Ergodic Capacity of Dual-hop Variable Gain Relaying over Mixture Gamma Fading Channels

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Abstract

The ergodic capacity of dual-hop variable gain amplify-and-forward wireless communication system is investigated over independent non-identical composite Nakagami-lognormal fading channels using mixture gamma distribution. A novel exact closed-form expression of ergodic capacity for the considered system is derived. Numerical and simulation results are shown and discussed to verify the accuracy of the analytical results under different scenarios, such as varying average signal to noise ratio and fading parameters of per hop.

1. Introduction

Cooperative relaying transmission has emerged as a promising technique for extending coverage, enhancing connectivity, and saving transmitter power in wireless communications networks. In the past few years, the ergodic capacity of various cooperative relaying systems, as an important performance metric, has been an active field of research over different fading channels, e.g., [1], [2] and the references therein.

To the best of our knowledge, no exact closed-form expression has been derived for the ergodic capacity of the dual-hop AF (amplify-and-forward) relaying transmission over composite Nakagami-lognormal (NL) fading channels. So far, Wu et al. [3] obtained an upper bound of ergodic capacity for dual-hop fixed-gain systems based on Jensen's inequality over NL fading channels using generalized-K (KG) distribution. Waqaret al.[1] proposed a general framework for analyzing the ergodic capacity of variable gain multihop relaying systems, but the presented results over identical KG fading channels is further approximated by using another Gamma–Gamma distributed random variable. The authors in [4] investigated the upper and lower bounds of the ergodic capacity for dual-hop AF relaying systems with both fixed and variable gain relaying over NL fading channels using $\mathcal{G}$ distribution. These expressions of capacity bounds still keep complicated and intractable since their probability density functions (PDFs) of average SNR (signal to noise ratio) include modified Bessel functions in the KG and $\mathcal{G}$ fading models.

Motivated by the aforementioned observations, we propose a new analytical result for the exact evaluation of the ergodic capacity of dual-hop variable gain AF relay networks over NL fading channels using Mixture Gamma (MG) distribution presented in [5]. This distribution is composed of a weighted sum of gamma distribution, and can avoid the above-mentioned problems. Some exact results obtained are possible by adjusting some parameters.

In this paper, we focus on a dual-hop variable gain AF relay networks and derive an exact closed-form expression of ergodic capacity over independent non-identical composite NL fading channels using MG distribution. With the new expression, various numerical and simulation results are shown to demonstrate the validity of the proposed analysis under different scenarios, such as varying average SNR and fading parameters of per hop.

This paper is organized as follows. Section 2 describes the system and channel models. In section 3, an exact expression of ergodic capacity is obtained. The performance evaluations and conclusions are presented in section 4 and 5, respectively.

2. System and Channel Model

We consider a wireless dual-hop AF relaying system over composite NL fading environment. The source node (**S**) communicates with the destination node (**D**) via a relaying node (**R**). The whole transmission is divided into two phase. In the first phase, **S** only transmits its signals to **R**, and in the second phase, **R** amplifies the received signal by a gain factor β and then forwards their amplified versions to **D**. Without loss of generality, we assume that the average powers of **S** and **R** are normalized to unity, and β is selected according to the instantaneous variable relay gain as $\beta^2 = 1/(|\mathrm{h}_1|^2 + N_0)$ in [6], Thus, the instantaneous end-to-end SNR, γ_{SRD}, at the destination can be expressed as in [6]

$$\gamma_{SRD} = \gamma_1\gamma_2/(\gamma_1 + \gamma_2 + 1) \tag{1}$$

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where $\gamma_i=\rho|h_i|^2$ is the instantaneous SNR of the ith-hop link, $|h_i|$ is the fading amplitude of the ith-hop link, $i\in(1,2)$, $\rho=1/N_0$ denotes the un-faded SNR, N_0 is the power of the additive white Gaussian noise (AWGN) component. Then, $\overline{\gamma_i}=\rho\mathbf{E}\left[|h_i|^2\right]=\rho\Omega_i$ denotes the average SNR of the ith-hop link, $\mathbf{E}(*)$ is the statistical expectation, Ω_i denotes the deviation of $|h_i|^2$. Due to assume the ith-hop link experiences NL fading, γ_i is a composite Gamma-lognormal distribution variable with the PDF approximated by [5]

$$f_{\gamma_i}(x)=\sum_{j=1}^{N}T_j x^{m_i-1}\exp(-M_j x) \tag{2}$$

where $T_j=c_j a_j/2\rho^{m_i}$, $M_j=b_j/\rho$, $a_j=2m_i^{m_i}w_j\exp[-m_i(\sqrt{2}\lambda_i t_j+\mu_i)]/\sqrt{\pi}\Gamma(m_i)$, $b_j=m_i\exp\left[-\left(\sqrt{2}\lambda_i t_j+\mu_i\right)\right]$, $c_j=\sqrt{\pi}/\sum_{j=1}^{N}w_j$ is the normalization factor, m_i is fading parameter in Nakagami-m fading and is integer, μ_i and λ_i are the mean and the standard deviation of lognormal shadowing, respectively, $\mu_i=\ln\Omega_i$, $\lambda_i=(\ln 10/10)\sigma_i$, σ_i denotes the standard deviation in dB. w_j and t_j are abscissas and weight factors for Gaussian-Hermite integration, w_j and t_j for different N values are available in [7, Table(25.10)].

3. Ergodic Capacity Analysis

For a dual-hop variable gain AF system with the single relay, the ergodic capacity can be obtained as [1]

$$\overline{C}=\frac{1}{2\ln 2}\mathbf{E}\left[\ln\left(1+\gamma_{SRD}\right)\right] \tag{3}$$

where the factor 1/2 accounts for the fact that the transmission process takes place in two orthogonal channels or time-slots.

Since an exact closed-form expression in (3) over MG fading channels is not mathematically tractable by directly using a traditional approach (i.e., finding the PDF of γ_{SRD}), we thus restructure (3) as [1]

$$\overline{C}=\frac{1}{2\ln 2}\{\underbrace{\mathbf{E}[\ln(1+\gamma_1)]}_{\overline{C_1}}+\underbrace{\mathbf{E}[\ln(1+\gamma_2)]}_{\overline{C_2}}-\underbrace{\mathbf{E}[\ln(1+\gamma_1+\gamma_2)]}_{\overline{C_3}}\} \tag{4}$$

Note that (4) provides an interesting information-theoretic result that states that the ergodic capacity of the dual-hop system is equal to the sum of the ergodic capacities of the source–relay link $\overline{C_1}$ and relay–destination link $\overline{C_2}$ minus the ergodic capacity $\overline{C_3}$ of the SIMO (Single Input Multiple Output) system, in which the relay acts as a transmitter, and the source and destination are the receivers. The advantage of (4) is now clear, because the methods of finding closed-form expressions for the ergodic capacities are already available in the open literature, i.e., using the PDF of γ_i. In the following, we will now derive new closed-form expressions for the ergodic capacities of the dual-hop variable-gain relay network over MG fading channels by using (4).

First, we find the closed-form expressions of $\overline{C_1}$ and $\overline{C_2}$. By using (2) and the integral expression, $I_n(\mu)=\int_0^\infty t^{n-1}\ln(1+t)\exp(-\mu t)dt=(n-1)!\exp(\mu)\sum_{k=1}^{n}\Gamma(-n+k,\mu)/\mu^k$ in [8], where $\mu>0, n=1,2,...$, $\Gamma(\bullet,\bullet)$ is the incomplete gamma function defined in [9, eq.(8.350.2)], then the closed-form expression of $\overline{C_1}$ can be written as

$$\overline{C_1}=\int_0^\infty \ln(1+x)f_{\gamma_1}(x)dx=\sum_{i=1}^{N}T_i\int_0^\infty x^{m_1-1}\ln(1+x)\exp(-M_i x)dx=\sum_{i=1}^{N}T_i I_{m_1}(M_i) \tag{5}$$

Similarly, the closed-form expression of $\overline{C_2}$ can be written as

$$\overline{C_2}=\int_0^\infty \ln(1+x)f_{\gamma_2}(x)dx=\sum_{j=1}^{N}T_j I_{m_2}(M_j) \tag{6}$$

Then, we find the closed-form expression of $\overline{C_3}$. Here, we let $z=\gamma_1+\gamma_2$. By using (2), the PDF of variable z can be obtained as

$$
\begin{aligned}
f_z(z) &= \int_0^z f_{\gamma_1}(x) f_{\gamma_2}(z-x)\,dx \\
&= \sum_{i=1}^{N}\sum_{j=1}^{N} T_i T_j \exp\left(-M_j z\right) \int_0^z x^{m_1-1}(z-x)^{m_2-1} \exp\left[-x\left(M_i - M_j\right)\right] dx
\end{aligned}
\tag{7}
$$

For (7), we consider two cases. One case is $M_i \neq M_j$, the other case is $M_i = M_j$. For the first case, the corresponding scenarios is that the fading parameters of per hop are independent and non-identical distribution, i.e., $m_1 \neq m_2$, or $\mu_1 \neq \mu_2$, or $\lambda_1 \neq \lambda_2$. Whereas, the other is that the fading parameters are independent and identical distribution, i.e., m_1=m_2, μ_1=μ_2 and λ_1=λ_2.

When $M_i \neq M_j$, by using the binomial expansion defined in [9, eq.(1.111)] and eq.(3.381) in [9], after applying some algebraic manipulations, (7) can be rewritten as

$$
f_z(z) = \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=0}^{m_2-1} \binom{m_2-1}{k} \frac{(-1)^k T_i T_j}{\left(M_i - M_j\right)^{m_1+k}} z^{m_2-k-1} \exp\left(-M_j z\right) \gamma\left[m_1 + k, \left(M_i - M_j\right) z\right]
\tag{8}
$$

where $\gamma(\bullet,\bullet)$ is the incomplete gamma function defined in [9, eq.(8.350.1)], .

With the help of the series expression of $\gamma(\bullet,\bullet)$ defined in [9, eq.(8.352.1)], and similar as (5), after applying some algebraic manipulations, the closed-form expression of $\overline{C_3}$ under the first case can be written as

$$
\overline{C_3} = \sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=0}^{m_2-1} \binom{m_2-1}{k} \frac{(-1)^k T_i T_j \Gamma\left(m_1+k\right)}{\left(M_i - M_j\right)^{m_1+k}} \left(I_{m_2-k}(M_j) - \sum_{l=0}^{m_1+k} \frac{\left(M_i - M_j\right)^l}{l!} I_{m_2+l-k}(M_i) \right)
\tag{9}
$$

When $M_i = M_j$, by using eq.(3.191.1) in [9], (7) can be rewritten as

$$
f_z(z) = \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{T_i T_j \Gamma(m_1)\Gamma(m_2)}{\Gamma(m_1+m_2)} z^{m_1+m_2-1} \exp\left(-M_j z\right)
\tag{10}
$$

Similarly, by using (10), the closed-form expression of $\overline{C_3}$ under the second case can be written as

$$
\overline{C_3} = \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{T_i T_j \Gamma(m_1)\Gamma(m_2)}{\Gamma(m_1+m_2)} I_{m_1+m_2}(M_j)
\tag{11}
$$

Finally, by substituting (5), (6) and (9) or (11) into (4), we can obtain the exact closed-form expression for the ergodic capacity of the dual-hop system over MG fading channels. To the best of our knowledge, this result is novel and simple without some more complicated special functions, such as meijer's G functions as in [1] and [3].

4. Numerical and Simulation Results

In this section, we present some numerical and simulation results to evaluate the ergodic capacity of the dual hop system over composite fading channels by using the MG distribution.

Fig.1 illustrates the ergodic capacity in (4) versus the un-faded SNR (ρ) under different fading scenarios. Without loss of generality, we assume μ_i=0, N =16 for MG distribution. As expected, the ergodic capacity increases with increasing ρ from Fig.1. At the same time, it is clear that there is an excellent match between our analytical expression and simulations over entire range of ρ.

Fig.1(a) shows the impact of multipath parameters on ergodic capacity. It can be seen from Fig.1(a) that the ergodic capacity increases with increasing m_i, (i=1,2), whereas, the effect of m_i on capacity becomes weaker as m_i get larger. Fig.1(b) shows the impact of shadowing parameter on ergodic capacity. As expected, the ergodic capacity decreases with increasing σ_i, where σ_i=0, 4, 8dB denote no shadowing, urban areas and typical macrocells, respectively.

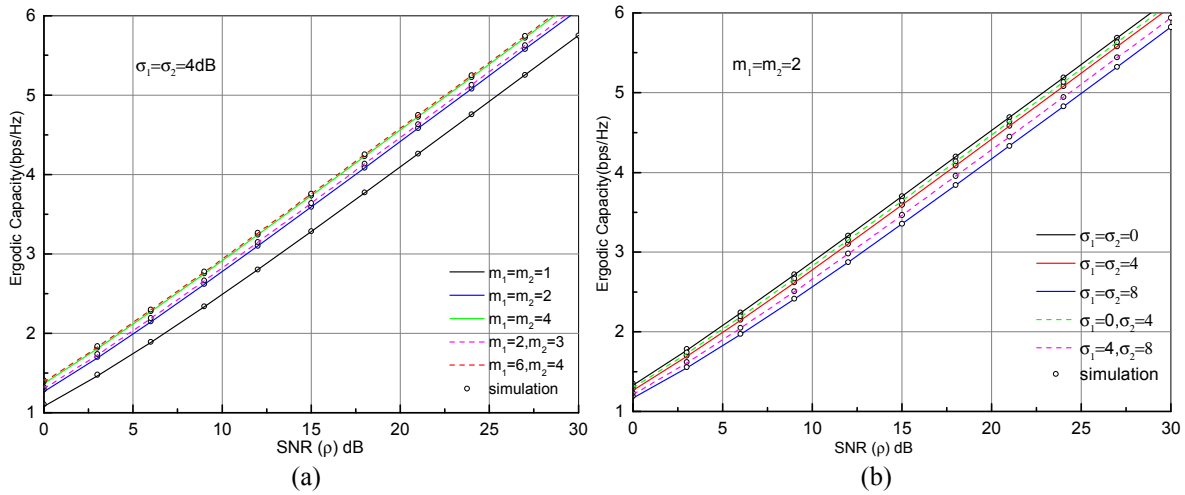

Fig.1 Ergodic capacity for the dual-hop system versus ρ under different fading parameters

5. Conclusion

In this paper, we investigated the ergodic capacity of dual-hop variable gain AF wireless system over independent non-identical composite NL fading channels approximated by using MG distribution. A novel exact closed-form expression of ergodic capacity is derived. We showed numerical and simulation results to verify the accuracy of the analytical results, and discussed the effect of the multipath and shadowing parameters on the ergodic capacity of the dual-hop variable gain AF wireless system.

6. Reference

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