Relaxation of some non-convex constraints for Q-factor optimization

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1 Extended Abstract

Design requirements of antennas often come with a range of goals and limitations on antenna performance like gain, efficiency, impedance bandwidth, and front-to-back ratio, under size and material constraints. To a-priori exclude incompatible goals already on the planning stadium is essential. A tool that provides this service for several interesting constraints is antenna current optimization based on stored energies. Utilizing the well-known relation between fractional impedance bandwidth and the Q-factor, a range of near and far-field requirements on small antennas can be expressed as a convex optimization problem [1, 2] to predict possible bandwidths given certain constraints. Most Q-factor related current optimizations are not convex problems, but rather a quadratically-constrained quadratic-programming (QCQP) type of problem. The class of QCQP contains NP-hard problems but also problems that are convex.

As an example, investigations of the partial directivity over Q-factor can be formulated as a convex problem for increasing demands on the partial directivity. However, investigating the Q-factor with constraint on the total-directivity has not been formulated as a convex problem. Similarly, adding the requirement of that the antenna should be tuned, or to have a given front-to-back ratio turns the minimization of the Q-factor into a non-convex problem. There are at least two different methods to relax lower bounds on the Q-factor, and more generally QCQP-problems to convex problems. One is the semi-definite relaxation method (SDR) see e.g. [3] and another is the reformulation linearization technique (RLT). The essential idea in SDR is to reformulate our unknown current, I, in the antenna optimization problems which often appears quadratically into a linear constraint. Consider for example a given radiation power which is a constraint of the type $I^H RI = 1$, where R is a given Hermitian positive semi-definite matrix. By the use of the identity $I^H RI = \text{tr}(RII^H)$, and defining $K = II^H$ we obtain the constraint $\text{tr}(RK) = 1$ with the condition rank $K = 1$. The relaxation is to drop the rank-condition. The SDR-method applies with small modifications to general quadratic forms, which allows us to investigate this larger class of antenna current optimization problems.

We have implemented antenna current optimization in particular with respect to Q-factor bounds given a total directivity. The SDR or trace-type optimization bounds are rank one for the tested cases as long as the directivity is large enough. For low directivities we find solutions with rank $K = 2$ that satisfy the constraints. This gives us a small solution space in which to find feasible minimizers. Similarly, constraints on the front-to-back ratio also exhibit solutions with rank one in the investigated cases. In the presentation we will show numerical examples comparing SDR with a non-linear heuristic solver. It is interesting to note that the SDR-problem and the heuristic solver largely show similar or identical behavior for the investigated cases, but that the SDR-method is faster.

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