The study on mapping algorithm for superposed signals in relaying system based on Decode and Forward Physical Network Coding

***Jiachun Liao**[*1], **Fanggang Wang**[2], **and Dongping Yao**[3]*

[1] School of Electronics and Information Engineering, Beijing Jiaotong University, Beijing, China, 12120104@bjtu.edu.cn
[2] State Key Lab of Rail Traffic Control and Safety, Beijing Jiaotong University, fgwang1@bjtu.edu.cn
[3] School of Electronics and Information Engineering, dpyao@bjtu.edu.cn

Abstract

Decode and forward (DF) Physical Network Coding (PNC) has shown its advantage in advancing the capacity performance of relaying system in the literature. However, there are many difficulties in the implement of DF PNC. For example, to derive useful information from superposed signals is even a problem in a system selecting Binary Phase Shift Keying (BPSK) as the baseband modulation scheme. Until now the only existing solution to it is based on the Minimum Euclidean distance criterion with a good performance and a very high computational complexity. In this paper, we deal with the problem and provide a new algorithm which has the equal performance with the existed method and a much lower computational complexity.

1. Introduction

Physical-layer network coding (PNC) has received much attention in academic and industrial circles from its emergence in 2006, for it doubles the capacity of relaying system comparing to the traditional relaying system. The advantage is from the thought of embracing the interferences instead of the traditional thinking- avoiding interferences, which is more agreeable with the wireless broadcast advantage [1, 2]. According to the superposed-symbol operation, PNC can be classified into three sorts, e.g., amplify and forward (AF) PNC, compress and forward (CF) PNC and decode and forward (DF) PNC. Most of the studies focus on the performance analysis of PNC in variable scenes [3, 4] and the PNC integration with other technologies like OFDM, MIMO [5, 6]. The research on the implementation of PNC is rare. Especially, the actualization of DF PNC has never been studied until the first prototype of DF PNC was proposed at the ICC conference in 2012 [7] called "FPNC based on OFDM" in the paper. Furthermore, the authors optimize the model and publish the improvement in 2013 [8]. In the literature, received BPSK symbols are mapped to bits with a method based on the Minimum Euclidean distance criterion, i.e., calculating the distances between superposed symbols and the constellation points, then selecting the information of the nearest point as the mapping bits. The technique has a good performance but a high computational complexity. In this paper a new technique for BPSK superposed symbol mapping to bits is provided, which has the equal performance with the existing method and has a much lower computational complexity.

The remainder of this paper is arranged as following. The system model is given in section 2 as well as the theoretical basis for the mapping. In section 3 An efficient method is proposed and the performance compare with the existing scheme are also presented. The section IV has summed the prior sections up.

2. Theoretical basis

The relaying system based on DF PNC is showed in Fig 1. BPSK is selected as baseband modulation scheme and the bit-to-symbol mapping is in Tab 1.

As Fig 1 indicates, the two sources send signals simultaneously in the first slot. After receiving the superposed signal, the relay derives the relationship of sources' information from it and transmits the relation in the latter slot. At last, two sources obtain the information from the peer according to the received information and their own information transmitted in first slot. An example is in Tab 2.

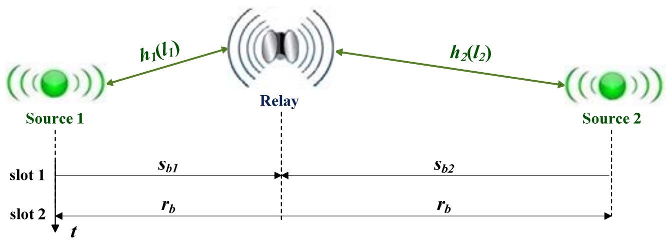

Fig 1 the relaying system based on DF PNC

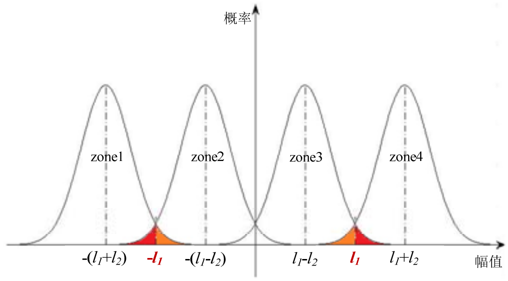

Fig 2 the statistical property of the received real signal

In this system, assume that the transmit power of two sources is 1 and phase synchronization has been realized. Let l_1 and l_2 in Fig 1 represent the amplitude fading factor, $x_i \in \{-1,1\}$ i=1,2 represents the signal sent by source i, and ω_R be the additive white Gaussian noise at the relay with random variables following the circularly-symmetric complex Gaussian (CSCG) distribution, denoted by $\omega_R \sim CN(0, 2\sigma^2)$. The received signal at the relay can be expressed as

Tab 1 Bit-Symbol mapping

Source 1		Source 2		Relay	
s_{b1}	x_1	s_{b2}	x_2	y	r_b
0	1	0	1	2	0
1	-1	0	1	0	1
0	1	1	-1	0	1
1	-1	1	-1	-2	0

Tab 2 An example of DF PNC

	Transmit bits	Received bits (the transmit bits of relay)	Bits from peer ($s_b \oplus r_b$)
Source 1	0 1 1 0	1 0 1 0	1 1 0 0
Source 2	1 1 0 0	1 0 1 0	0 1 1 0

$$\begin{aligned} y &= y_1 + y_2 \\ &= l_1 x_1 + l_2 x_2 + \omega_R \end{aligned} \tag{1}$$

So, the received signal is also a random variable following complex Gaussian distribution. i.e., the real signal y_r has Gaussian distribution with non-zero mean and its orthogonal signal y_i has Gaussian distribution with zero mean, denoted by $y_r \sim N(l, \sigma^2)$, $y_i \sim N(0, \sigma^2)$ respectively, where $l \in \{l_1+l_2, -(l_1+l_2), l_1-l_2, -(l_1-l_2)\}$.

In theory, the transmit signal would be mapped to real domain of the received signal completely, so only the received real signal would be analyzed later. Since the sources have the equal probability for sending bit 0 and bit 1, according to the nature of Gaussian distribution, the probability density function for received real signal would be express as

$$\begin{aligned} f(y_r) = 1/4\sqrt{2\pi}\sigma \cdot \Big(& \exp\left(-\left(y_r-(l_1+l_2)\right)^2/2\sigma^2\right) + \exp\left(-\left(y_r+(l_1+l_2)\right)^2/2\sigma^2\right) \\ & + \exp\left(-\left(y_r-(l_1-l_2)\right)^2/2\sigma^2\right) + \exp\left(-\left(y_r+(l_1-l_2)\right)^2/2\sigma^2\right)\Big) \end{aligned} \tag{2}$$

Where, $l_1 > l_2 > 0$ and Fig 2 illustrates the statistical property of the received real signal under the conditions.

Based on the Maximum A Posteriori (MAP) criterion, the mapping bit can be expressed as

$$r_b = i \oplus j = \max_{i,j\in\{0,1\}} \{P(s_b = i, j \mid y_r)\} \tag{3}$$

For the probabilities for sources transmitting bit 0 and bit 1 are the same, and the send behaviors are mutual independent for sources, i.e., $\forall\ i\, j \in \{0,1\} \Rightarrow P(i,j)=1/4$, and $P(i,j|\,y_r) = [P(y_r|i,j)\ P(i,j)]/P(y_r)$, Equation (3) is equivalent to Equation (4).

$$r_b = i \oplus j = \max_{i,j\in\{0,1\}} \{P(y_r \mid i,j)\} \tag{4}$$

According to the statistic property of the received real signal, the further express is as equation (5) showed.

$$r_b = i \oplus j = \min_{i,j} \begin{Bmatrix} |y_r-(l_1+l_2)|, |y_r+(l_1+l_2)| \\ |y_r-(l_1-l_2)|, |y_r+(l_1+l_2)| \end{Bmatrix} \tag{5}$$

It means MAP is equal to the Minimum Euclidean distance criterion in the case. Furthermore, as they are on the real axis, the mapping could be done by the simple threshold judgment method.

In the realization, the signals transmitted by different sources could not be mapped to the real domain of the received signal for various phase shifts from different channels. I.e., the threshold judgment scheme could not be applied in the mapping operation directly. So, The Minimum Euclidean distance criterion is adopted by the existing effective mapping method. Although it has a good performance, the computation complexity is very high. In section 3, a low computation complexity scheme is proposed with equivalent performance.

3. A simple mapping scheme for superposed signal

The existing mapping method mentioned in section 2 can be expressed as equation (6), i.e., to perform mapping operation once, four distance calculations and three comparing operations are required.

$$r_b = \min\{|\,y-(h_1+h_2)\,|, |\,y+(h_1+h_2)\,|\} > \min\{|\,y-(h_1-h_2)\,|, |\,y+(h_1-h_2)\,|\} \tag{6}$$

Where, h_1 and h_2 represent the channel parameters of two uplink links respectively, which include the information about the phase shift and amplitude fading.

Similar with the expression in section 2, the received superposed signal is expressed as

$$
\begin{aligned}
y &= y_1 + y_2 \\
&= h_1 x_1 + h_2 x_2 + \omega_R
\end{aligned} \quad (7).
$$

The only difference is that the channel parameters are complex. So, the received signal follows complex Gaussian distribution with non-zero mean. I.e., the real signal y_r is independent of its orthogonal signal y_i, and $y_r \sim N(h_r, \sigma^2)$, $y_i \sim N(h_i, \sigma^2)$, where $h_r \in \{h_{1r}+h_{2r}, -(h_{1r}+h_{2r}), h_{1r}-h_{2r}, -(h_{1r}-h_{2r})\}$, $h_i \in \{h_{1i}+h_{2i}, -(h_{1i}+h_{2i}), h_{1i}-h_{2i}, -(h_{1i}-h_{2i})\}$.

Reference to the analysis in section 2, while the signals sent by two sources are both mapped in the same domain of the received signal, the mapping bits r_b can be derived from the signal in the corresponding domain with the simple threshold judgment method. In addition, the thresholds are listed in Tab 3 for all cases.

Tab 3 the list of thresholds for judging superposed signal

Condition		Threshold 1	Threshold 2	$r_b = 0$	$r_b = 1$
$h_{1r}h_{2r} > 0$	$\lvert h_{1r}\rvert \le \lvert h_{2r}\rvert$	$-\lvert h_{2r}\rvert$	$\lvert h_{2r}\rvert$	$(-\infty, -\lvert h_{2r}\rvert) \cup (\lvert h_{2r}\rvert, +\infty)$	$(-\lvert h_{2r}\rvert, \lvert h_{2r}\rvert)$
	$\lvert h_{1r}\rvert > \lvert h_{2r}\rvert$	$-\lvert h_{1r}\rvert$	$\lvert h_{1r}\rvert$	$(-\infty, -\lvert h_{1r}\rvert) \cup (\lvert h_{1r}\rvert, +\infty)$	$(-\lvert h_{1r}\rvert, \lvert h_{1r}\rvert)$
$h_{1r}h_{2r} < 0$	$\lvert h_{1r}\rvert \le \lvert h_{2r}\rvert$	$-\lvert h_{2r}\rvert$	$\lvert h_{2r}\rvert$	$(-\lvert h_{2r}\rvert, \lvert h_{2r}\rvert)$	$(-\infty, -\lvert h_{2r}\rvert) \cup (\lvert h_{2r}\rvert, +\infty)$
	$\lvert h_{1r}\rvert > \lvert h_{2r}\rvert$	$-\lvert h_{1r}\rvert$	$\lvert h_{1r}\rvert$	$(-\lvert h_{1r}\rvert, \lvert h_{1r}\rvert)$	$(-\infty, -\lvert h_{1r}\rvert) \cup (\lvert h_{1r}\rvert, +\infty)$

On the other hand, while the transmit signals of two sources are mapped in different domains of the received signal, we can obtain each information from the corresponding domains with the simple 0-threshold judgment method, which is the conventional scheme for BPSK demodulation. Then, getting the mapping bits r_b with a simple XOR operation.

For the phase shifts brought in by the uplinks are various, the sending signals may be mapped in the same domain, and it is also probably that being mapped in different domains. So, the first step of the algorithm designed in the paper is to determine the mapping function through analyzing the estimated values of the channel parameters. Then, the effective signal domains would be picked out and the mapping methods are also determined with the following principles: While the greater projection values of the transmit signals are in the same domain, the mapping bits r_b are determined with thresholds judgment method for the superposed signal, and the data in Tab 3 would be used. Otherwise, the transmit information i, j of two sources is obtained with the zero threshold judgment method operating in corresponding domains. Therefore, the mapping bits r_b could be obtained by $r_b = i \oplus j$.

In Fig 3, the main process of the mapping algorithm is presented. Moreover, the detail descriptions of the two threshold judgment method are showed by Fig 4 and Fig 5 respectively. Clearly, the parameter *threshold* is decisive for determining the effective signal domains correctly, which is the key to mapping successfully.

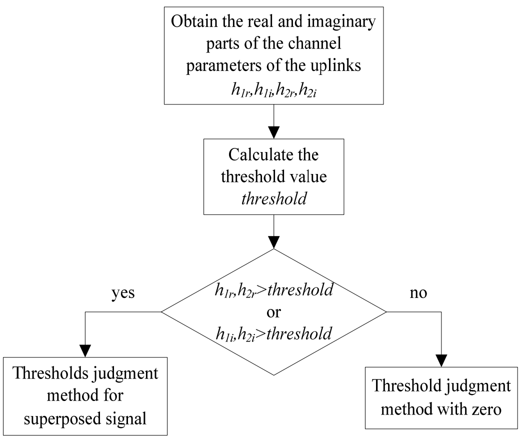

Fig 3 The main process for mapping

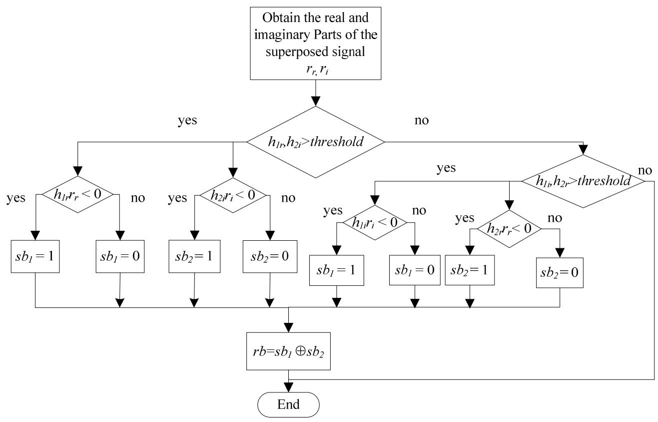

Fig 4 0-threshold judgment method

The parameter *threshold* should help to pick out the greater projection values, also avoid the fault caused by mistaking the noise as the effective signals, which could be derived by equation (8) where the value of κ is determined by the channel condition that the system works in.

For the noise signals in real and orthogonal signal domains are both Gaussian distribution, the greater the value of κ is, the lower the error probability is. However, the greater the value of κ is, the smaller the value of (min{$|h_1|$,

$|h_2|\}/\sqrt{2}) - \kappa$ is. I.e., as the value of κ increases, the ability recedes to find the effective domain. So, the greater value of κ should be selected in the range where the values are much less than the value of $(\min\{|h_1|, |h_2|\}/\sqrt{2}) - \kappa$.

$$threshold = \max\left\{\left(\min\{|h_1|,|h_2|\}/\sqrt{2}-\kappa\right),\kappa\right\} \tag{8}$$

In the simulation, various phase shift and noise would be brought in and the Signal to Noise Ratio (SNR) is in the range [-2, 20]*dB*. Let $\kappa=2\sigma$ to make sure there is a high probability of selecting the effective signal domains correctly. The simulating result is presented in Fig 6. Obviously, the threshold judgment method proposed by this paper is equivalent with the existing Minimum Euclidean distance method in term of performance. Otherwise, while the effective signal domains are determined through 1/2 comparing operations to the channel parameters, only 1/2 comparing operations are required for every received symbol being mapped to information bits. The computation complexity is much lower than the one of four distance calculation and three comparing operations for every received symbol required by the existing Minimum Euclidean distance method.

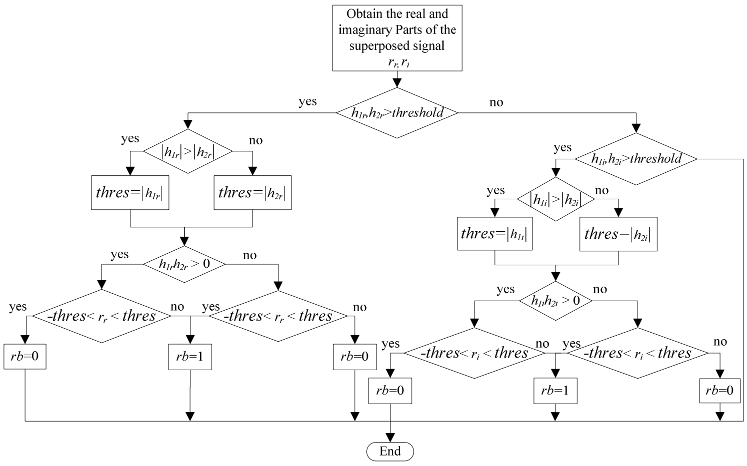

Fig 5 Threshold judgment method for superposed signal

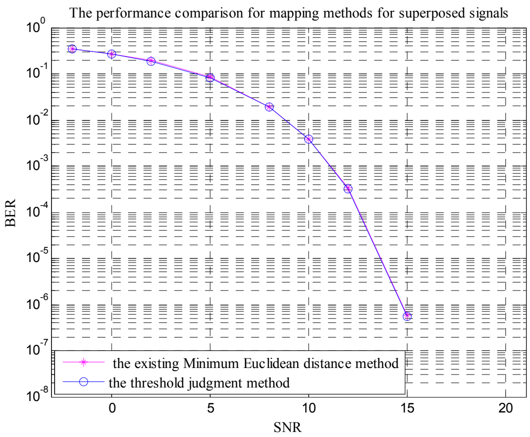

Fig 6 the performance comparison

4. Conclusion

In this paper, the theoretical basis is illustrated for deriving relationships of transmit information from received superposed signals firstly. Then, a simple but efficient mapping scheme is proposed, which has the virtually identical performance with the existing Minimum Euclidean distance method proved by the simulating result, but the required computation complexity is much lower than the one required by the existing method. Also, the theoretical explanation for the parameter *threshold* is provided which is the key of the simple method.

5. References

[1] S. Zhang, S. C. Liew, and P. P. Lam, “Hot topic: Physical-layer network coding,” in *Proc. of ACM Mobicom*, 2006, pp. 358–365.

[2] S. C. Liew, S. Zhang, and L. Lu, “Physical-layer network coding: tutorial, survey, and beyond,” *Physical Communication*, vol. 6, pp. 4–42, 2013.

[3] S. Ikki and S. Aissa, “Performance analysis of two-way amplify-and-forward relaying in the presence of co-channel interferences,” *IEEE Trans. Commun.*, vol. 60, no. 4, pp. 933–939, 2012.

[4] B. Li, G. Wang, H. Yang, and R. Liu, “Performance of physical-layer network coding in asymmetric Rayleigh fading two-way relay communication systems,” *Information Technology Journal*, vol. 12, no. 9, pp.1804–1810, 2013.

[5] J. Joung and S. A.H., “Multiuser two-way amplify-and-forward relay processing and power control methods for beamforming systems,” *IEEE Trans. Signal Process.*, vol. 58, no. 3, pp. 1833–1846, 2010.

[6] F. Wang, X. Yuan, S. Liew, and D. Guo, “Wireless MIMO switching: Weighted sum mean square error and sum rate optimization,” *IEEE Trans. Inform. Theory*, vol. 59, no. 9, pp. 5297–5312, 2013.

[7] L. Lu, T. Wang, S. C. L. Liew, and S. Zhang, “Implementation of physical-layer network coding,” in *IEEE International Conference on Communications*, 2012, pp. 4734–4740.

[8] L. Lu, L. You, Q. Yang, T. Wang, M. Zhang, S. Zhang, and S. C. Liew, “Real-time implementation of physical-layer network coding,” in *SRIF 2013 - Proceedings of the 2nd*, 2013, pp. 71–76.