmitter is used to invert the channel, as in the previous section. However, we make the further assumption that, to decrease the transmission burden on the feedback channel, CSI is fed back from the receiver to the transmitter only every B time steps, say at time steps iB, $i = 0, 1, 2 \ldots$. Since the channel changes in time, the information about the channel gets old in between two consecutive CSI transmissions, and channel inversion becomes mismatched.

To highlight the effect of information aging, we show some illustrative results in Figures 6 and 7. In the case of AS-CCM, for the parameters chosen ($\sigma^2 = 0.5$, $K = 10$ dB, and $f_{max,d} = 10^{-2}$ Hz), the system is anytime reliable up to $B = 9$, where, as shown in Figure 6, there are bits that cannot be correctly decoded. From $B = 1$ to $B = 8$, the system offers uniform performance in terms of BER and somehow resists information aging. For AB-CCM, as shown in Figure 7, we show a similar situation. For the set of simulated parameters ($\sigma^2 = 0.25$, $K = 7$ dB, and $f_{max,d} = 10^{-2}$ Hz), the system is anytime reliable from $B = 1$ to $B = 3$.

4 Conclusions

In this article we have shown that two chaos-based AR systems, proved to be efficient in AWGN, can also work in typical fading channels, when there is CSI either at the RX or the TX. In this latter case, we have also verified how, in presence of CSI information aging, the AR property is still preserved, provided that the aging is kept within bounds. Further studies will cast the theoretical grounds to formally characterize the effects of information aging in this context.

References

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