

Unified Treatment of Integrals for Curvilinear Elements in both Finite and Boundary Element Methods

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Abstract

In this paper we present the implementation of schemes for handling numerical quadrature for modeling curvilinear elements in both integral and differential equation formulations. The proposed integration schemes are based on singularity cancellation methods that can be extended to curvilinear elements for higher order electromagnetic modeling. The key of the proposed approaches is the introduction of a tangent element. The tangent element not only incorporates fundamental geometry information about the surface, but is also useful in describing bases defined on the element, and facilitates the handling of quadrature rules, especially for singular or near-singular integrals.

1. Introduction

Higher order modeling in electromagnetics is a topic of considerable recent interest (see e.g. [1,2]); the main aim is to provide more accurate representations of the fields or equivalent surface currents associated with them. However, the potential benefits of higher order modelling are unrealized unless careful attention is given to the accuracy of the underlying numerical quadrature schemes. One of the more successful emerging schemes is the singularity cancellation method (see e.g. [3,4]), in which transformations of the integration variables are chosen such that the Jacobian regularizes the integrand.

This paper attempts to address this issue by discussing in some detail how high order quadrature schemes can be applied to the modelling of surfaces by curvilinear triangular patches with the electric field integral equation (EFIE). Moreover we illustrate the treatment of partial differential equations via finite element methods (FEM) with curvilinear elements to model fields satisfying the vector Helmholtz equation. In both cases the tangent element concept is developed, showing its usefulness in defining bases, implementing quadrature rules, and in adapting linear element concepts to curvilinear elements.

2. References

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