Analysis, Design, and Sensitivity Study of Substrate Integrated Waveguide Circuits by Using Equivalent Circuit Models

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Abstract

The design of complex substrate integrated waveguide (SIW) circuits requires the use of sophisticated full-wave simulation tools. When the complexity of the circuit increases, the computational effort required for the analysis and especially for the optimization of complex SIW circuits becomes prohibitive. For this reason the use of equivalent circuit models is very beneficial. This paper presents the derivation of parametric and multimodal equivalent circuit models of SIW discontinuities, based on the boundary integral-resonant mode expansion (BI-RME) method. To proof the validity of the models, the use of these equivalent circuits for the modeling and design of some SIW circuits, as well as for fast sensitivity study, is presented.

1. Introduction

There is an ever growing interest in the last years for the development of components and systems based on the SIW technology, in a variety of frequency bands, ranging from few GHz to hundreds of GHz [1,2]. SIW structures are integrated waveguide-like structures fabricated by using two rows of metal vias or slots in a grounded dielectric substrate (Fig. 1). The electromagnetic characteristics of SIW structures are similar to the standard metallic waveguide, but SIW technology presents the advantage of low fabrication cost, small weight and size, and easy integration with planar transmission lines and with active devices. Due to the large design flexibility, SIW technology allows to realize complex circuits and complete systems on a single dielectric substrate, according to the so-called Systems on Substrate (SoS) approach [3,4].



Fig. 1.Substrate integrated waveguide structure.

The potential of SIW technology can be fully exploited in the design of microwave and mm-wave components, provided that flexible, fast, and reliable full-wave electromagnetic solver are available. Different analysis techniques have been proposed for the modeling of SIW structures in the past decade, based on the application of the finite-difference time domain (FDTD) method [5], of the finite-difference frequency domain (FDFD) method [6], and of the numerical calibration technique [7]. These modeling techniques are very flexible, and allow the analysis of complex structures with arbitrary geometry. A particularly efficient technique for the modeling and design of SIW components and circuits is based on the boundary integral-resonant mode expansion (BI-RME) method [8]. The BI-RME method provides the frequency response of SIW components through the generalized admittance matrix, expressed in the form of a pole expansion in the frequency domain. For this reason, the BI-RME method allows a broadband characterization of SIW components by a single electromagnetics analysis and, consequently, it proved to be very fast.

However, when the complexity of the circuit increases, the computational burden becomes prohibitively large for any full-wave computational technique. A convenient solution to overcome this problem is based on the segmentation technique of the circuit into elementary building blocks and the use of equivalent circuit models. This approach leads to a fast and accurate analysis of SIW circuits, if high quality equivalent models are adopted. The use of equivalent circuit models can result extremely useful for optimization purposes, where several repeated analyses are required. Furthermore, SIW equivalent models find another useful application in the case of sensitivity analysis, to assess the suitability of a manufacturing process or to estimate the fabrication yield in the case of mass production.

This paper presents an outline of the BI-RME method for the full-wave modeling of arbitrary SIW components, and the derivation of multimodal and parametric equivalent circuit models of SIW discontinuity, directly from the results of the BI-RME method, without any initial guess or fitting procedure. Finally, the application of equivalent circuit models to the analysis, design, sensitivity study of SIW circuits is discussed.

2. Full-wave Modeling of SIW Circuits by the BI-RME Method

The BI–RME method, originally developed for the modeling of planar waveguide components, has also been applied to the analysis of SIW interconnects and components [8]. Since the BI–RME method applies to completely shielded structures, and SIW components are semi–unbounded structures (Fig. 1), it is needed to modify the geometry of an SIW component by adding fictitious metal walls outside the circuit [8]. It has been proved that, when radiation leakage is negligible, the effect of the fictitious metal walls is very small, and they practically do not modify the propagation characteristics of the SIW structures or the frequency response of the SIW components.

In the case of lossless structures, the formulation of the admittance matrix of the circuit can be expressed by the following equation

$$\mathbf{Y}(\omega) = \frac{1}{j\omega}\mathbf{A} + j\omega\mathbf{B} + j\omega^{3}\mathbf{C}\left(\mathbf{\Omega}^{4} - \omega^{2}\mathbf{\Omega}^{2}\right)^{-1}\mathbf{C}^{T}$$
(1)

where $\omega = 2\pi f$ is the angular frequency, **A** and **B** are matrices related to the low-frequency behavior of the admittance matrix, Ω is a *M*-element diagonal matrix, whose entries are the resonance angular frequencies of the first *M* modes of the cavity obtained by short-circuiting the ports of the SIW circuit, and **C** is a matrix related to the coupling between the ports modes and the resonant cavity modes. Matrices **A**, **B**, **C** and Ω are frequency independent matrices and, therefore, the expression of the **Y** matrix in (1) exhibits an explicit dependency on the angular frequency ω . Consequently, once these matrices are known, this representation of the **Y** matrix allows to calculate the frequency response of the circuit at any frequency of interest in a negligible time, with no need of frequency–by–frequency full–wave analyses.

If conductor and dielectric losses in the SIW component are low but not negligible, formulation (1) of the admittance matrix can be modified to account for conductor and dielectric losses, by using a perturbation approach [8]. When considering a lossy SIW component with metal conductivity σ_c and filled with a dielectric medium with relative dielectric permittivity ε_r and conductivity σ_d , equation (1) is modified as follows

$$\mathbf{Y}(\boldsymbol{\omega}) = \frac{1}{j\boldsymbol{\omega}}\mathbf{A} + \frac{\sigma_d}{\varepsilon_0\varepsilon_r}\mathbf{B} + j\boldsymbol{\omega}\mathbf{B} + \boldsymbol{\omega}^2\mathbf{C}\left(\mathbf{\Omega}^3\mathbf{Q} + j\boldsymbol{\omega}\,\mathbf{\Omega}^2 - \boldsymbol{\omega}^2\mathbf{\Omega}\,\mathbf{Q}\right)^{-1}\mathbf{C}^T + j\boldsymbol{\omega}^3\mathbf{C}\left(\mathbf{\Omega}^4 + j\boldsymbol{\omega}\,\mathbf{\Omega}^3\,\mathbf{Q}^{-1} - \boldsymbol{\omega}^2\mathbf{\Omega}^2\right)^{-1}\mathbf{C}^T \quad (2)$$

where \mathbf{Q} is an *M*-element diagonal matrix, whose entries are the quality factors of the cavity modes [8]. Radiation loss can be accounted for by including in the full-wave analysis additional ports, located at the sides of the SIW circuits, which are subsequently connected to matched loads [8].

In order to validate this method, the BI-RME modeling of an SIW filter is considered [8]. The filter, shown in Fig. 2*a*, comprises three centered posts with different diameter (dimensions in mm: w=21.06, d=2, s=4, $d_1=2$, $d_2=0.5$, $s_1=14$, h=1; dielectric substrate with $\varepsilon_r=2$, $\sigma_d=0.001$ S/m, $\sigma_c=4\cdot10^7$ S/m). The scattering parameters, obtained by using the BI-RME method, are compared with those calculated with the commercial code HFSS (Fig. 2*b*). In order to better appreciate the accuracy of the proposed method, a detail of the insertion loss in the pass-band is shown in the inset of Fig. 2*b*. The required computing time was 23 sec for the wideband analysis in the entire frequency band when using the BI-RME method, and 30 sec per frequency point in the case of HFSS. It is noted that the BI-RME method provides a substantial reduction in computing time, thus representing a powerful tool for the efficient design of complex SIW circuits.



Fig. 2. Modeling of a two-pole SIW filter [8]: (a) geometry of the filter; (b) scattering parameters versus frequency (solid lines: BI-RME method; circles: HFSS).

3. Derivation of Equivalent Circuit Models of SIW Discontinuities

The modeling of SIW components by the BI-RME method permits to directly derive the layout of the equivalent circuit and the value of its components, avoiding any initial guess or fitting procedure [9,10]. To this aim, in the case of lossless SIW discontinuities, the expression of the *ij*-th entry of the **Y** matrix in (1), after extracting the term for $\omega \rightarrow \infty$ from the summation, results in

$$Y_{ij}(\omega) = \frac{1}{j\omega} A_{ij} + j\omega \left[B_{ij} - \sum_{m=1}^{M} \frac{C_{im} C_{jm}}{\omega_m^2} \right] + j\omega \sum_{m=1}^{M} \frac{C_{im} C_{jm}}{\omega_m^2 - \omega^2}$$
(3)

Expression (3) of the Y matrix can be identified as the parallel combination of an inductor, a capacitor, and M series LC-resonators. The value of the lumped elements can be analytically obtained from expression (3):

$$L_{0,ij} = \frac{1}{A_{ij}} \qquad \qquad C_{0,ij} = B_{ij} - \sum_{m=1}^{M} \frac{C_{im} C_{jm}}{\omega_m^2} \qquad \qquad L_{m,ij} = \frac{1}{C_{im} C_{jm}} \qquad \qquad C_{m,ij} = \frac{C_{im} C_{jm}}{\omega_m^2} .$$
(4)

The equivalent circuit model of a lossless two-port SIW component is shown in Fig. 3. Once each entry of the admittance matrix **Y** is represented as a network of capacitors and inductors (Fig. 3*b*), the equivalent model of the SIW discontinuity can be represented by using a π -type lumped element circuit, as discussed in [9]. The model can be extended to lossy circuit by using a similar approach, by adding a resistor in parallel to L_0 and C_0 , and replacing the *LC*-resonators with *RLC*-resonators [10].



Fig. 3. Equivalent circuit model derived directly from the BI-RME modeling: (a) π -type equivalent circuit; (b) circuit model of Y_{ij} .

A fundamental step for the real applicability of the proposed method is the determination of parametric and multimodal equivalent circuit models, where the values of the lumped elements depend on the geometrical dimensions of the component [9]. In fact, once the equivalent circuit model is available, the direct synthesis of a component can be performed in a short time by using conventional circuit CAD tools, with no need of electromagnetic full–wave analysis codes.

The parametric equivalent circuit models can be obtained by repeated analyses performed by the BI-RME method, by slightly changing some relevant geometrical dimensions. The dependency of the values of inductance and capacitance from the geometrical dimensions are expressed in the form of low-order polynomial functions, which are determined by interpolation.

4. Application to the Analysis, Design, and Sensitivity Study of SIW Circuits

The use of equivalent circuit models represents a very efficient approach for the analysis of SIW components with high complexity, as the required computing time is negligible. In general, the use of multimodal equivalent circuit models is needed, to correctly account for the effect of higher order modes that do not propagate but play a significant role whenever the circuit presents closely spaced discontinuities.

In order to validate the accuracy of this approach, a four-pole filter is considered (Fig. 4) [11]. The analysis is performed by subdividing the filter into a number of iris-type discontinuities and SIW sections. The discontinuities are modeled by lumped-element equivalent circuits, with three odd modes per port and one LC-series resonator. Fig. 4b shows the frequency response calculated by using the lossless models (gray lines) and the experimental results from [11] (black line).



Fig. 4. Four-pole SIW filter: (a) photograph of the prototype; (b) scattering parameters of the filter (gray lines: BI-RME method; black lines: measures).

The use of parametric equivalent models can be adopted in a very effective way for optimization purposes: in this case, the optimization process of SIW components and circuits can be based on circuit simulators, with no need of full-wave electromagnetic solvers. In addition, single-mode equivalent circuit models can be adopted, in conjunction with space-mapping mapping optimization routines [11]: the equivalent models are used for the coarse simulations and a commercial software is adopted for the fine simulations.

SIW equivalent circuit models find another important application in the sensitivity analysis of SIW components, to estimate the effects of manufacturing inaccuracies and the resulting fabrication yield, in the case of mass production. In this case, for the given statistical characteristics of the manufacturing process, hundreds or thousands analyses need to be performed, with an extremely high computational cost when using full-wave electromagnetic solvers. These analyses can be performed in a very short time by adopting the equivalent circuits, while preserving a very good accuracy.

5. Conclusion

The BI-RME method represents a fast and accurate technique for the full-wave electromagnetic modeling of SIW components and circuits. The use of the BI-RME method also allows the direct derivation of multimodal equivalent circuit models, which can be adopted for the analysis, optimization, and sensitivity study of complex SIW circuits.

6. References

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