# Spectrum-Efficient Superimposed Pilot Design Based on Structured Compressive Sensing for Downlink Large-Scale MIMO Systems

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## Abstract

Large-scale multiple-input multiple-output (MIMO) with high spectrum and energy efficiency is a very promising key technology for future 5G wireless communications. Although most research only considers training and channel estimation in the uplink based on the assumption of time division duplexing (TDD) protocol, downlink training and channel estimation is also necessary, especially for the dominated frequency division duplexing (FDD) protocol. Unlike conventional orthogonal pilots whose overhead prohibitively increases with the number of transmit antennas, we propose a spectrum-efficient superimposed pilot design based on the emerging theory of structured compressive sensing. The frequency-domain pilots of different transmit antennas share common subcarriers instead of orthogonal subcarriers, and accordingly we propose the structured subspace pursuit (SSP) algorithm to simultaneously recover multiple channels by exploiting the spatial and temporal correlations of large-scale MIMO channels. Simulation results verify that the proposed scheme can approach performance bound of the exact least square algorithm.

## 1. Introduction

Large-scale multiple-input multiple-output (MIMO) employing large number of antennas at the base stations (BS) to simultaneously serve multiple users can increase the spectrum efficiency and energy efficiency by orders of magnitude, which makes it a very promising key technology for future 5G wireless communications [1].

In large-scale MIMO systems, the BSs and users must know the channel state information (CSI) for signal detection, precoding, resource allocation, etc., but accurate channel estimation is quite challenging for large-scale MIMO, especially in the downlink where the channels coming from a large number of transmit antennas have to be reliably distinguished at first and then accurately estimated [2]. Up to now, most research avoids this challenging problem by assuming time division duplexing (TDD) protocol, where the acquired CSI at the BS in the uplink can be directly feedback to users by using the channel reciprocity property, and then CSI acquisition is not required any more in the downlink [3]. However, the CSI obtained in the uplink may not be accurate or even outdated for the downlink in TDD systems, which might cause a significant performance loss. Moreover, as frequency division duplexing (FDD) still dominates current wireless cellular systems, downlink CSI acquisition must be required due to the channel reciprocity does not exist for FDD systems. Thus, the seldom addressed problem of downlink training and channel estimation is also very important for large-scale MIMO systems.

In this paper, unlike standardized orthogonal pilots whose overhead prohibitively increases with the number of transmit antennas, we propose a spectrum-efficient superimposed pilots design based on the emerging theory of structured compressive sensing (CS) [4]. The pilots of different transmit antennas share common subcarriers instead of orthogonal subcarriers in the frequency domain, and the proposed structured subspace pursuit (SSP) algorithm derived from the classical SP algorithm [5] is used for accurate estimation of multiple channels by exploiting the spatial and temporal correlations of large-scale MIMO channels. In this way, the pilot overhead can be significantly reduced.

## 2. Spatial and Temporal Correlations of Large-Scale MIMO Channels

In typical large-scale MIMO systems, the BS employs $M$ antennas to simultaneously serve $U$ single-antenna users. Usually, $M$ and $U$ are very large in large-scale MIMO systems, e.g., $M = 64$ and $U = 16$ were considered in [2].

The channel impulse response (CIR) between the $m$th transmit antenna and one specific user can be expressed as $\mathbf{h}_m = \left[h_m(0), h_m(1), \cdots, h_m(L-1)\right]^T$, where $L$ is the maximum channel delay spread. Due to the sparsity of wireless channels [6], the number of nonzero element $K$ in the CIR is much less than $L$, i.e., $K << L$. Meanwhile, MIMO channels appear spatial correlation due to the close antenna geometry, e.g., CIRs between different transmit-receive pairs share very similar path delays [6], which are referred as the spatial common sparsity of MIMO channels. Thus, we

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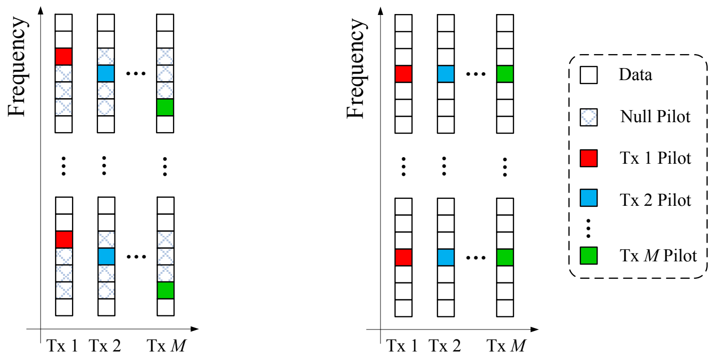
have $S_1^\Gamma = S_2^\Gamma = \cdots = S_M^\Gamma$, where $S_m^\Gamma = \{\tau : |h_m(\tau)| > 0\}_{\tau=0}^{L-1}$ denotes the support of $\mathbf{h}_m$. In addition, MIMO channels also appear temporal correlation, e.g., during several adjacent OFDM symbols, the path gains may be quite different, while path delays remain nearly unchanged [7]. Consequently, such temporal correlation results in the temporal common sparsity of MIMO channels, so we have $S_{m,i}^\Gamma = S_{m,i+1}^\Gamma = \cdots = S_{m,i+R-1}^\Gamma$, where the subscript $i$ denotes the $i$th OFDM symbol.

## 3. Superimposed Pilot Design and Structured Joint Sparse Channel Estimation

### 3.1 Superimposed Pilot Design

In contrast to standardized orthogonal pilots widely used in MIMO systems, the proposed superimposed pilot design allows pilots of different transmit antennas to share the common subcarriers as illustrated in Figure 1. Without loss of generality, the pilot index can be denoted as $\Omega$, which uniformly decimated from [0, $N$ - 1] as the conventional comb-type pilots, here $N$ denotes the discrete Fourier transform (DFT) size of the OFDM symbol. Meanwhile, in order to distinguish channels associated with different transmit antennas, pilot sequences of different transmit antennas differ one from another, i.e., $\mathbf{s}_m \neq \mathbf{s}_n$ if $m \neq n$, and this can be easily realized by generating these pilot sequences according to the identically and independently distributed (i.i.d.) random Bernoulli distribution ($\pm 1$). In contrast to the standardized orthogonal pilots with the total number of pilots $N_{p\_total} = M N_p$, where $N_p$ denotes the number of pilots per antenna, the pilot number for the proposed scheme is significantly reduced to $N_{p\_total} = N_p$ due to the superimposed pilot design.




(a) Conventional orthogonal pilots (b) Proposed superimposed pilots

**Figure 1: Comparison of the conventional orthogonal pilots and the proposed superimposed pilots**

At the receiver, after the the cyclic prefix removal and DFT, the received pilot sequence $\mathbf{y}$ coming from $M$ different transmit antennas can be expressed as

$$\mathbf{y} = \sum_{m=1}^{M} \mathrm{diag}\{\mathbf{s}_m\} \mathbf{F}\big|_\Omega \begin{bmatrix} \mathbf{h}_m \\ \mathbf{0}_{(N-L)\times 1} \end{bmatrix} + \mathbf{w} = \sum_{m=1}^{M} \mathbf{S}_m \mathbf{F}_L\big|_\Omega \mathbf{h}_m + \mathbf{w}, \tag{1}$$

where $\mathbf{F}$ is a DFT matrix of size $N \times N$, $\mathbf{F}_L$ of size $N \times L$ is a partial DFT matrix consisted of the first $L$ columns of $\mathbf{F}$, and $\mathbf{F}_L\big|_\Omega$ denotes the sub-matrix by selecting the rows of $\mathbf{F}_L\big|$ according to $\Omega$, $\mathbf{S}_m = \mathrm{diag}\{\mathbf{s}_m\}$, and $\mathbf{w}$ is the additive white Gaussian noise (AWGN). Eq. (1) can be also rewritten in a more compact form as

$$\mathbf{y} = \mathbf{\Phi}\mathbf{h} + \mathbf{w}, \tag{2}$$

where $\mathbf{h} = [\mathbf{h}_1^T, \mathbf{h}_2^T, \cdots, \mathbf{h}_M^T]^T$ has the size $ML \times 1$,and $\mathbf{\Phi} = \left[\mathbf{S}_1 \mathbf{F}_L\big|_\Omega, \mathbf{S}_2 \mathbf{F}_L\big|_\Omega, \cdots, \mathbf{S}_M \mathbf{F}_L\big|_\Omega\right]$ has the size $N_p \times ML$.

For large-scale MIMO systems, we usually have $N_p << ML$ due to the large number of transmit antennas $M$ and the limited number of pilots $N_p$ which implies that we cannot recover the channel $\mathbf{h}$ from a underdetermined problem (2). However, we observe that $\mathbf{h}$ is a sparse signal since $\{\mathbf{h}_m\}_{m=1}^M$ are sparse, and this observation inspires us to reconstruct the high-dimension sparse signal $\mathbf{h}$ from the low-dimension received pilot vector $\mathbf{y}$ under the framework of CS theory. Moreover, spatial and temporal correlations of wireless MIMO channels can be integrated in the classical CS framework for expected performance enhancement, which is the topic of the following subsection 3.2.

## 3.2 Joint Sparse Channel Estimation Based on Structured CS

The spatial and temporal correlations of wireless MIMO channels motivate us to exploit the structured CS framework developed from the classical CS theory to simultaneously reconstruct multiple channels. Considering (2) for $R$ adjacent OFDM symbols with the same pilot pattern, which is quite common in practice, we have

$$\mathbf{Y} = \mathbf{\Phi}\mathbf{H} + \mathbf{W}, \tag{4}$$

where $\mathbf{Y} = [\mathbf{y}_k, \mathbf{y}_{k+1}, \cdots, \mathbf{y}_{k+R-1}]$, $\mathbf{H} = [\mathbf{h}_k, \mathbf{h}_{k+1}, \cdots, \mathbf{h}_{k+R-1}]$, and $\mathbf{W} = [\mathbf{w}_k, \mathbf{w}_{k+1}, \cdots, \mathbf{w}_{k+R-1}]$.

According to the MIMO channel property as addressed in Section 2, $\mathbf{H}$ has the inherent structured sparsity both in the spatial and temporal dimensions. Based on the classical subspace pursuit (SP) algorithm for recovery of a single sparse vector [5], we propose the structured SP (SSP) algorithm as listed below to simultaneously recover multiple vectors with structured sparsity.

**Inputs**: Noisy measurement matrix $\mathbf{Y}$ and sensing matrix $\mathbf{\Phi}$ in (4)

**Output**: Estimated CIR matrix $\widehat{\mathbf{H}}$

**Initialization**: $\Omega \leftarrow \varnothing$, $k \leftarrow 1, \mathbf{V} \leftarrow \mathbf{Y}$

**While** $k \le K$

$\mathbf{Z} \leftarrow \mathbf{\Phi}^H \mathbf{V}$;

$g_1(\tau) \leftarrow \sum_{r=1,i=0}^{R,M-1} \left| z^{(\tau+iL,r)} \right|^2, \quad 0 \le \tau \le L-1$,

where $\mathbf{g}_1 = [g_1(0), g_1(1), \cdots, g_1(L-1)]^T$, and $z^{(m,n)}$ is the $m$th row and $n$th column element of $\mathbf{Z}$;

$\Omega \leftarrow \Omega \cup \text{supp}\{\mathbf{g}_1\rangle_K\}$;

$\Gamma \leftarrow \Omega \cup [\Omega + L] \cup \cdots \cup [\Omega + L(M-1)]$;

$\widehat{\mathbf{H}}_\Gamma \leftarrow \mathbf{\Phi}_\Gamma^\dagger \mathbf{Y}$, where $\mathbf{\Phi}_\Gamma$ denotes the sub-matrix by selecting the columns of $\mathbf{\Phi}$ according to $\Gamma$;

$g_2(\tau) \leftarrow \sum_{r=1,i=0}^{R,M-1} \left| \hat{h}^{(\tau+iL,r)} \right|^2$;

where $\mathbf{g}_2 = [g_2(0), g_2(1), \cdots, g_2(L-1)]^T$, and $\hat{h}^{(m,n)}$ is the $m$th row and $n$th column element of $\widehat{\mathbf{H}}$;

$\Omega \leftarrow \text{supp}\{\mathbf{g}_2\rangle_K\}$;

$\Gamma \leftarrow \Omega \cup [\Omega + L] \cup \cdots \cup [\Omega + L(M-1)]$;

$\widehat{\mathbf{H}}_\Gamma \leftarrow \mathbf{\Phi}_\Gamma^\dagger \mathbf{Y}$;

$\mathbf{V} \leftarrow \mathbf{Y} - \mathbf{\Phi}^H \widehat{\mathbf{H}}$;

$k \leftarrow k+1$;

**end**

**Algorithm 1: The proposed SSP algorithm**

In contrast to the classical SP algorithm which reconstructs a high-dimension sparse vector from a noisy measurement vector, the proposed SSP algorithm reconstructs a high-dimension sparse matrix composed of multiple sparse vectors sharing the common support due to the spatial and temporal correlations of MIMO channels. Consequently, all vectors of the sparse matrix sharing the common support are updated in each iteration, while only one vector is updated in the conventional SP algorithm.

## 4. Simulation Results

A simulation study was carried out to investigate the performance of the proposed solution. The proposed SSP algorithm, the conventional SP algorithm, and exact least square (LS) algorithm with perfectly known support of the sparse channel were compared using the proposed superimposed pilot scheme.

Figure 2 shows the mean square error (MSE) performance comparison in a large-scale MIMO system with $M$ = 64 transmit antennas at the BS. The system bandwidth is 7.56 MHz, the size of the OFDM symbol is $N$ = 4096, the cyclic prefix length is $N_g$ = 256, and $N_p$ = 800 pilots are uniformly-spaced in the frequency domain. The 6-path International Telecommunication Union (ITU) Vehicular B channel model [7] with $L$ = 153 was considered. From Figure 2, it can be observed that the conventional SP algorithm cannot work due to $N_p \ll ML$. On the contrary, the proposed SSP algorithm with $R$ = 4 works better than that with $R$ = 1, and both of them perform well and approach to the exact LS method (performance bound). For the proposed superimposed pilot design, the total $N_{p\_total}$ = 800 pilots occupy 19.53% of the total $N$ = 4096 subcarriers, and the equivalent average pilot overhead per transmit antenna is just

$N_{p\_avg}$ = 12.5 (only 0.30% of the total $N$ = 4096 subcarriers) compared with the conventional orthogonal pilots. Such low pilot overhead is almost impossible for the conventional algorithms to realize accurate channel estimation.

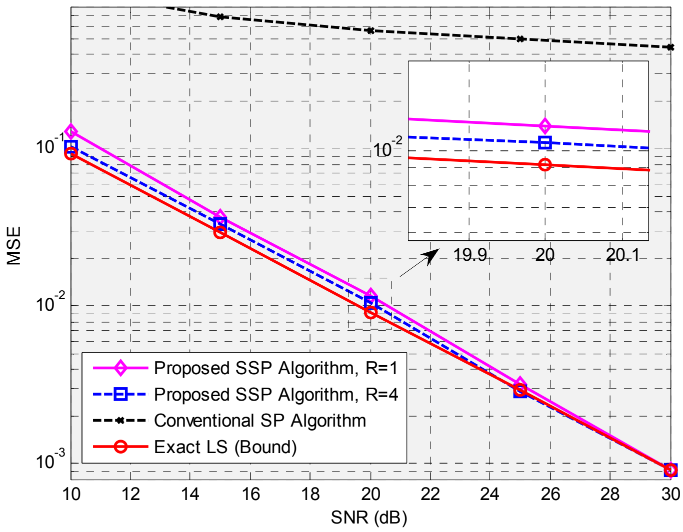



**Figure 2: MSE performance comparison over the ITU Vehicular B channel**

It is worth noting that sensing matrix $\mathbf{\Phi}$ depends on the pilot position $\Omega$ and pilot sequences $\{\mathbf{s}_m\}_{m=1}^{M}$. Although simulation results indicate that the specific $\Omega$ and $\{\mathbf{s}_m\}_{m=1}^{M}$ as mentioned in this paper have reliable performance due to the near-orthogonal columns of $\mathbf{\Phi}$, the optimal design of the pilot position and pilot sequence remains an interesting problem to be studied in the future.

## 5. Conclusion

This paper focuses on the downlink training and channel estimation for large-scale MIMO systems. In contrast to standardized orthogonal pilots with the prohibitive overhead increasing with the number of transmit antennas, the proposed superimposed pilot design based on structured CS can efficiently solve the pilot overhead problem. At the receiver, the proposed SSP algorithm can exploit the spatial and temporal correlations of large-scale MIMO channels for simultaneous recovery of multiple channels. Moreover, the proposed superimposed pilot design and the corresponding channel estimator can be applied in the uplink too, and conventional small-scale MIMO can also adopt the proposed scheme to reduce the pilot overhead and improve the channel estimation performance. The remained problem to be solved next is the optimal design of the pilot position and pilot sequence for large-scale MIMO systems.

## 6. References


1. T. L. Marzetta, “Noncooperative cellular wireless with unlimited numbers of base station antennas,” *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3590-3600, Nov. 2010.
2. F. Rusek, D. Persson, B. K. Lau, E. G. Larsson, T. L. Marzetta, O. Edfors, and F. Tufvesson, “Scaling up MIMO: Opportunities and challenges with very large arrays,” *IEEE Signal Process. Mag.*, vol. 30, no. 1, pp. 40-60, Jan. 2013.
3. F. Fernandes, A. E. Ashikhmin, and T. L. Marzetta, “Inter-cell interference in noncooperative TDD large scale antenna systems.” *IEEE J. Sel. Areas Commun.*, vol. 31, no. 2, pp. 192-201, Feb. 2013.
4. M. Duarte and Y. Eldar, “Structured compressed sensing: from theory to applications,” *IEEE Trans. Signal Process.*, vol. 59, no. 9, pp. 4053-4085, Sep. 2011.
5. W. Dai and O. Milenkovic, “Subspace pursuit for compressive sensing signal reconstruction,” *IEEE Trans. Inf. Theory*, vol. 55, no. 5, pp. 2230-2249, May 2009.
6. Y. Barbotin and M. Vetterli, “Estimation of sparse MIMO channels with common support,” *IEEE Trans. Commun.*, vol. 60, no. 12, pp. 3705-3716, Dec. 2012.
7. L. Dai, J. Wang, Z. Wang, P. Tsiaflakis, and M. Moonen, “Spectrum and energy-efficient OFDM based on simultaneous multi-channel reconstruction,” *IEEE Trans. Signal Process.,* vol. 61, no. 23, pp. 6047-6059, Dec. 2013.