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Finding the polarizability of radially anisotropic multilayer circular cylinder

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Abstract

In this article, we have studied the electrostatic response of a Polarly Radially Anisotropic (PRA) multilayer circular cylinder. It consists of different components of the permittivity in radial and tangential directions for each layer. We have considered the familiar model of quasi-static for obtaining the polarizability and effective permittivity of the PRA multilayer circular cylinder. We also have studied the behavior of polarizability of PRA multilayer circular cylinder, as a function of the number of anisotropic alternating layers by using a numerical approach.

1 Introduction

The electromagnetic scattering by a multilayer anisotropic circular cylinder has been actively studied research topic in recent past [1, 2]. More often, the anisotropy has been considered with respect to the Cartesian coordinates using cylindrical geometry, and its scattering applications from (plasmonic) PRA circular cylinders have been computed [3]. Henrik et. al. reported that the PRA circular cylinder has advanced application in plasmonic cloaking such that the cloaked object has been covered by PRA cylindrical shell [4].

We have continued with the study of anisotropic cylindrical geometry and introduced an approximate quasi-static approach based on anisotropic permittivity. Dyadic representation of radially anisotropic permittivity is [5].

$$\overline{\overline{\varepsilon}} = \varepsilon_0 \left[\varepsilon_{rad}\mathbf{u}_{rad}\mathbf{u}_{rad} + \varepsilon_{tan}(I - \mathbf{u}_{rad}\mathbf{u}_{rad})\right] \tag{1}$$

where ε_0 is the permittivity of free space, ε_{tan} and ε_{rad} represents the relative tangential and radial components of permittivity, respectively. I is the unit dyadic, and $\mathbf{u}_{rad}$ represents the unit vector in the radial direction. In our manuscript, we have presented the PRA multilayer circular cylinder and calculated mathematical expression of polarizability and effective permittivity.

2 Formation

Let us consider a PRA multilayer circular cylinder with radius a_k and alternative sequence of relative permittivity ε_1, ε_2, immersed in the free space, that is presented in Fig. 1. The cylinder may have an arbitrary axial permittivity component ε_z, but since the axial electric field has not been excited, so this component has been omitted in our analysis. The radius of the outer cylinder is fixed and equal to a_1 and the internal radii may be written as [6, 7, 8]

$$a_k = \frac{N-(k-1)}{N} a_1 \tag{2}$$

The number of layers, N is arbitrary and $k = 1, 2, 3......N$.

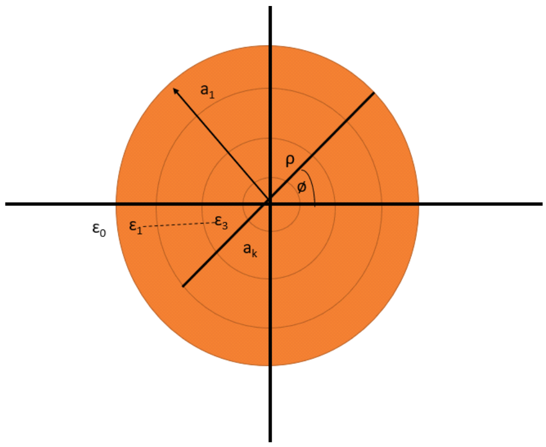

Figure 1. Geometry represents the PRA multilayer circular cylinder immersed in a free space.

To solve for the electric potential, the geometry has been excited by the electric field $\mathbf{E} = E\hat{\mathbf{x}}_0$ and Laplace's equation has been solved for cylindrical coordinates. The solution to the Laplace's equation in an arbitrary k_{th} subregion is as follows [9]

$$\Phi_k = B_k \rho^{\gamma} \cos\phi + C_k \rho^{-\gamma} \cos\phi \tag{3}$$

where the coefficient B_k represents the amplitude of the constant field component whereas the coefficient C_k describes the dipole-type contribution to the field. To solve the unknowns coefficients B_k and C_k, transmission-line method has been used [10]. The boundary conditions applied between any two adjacent subregions k and $k+1$ can be written as

$$\begin{pmatrix} B_k \\ C_k \end{pmatrix} = [P_k] \begin{pmatrix} B_{k+1} \\ C_{k+1} \end{pmatrix} \tag{4}$$

where

$$[P_k] = \frac{1}{2\gamma_k \varepsilon_k} \begin{pmatrix} P_{k11} & P_{k12} \\ P_{k21} & P_{k22} \end{pmatrix} \tag{5}$$

and:

$$\begin{aligned} P_{k11} &= (\gamma_k \varepsilon_k + \gamma_{k+1}\varepsilon_{k+1})\, a_{k+1}^{\gamma_{k+1}-\gamma_k} \\ P_{k12} &= (\gamma_k \varepsilon_k - \gamma_{k+1}\varepsilon_{k+1})\, a_{k+1}^{-\gamma_{k+1}-\gamma_k} \\ P_{k21} &= (\gamma_k \varepsilon_k - \gamma_{k+1}\varepsilon_{k+1})\, a_{k+1}^{\gamma_{k+1}+\gamma_k} \\ P_{k22} &= (\gamma_k \varepsilon_k + \gamma_{k+1}\varepsilon_{k+1})\, a_{k+1}^{-\gamma_{k+1}+\gamma_k} \end{aligned}$$

It is easily verifiable, when $\gamma_k = \gamma_{k+1} = 1$, the above expression is reduced into the case of isotropic multilayer circular cylinder.

When we have considered all layers, we reached the following explicit relationship

$$\begin{pmatrix} B_0 \\ C_0 \end{pmatrix} = \prod_{k=0}^{N-1} [P_k] \begin{pmatrix} B_N \\ C_N \end{pmatrix} = [P] \begin{pmatrix} B_N \\ 0 \end{pmatrix} \tag{6}$$

with

$$[P] = \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} \tag{7}$$

As we know, that there is no reflected field component in the core region, i.e., $C_N = 0$. Here, the expression of polarizability is [9]

$$\alpha_P = \frac{2V\varepsilon_0}{a_1^2} \frac{P_{21}}{P_{11}} \tag{8}$$

where, ε_0 is the vacuum's permittivity and V represents the volume of the cylinder, i.e., $V = \pi r^2 h$. This also allows us to obtain the expression of an effective permittivity for multilayer PRA circular cylinder

$$\varepsilon_{eff} = \varepsilon_0 + \frac{\frac{\alpha_P}{V}}{1 - \frac{\alpha_P}{2\varepsilon_0 V}} \tag{9}$$

The polarizability of PRA circular cylinder in free space can be computed by taking $N = 1$; for this trivial case we have:

$$\alpha_P = 2V\varepsilon_0 \frac{(\gamma_0\varepsilon_0 - \gamma_1\varepsilon_1)}{(\gamma_0\varepsilon_0 + \gamma_1\varepsilon_1)} \tag{10}$$

For the case $N = 2$, the polarizability of two concentric PRA circular cylinders can be derived.

$$\alpha_P = 2V\varepsilon_0 \frac{(\gamma_0\varepsilon_0 - \gamma_1\varepsilon_1)(\gamma_1\varepsilon_1 + \gamma_2\varepsilon_2) + (\gamma_0\varepsilon_0 + \gamma_1\varepsilon_1)(\gamma_1\varepsilon_1 - \gamma_2\varepsilon_2)\left(\frac{a_2}{a_1}\right)^{2\nu_1\gamma}}{(\gamma_0\varepsilon_0 + \gamma_1\varepsilon_1)(\gamma_1\varepsilon_1 + \gamma_2\varepsilon_2) + (\gamma_0\varepsilon_0 - \gamma_1\varepsilon_1)(\gamma_1\varepsilon_1 - \gamma_2\varepsilon_2)\left(\frac{a_2}{a_1}\right)^{2\nu_1\gamma}} \tag{11}$$

Similarly, we can find the polarizability of multilayer PRA circular cylinder consisting of an arbitrary number of layers.

3 Numerical Results and Discussion

Numerical results for the polarizability of a multilayer PRA circular cylinder with respect to the number of layers and placed in free space have been described in this section. For this purpose equation (8)-(11) have been implemented using Matlab code. We have fixed the following parameters; radius of the inner core= $a_2 = 0.70$, outer shell $a_1 = 1$, the anisotropy ratio $\gamma_0 = 2$. For all cases the anisotropy ratio is $\gamma = \{3, 4\}$ such that for two concentric PRA cylindrical layers $\gamma_1 = 3$ and $\gamma_2 = 4$ and vice versa. Similarly, values of the permittivity are $\varepsilon = \{2, 4\}$: for all cases, two concentric PRA cylindrical layers with $\varepsilon_1 = 2$ and $\varepsilon_2 = 4$ vice versa.

In Fig. 2 we can see that for a variable number of layers, the value of the polarizability when the permittivity of the cover layer $\varepsilon_2 = 4$ has more weight as compared to the case when permittivity of cover layer is $\varepsilon_2 = 2$. Similarly, for the case of two layers, the same trend is observed for cover layers with permittivity $\varepsilon_2 = 4$ and $\varepsilon_2 = 2$ respectively. The polarizability of isotropic multilayer circular cylinder placed in free space, by inserting anisotropic ratios $\gamma_0 = \gamma_1 = \gamma_2 = 1$ is shown in Fig. 3.

During our numerical test, we have observed polarizability of isotropic multilayer circular cylinder and PRA multilayer circular cylinder with arbitrary number of layers by using two alternating values of anisotropic permittivity. We are hopeful that our improved approach will help researchers in developing applications. We will continue this model of PRA multilayer circular cylinder for cloaking application.

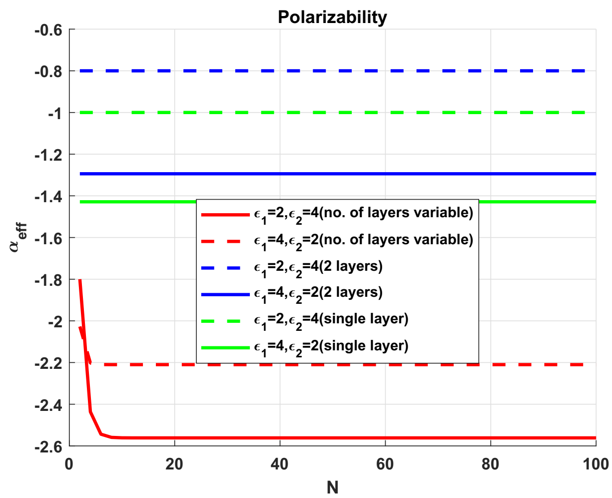

Figure 2. Normalized polarizability of PRA circular cylinder as a function of the number of layers, with the following parameters: ε_1=2; ε_2=4; γ_0=2, γ_1=3 and γ_2=4.

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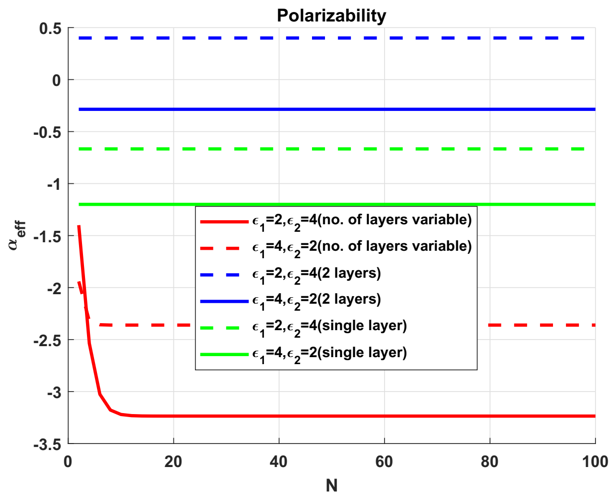

Figure 3. Normalized polarizability of an isotropic circular cylinder as a function of the number of layers, with the following parameters: ε_1=2; ε_2=4; $\gamma_0 = \gamma_1 = \gamma_2 = 1$.

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