Application of Stochastic Differential Equations for the Performance Evaluation of Broadband Satellite Communication Systems

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Abstract

In this paper, a general framework for the application of Stochastic Differential Equations (SDEs) for the accurate evaluation of broadband satellite communication networks is presented. Following the tendency that the operating frequencies in modern satellite communication systems have migrated to Ka-band and above, it is very important to model the radiowave propagation phenomena. The dynamics of rain attenuation as well as the generation of rain attenuation time series of rain attenuation are required for the prediction of the satellite communication systems' performance. Here we present the mathematical framework for the description of correlated rain attenuations induced on multiple links with SDEs. Moreover, an expression for the channel estimation that can be used for the evaluation of Fade Mitigation Techniques (FMTs) is also presented. In the numerical results Section, two satellite systems employing spatial diversity (site and orbital) are examined and the new channel estimation expression is investigated.

1. Introduction

Broadband satellite communications for data delivery or broadcasting services will operate at frequencies higher than 20 GHz. Their aim is to have a higher available bandwidth in order to be able to transmit with higher data rates erroneously at a prescribed high availability [1]. For high frequency bands, the atmosphere causes the attenuation of the signal through precipitation, clouds and atmospheric gases [2]. The mitigation of the propagation impairments is realized through the adoption of diversity and adaptive techniques [2]. Specifically, in order to evaluate the performance of the adaptive techniques and predict the outage statistics in broadband satellite communication systems, the dynamics of the atmospheric effects are needed. The temporal and the spatio-temporal characteristics of the atmospheric channel can be modeled and reproduced through the employment of a mathematical framework based on Stochastic Differential Equations (SDEs) [3-8]. SDEs can describe the stochastic processes of the satellite channels and can be also used for the generation of time series of atmospheric attenuation. Time series generators are very useful for the satellite communication systems simulation in order to assess their performance, especially in the usual case that there are no experimental data for the specific location and specific characteristics of the satellite link.

However, for the description of satellite channel, SDEs have been firstly used for the description of rain attenuation as a stochastic process in the work of Maseng-Bakken [3]. In [3], using the assumption that rain attenuation induced on single link follows in long-term the lognormal distribution, a first order SDE is used for the characterization of the satellite channel under rain. In the same reference, using the Fokker-Planck equation the Transition Probability Distribution Function (TPDF) is found. Although, in [3] the SDE is given, time series of rain attenuation are generated through low-pass filtering Gaussian noise. In [5], the strong analytical solution of the SDE proposed in [4] is derived, enabling fast and accurate simulation of rain attenuation time series. Moreover, in [5] the authors have proposed the introduction of hitting times for modeling the rain attenuation dynamics. SDE approach has also been introduced and analytically solved for rain attenuation modeling in tropical regions [6].

The aforementioned references refer to single dimensional SDEs. However, in order to be able to generate rain attenuation for multiple links, separated in various domains, multi-dimensional SDEs must be used. In [7], a multi-dimensional system of SDEs is proposed. It is based on the assumptions of Maseng-Bakken model for the generation of rain attenuation time series. With the proposed model rain attenuation time series on correlated, in space, multiple links can be generated. In [8], two-dimensional SDEs are used for the modeling of dynamic frequency scaling in short-term given that downlink and uplink use different frequencies. The use of mathematical framework of the SDEs apart from the generation of time series of attenuation, it enables the accurate calculation of figures of merit, metrics, which can be used for the design of broadband satellite communication systems. In this paper, the general framework of multi-dimensional SDEs is presented for the modeling of rain attenuation in complex radio systems operating at Ka-band and above. The expressions for the definition and further solving the SDEs are presented, as well as an expression for

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estimating the channel in a single link satellite configuration after a time period. Moreover, some figures of merit that can be calculated with the SDE framework are given.

2. Rain Attenuation Modeling Framework

Assume that there are n millimeter wave links (either satellite or terrestrial) which differ either on spatial domain or on the link characteristics, such as frequency, the rain attenuation vector induced into the multiple links is given $\mathbf{a}_t = \left[a_t^1, \ldots, a_t^n\right]^T$. For the modeling of the time series of rain attenuation which are correlated, the 1st order multi-dimensional SDEs are used:

$$d\mathbf{a}_t = \mathbf{F}(\mathbf{a}_t)\,dt + \mathbf{Z}(\mathbf{a}_t)\,d\mathbf{W}_t \tag{1}$$

where

$$\begin{aligned} \mathbf{F}(\mathbf{a}_t) &= \left[F_1\left(a_t^1, \ldots, a_t^n\right), \ldots, F_n\left(a_t^1, \ldots, a_t^n\right)\right]^T \\ \mathbf{Z}(\mathbf{a}_t) &= \left[z_{ij}\left(a_t^1, \ldots, a_t^n\right)\right]_{1 \le i, j \le n} \\ \mathbf{W}_t &= \left[W_t^1, \ldots, W_t^n\right]^T \end{aligned} \tag{2}$$

with $\mathbf{W}_t$ the n-dimensional Brownian Motion, which means that $W_t^1, \ldots, W_t^n$ are independent Brownian Motions [3]. Now considering the assumptions of M-B model and a covariance matrix ($\mathbf{C_X}$) between rain attenuation in various links the elements of coefficients **F** and **Z** are:

$$F_i(\mathbf{x}) = x_i\left[-\beta_i \ln \frac{x_t^i}{a_{m_i}} + \frac{1}{2}\sum_{j=1}^{n} s_{ij}^2\right] \tag{3}$$

$$z_{ij}(\mathbf{x}) = x_i s_{ij} \tag{4}$$

In (3), the parameter β_i is the dynamic parameter of rain attenuation at link i, a_{m_i} is the median value of the lognormal distribution of rain attenuation induced at link i and s_{ij} are the elements of matrix **S** which is computed through the decomposition of matrix **G**: $\mathbf{G} = \mathbf{S}\mathbf{S}^T$. The elements of matrix **G** are:

$$[G]_{ij} = \left(\beta_i + \beta_j\right)[C_X]_{ij} \tag{5}$$

with $[C_X]_{ij}$ are the elements of the covariance matrix $\mathbf{C_X}$. Considering only one link, the SDE collapses to one dimension:

$$da_t = a_t \beta\left[\sigma_a^2 - \ln\left(\frac{a_t}{a_m}\right)\right]dt + a_t\sqrt{2\beta}\sigma_a dW_t \tag{6}$$

which is the M-B model. The TPDF for the single case is [7]:

$$p\left\{A(t), t \mid A(t_0), t_0\right\} = \frac{1}{\sqrt{2\pi}\, S_{a0}(\Delta t)\, A(t)} \exp\left\{-\frac{\left[\ln A(t) - \ln A_{m0}(\Delta t)\right]^2}{2 S_{a0}^2(\Delta t)}\right\} \tag{7}$$

with $A_{m0}(\Delta t) = \alpha_m^{\left[1-\exp(-\beta\Delta t)\right]} A(t_0)^{\exp(-\beta\Delta t)}$, $S_{a0}(\Delta t) = \sigma_a\sqrt{1-\exp(-2\beta\Delta t)}$ and $\Delta t = t - t_0$.

An estimation of the channel after time Δt from the initial time instance is:

$$E[A^t \mid A^0] = a_m \exp\left[\ln\left[\frac{A^0}{a_m}\right]e^{-\beta_A \Delta t} + \frac{1}{2}\left(\sigma_A\right)^2\left(1 - e^{-2\beta_A \Delta t}\right)\right] \tag{8}$$

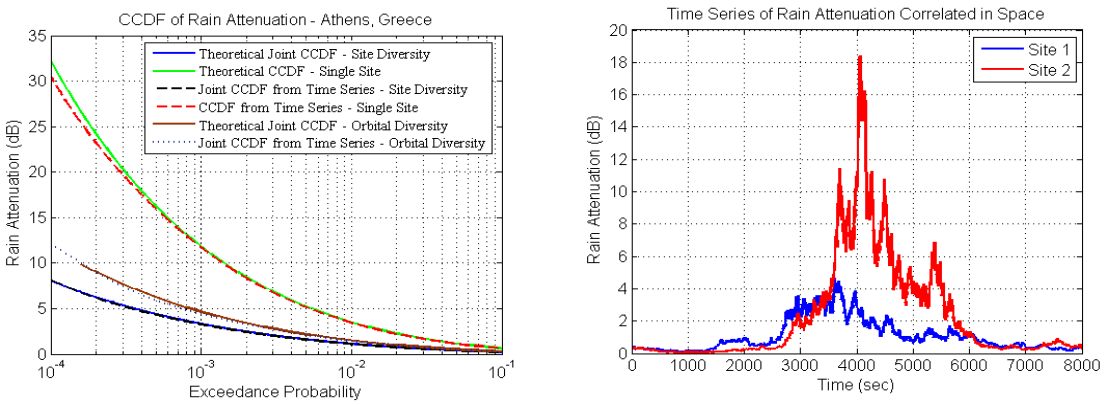
2.1 Figures of Merit

In the physical layer, the most important metrics for the performance of satellite communication systems are:

- Signal-to-noise ratio, (SNR) is a term for the power ratio between the signal of the useful information and the noise power and consists of a deterministic part and a stochastic part (large scale fading and multipath). From its evaluation we can calculate the outage statistics.
- Signal-to-noise plus interference ratio, (SNIR), is the ratio of the wanted signal to the total power of the interfering signals and noise that is evaluated at a specific point of the satellite channel. If we have dual polarization channels, the depolarization powers may be considered as extra interfering channels, yielding the SNIDR, signal-to-noise plus depolarization and interference ratio.
- Bit-error-ratio, (BER), for example the transmission BER, the number of erroneous bits received, divided by the total number of bits transmitted and the information BER, the number of erroneous (decoded) corrected divided by the total number of decoded bits. Its instantaneous value depends on the satellite channel state information.
- Ergodic Capacity, is mostly used for fast varying fading channels, either flat or frequency selective and is given as the average Shannon Capacity over the fading wireless channel.
- Outage Capacity, is used for slowly varying channel where the instantaneous signal-to-noise ratio is assumed to be constant for a large number of transmitting symbols. Outage capacity is characterized by an achieving threshold for a given outage probability and in most of the time is more practical than the average capacity.

The above described general simulation framework of rain attenuation fading channels can be easily implemented using MATLAB software. The suggested radio channel framework suits very well for computer simulations and it is universal since a single structure allows the modeling of a great variety of different processes by a simple variation of the system parameters and excitations.

3. Numerical Results and Discussion



(a) (b)

Fig. 1 a) Outage probability statistics for a single link and site and orbital diversity and orbital diversity configurations, b) time series or rain attenuation correlated in spatial domain

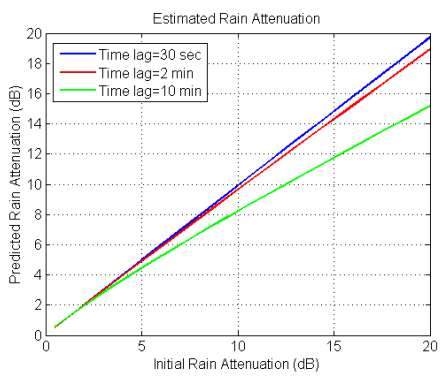


Fig. 2 Estimated rain attenuation values versus initial rain attenuation for various time lags.

In this numerical results Section, we calculate the outage probability of rain attenuation for a dual site configuration and dual orbital configuration are shown for Ground stations located in Athens, GR. In the dual site configuration the separation distance between the Earth stations is 30 km, while for the dual orbital diversity configuration the separation angle between the two slant paths is 60°. The elevation angle of all links is 30° and the

operating frequency 30 GHz. The results are shown in Fig. 1a. In the same sub-Figure, the single link exceedance probability is also shown. For all the configurations, apart from the exceedance probability derived from the time series, the theoretical ones are also shown. It can be observed that the simulated exceedance probability coincides with the theoretical one. In Fig. 1b, the correlated time series of rain attenuation during a rain event is shown for the dual site configuration.

For the investigation of the channel estimation, we consider the dynamic parameter of rain attenuation equal to $2*10^{-4}$ sec^{-1} and independent to the link characteristics. For initial rain attenuation equal to 1 and 4 dB, the estimated channel is calculated for various time delays up to 120 sec. The frequency of the link is 40 GHz and elevation angle equal to 20°. The resulted by (8) satellite channel estimation is shown in Fig. 2. This formula can be directly used for the evaluation of adaptive modulation and coding techniques in satellite communication systems.

4. Conclusions

In this paper, the general framework of the application of SDEs on rain attenuation modeling is briefly presented. More particularly, in the introduction a detailed survey is given presenting the models based on SDEs. Furthermore, the general expressions of the multi-dimensional SDEs in order to describe correlated rain attenuation time series are given and the most important figures of merit for satellite communication systems are presented. In the numerical results, it is shown that the modeling with multi-dimensional SDEs reproduce the outage probability of satellite diversity systems.

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