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Rumsey's Reaction Generalized

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This paper is dedicated to the memory of Professor Jean Van Bladel (1922–2018).

The concept of reaction between two electromagnetic sources was introduced by V.H. Rumsey in 1954 as "a physical observable like mass, length, charge, etc." [1]. Assuming two sets of monochromatic time-harmonic electric and magnetic current sources $\mathbf{J}^a_{eg}, \mathbf{J}^a_{mg}$ and $\mathbf{J}^b_{eg}, \mathbf{J}^b_{mg}$, the reaction of sources b on the sources a through the fields $\mathbf{E}^b_g, \mathbf{H}^b_g$ created by the sources b was defined by

$$< ab >= \int_{V_a} R^{ab} dV, \quad R^{ab} = \mathbf{J}^a_{eg} \cdot \mathbf{E}^b_g - \mathbf{J}^a_{mg} \cdot \mathbf{H}^b_g. \tag{1}$$

where R^{ab} is the scalar-valued reaction density. The boldface quantities with subscript $()_g$ denote Gibbsian vectors to distinguish them from the corresponding one-form and two-form quantities below.

On the other hand, the force exerted by the electric field b on the electric charge a was defined by Rumsey in terms of a vector-valued reaction as $(ab) = \int_{V_a} \mathbf{E}^b_g \rho^a_e dV$ [1].

Over the years following its introduction, the scalar-valued reaction concept (1) has found application in solving numerous electromagnetic problems. For example, impedance parameters of multiport networks, resonant frequencies of cavities, cut-off frequencies of waveguides, input impedances of antennas and scattering cross sections of obstacles could be shown to be proportional to reaction quantities, which helped finding simple numerical solutions to practical problems.

The topic of the present talk is to generalize the reaction concept so that the scalar and vector reaction concepts would fall under the same definition. This can be accomplished by applying the four-dimensional formalism. Following the notation introduced by Deschamps [2, 3], the basic field quantities are the two two-forms

$$\Phi = \mathbf{B} + \mathbf{E} \wedge \varepsilon_4, \quad \Psi = \mathbf{D} - \mathbf{H} \wedge \varepsilon_4, \tag{2}$$

and the basic source quantities are the two three-forms

$$\gamma_e = \rho_e - \mathbf{J}_e \wedge \varepsilon_4, \quad \gamma_m = \rho_m - \mathbf{J}_m \wedge \varepsilon_4. \tag{3}$$

Here, ε_4 denotes the temporal basis one-form, while $\mathbf{B}, \mathbf{D}, \mathbf{J}_e, \mathbf{J}_m$ are spatial two-forms, $\mathbf{E}, \mathbf{H}$ are spatial one-forms and ρ_e, ρ_m are spatial three-forms.

Expressing the scalar reaction density of (1) in terms of 4D field and source quantities and requiring that the concept be independent of any particular choice of basis, one can show that the scalar reaction must actually be generalized to a one-form reaction concept. While the temporal component of the reaction one-form yields the scalar reaction by Rumsey, its spatial component combines the four force terms between electric and magnetic sources and electric and magnetic fields in one expression, which can be interpreted in terms of a corresponding Gibbsian vector reaction.

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References

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[3] I.V. Lindell, *Differential Forms in Electromagnetics*, Hoboken NJ: Wiley and IEEE Press, 2004.