

Efficient Method of Moments Analysis of Metasurface Antennas

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Abstract

This paper presents an efficient approach for the method of moments (MoM) analysis of metasurface (MTS) antennas. Instead of simulating the actual structure, made of several thousands of subwavelength patches, the presence of the MTS is accounted for using an impedance boundary condition in the integral equation. Then, when the MTS antenna exhibits rotational symmetry, the integral equation can be efficiently solved adopting a formulation for bodies of revolution. On the other hand, a novel type of basis functions can also be applied to the analysis of planar MTSs without any particular symmetry. The advantages of such basis, in both the space and spectral-domain, are outlined. Indeed, their closed-form spectrum allows one to efficiently compute the MoM impedance matrix using the spectral-domain approach. More importantly, these basis functions represent the global evolution of the surface current density in an effective manner. In both approaches, one obtains a drastic reduction in the number of unknowns, with respect to the cases in which the actual structure has been meshed with sub-entire domain basis.

1. Introduction

The application of metasurfaces (MTS) [1, 2] to the design of microwave circuits and antennas has undergone an unprecedented development during the last few years. More precisely, MTSs have been exploited to control the propagation of surface waves [3-6] or to transform surface waves into leaky-waves [7-9], obtaining radiation. A MTS leaky wave antenna typically consists of a thin metamaterial layer, which is made of several thousands of subwavelength patches printed on a dielectric slab. Moreover, it is interesting to notice that the patches' geometrical parameters tend to exhibit a smooth variation with respect to the wavelength.

Naturally, a crucial step for designing these structures will be the choice of an efficient analysis method. Method of moments (MoM) is particularly suitable for analyzing MTS antennas: the radiation condition is already accounted for in the integral equation and the use of appropriate Green's functions allows one to limit the presence of unknowns to the metallizations. On the other hand, the small size of the patches leads to fine mesh details that have a negative impact on the condition number of the MoM impedance matrix. This calls for the use of efficient pre-conditioners to get iterative methods to converge, or to obtain an accurate solution in a reasonable number of iterations. For instance, one could use the fast iterative method in [10, 11], compressive domain decomposition [12, 13] or a combination of both. Nevertheless, even when exploiting fast methods, it is difficult to reduce the computation time in such a way that more than a few parameters are included in the optimization step (see, for instance, [9, Sec. VII]). This is the reason why an alternative to the full-wave analysis of the actual structure has been investigated. Indeed, since the patches' geometrical parameters present a smooth spatial evolution, the tensor impedance boundary condition (IBC) associated with the local problem will also be smooth. Therefore, in the first instance, one could analyze the equivalent problem in which the actual patches have been substituted with a slowly varying IBC ($\underline{\underline{Z}}_s$). The introduction of $\underline{\underline{Z}}_s$ leads to a drastic reduction in the number of unknowns required to discretize the problem, thus increasing the computational efficiency.

In the following section, the efforts carried out in this direction at the University of Siena are briefly described. First, it is explained that a MTS antenna with rotational symmetry can be efficiently analyzed using a body of revolution (BoR) formulation [14]. Then, a complementary approach is presented for planar MTS antennas that do not exhibit any particular symmetry. Finally, in the last section, conclusions are drawn.

2. Method of Moments Analysis of Metasurface Antennas

Let us first consider the special case of axially symmetric MTS structures, called bodies of revolution (BoR). In this case, the computational burden can be tremendously reduced since the three dimensional problem is reduced to a two dimensional one. Such advantage opens the possibility of analyzing and accurately synthesizing radiating objects of hundreds of wavelengths, with simulation times of the order of few tens of seconds per frequency. This allows one to

couple a MoM-BoR full-wave solver with a multi-parametric optimization process. Nevertheless, it is important to notice that composite metal-dielectric materials require a special treatment. In such cases, the simultaneous presence of electric and magnetic currents at the dielectric/air interface complicates the integral kernels and introduces additional difficulties in the MoM reaction integral. The nature and possible solutions to these difficulties have been described in detail in [15].

On the other hand, in the case of a planar MTSs on an infinite grounded slab without any particular symmetry, one can impose the relevant impedance boundary condition in the electric field integral equation (EFIE), which leads to

$$\hat{n} \times \left[\iint_{S'} \bar{G}^{EJ}(\bar{\rho}, \bar{\rho}') \cdot \vec{J}_s(\bar{\rho}') dS' - \underline{\underline{Z}}_S(\bar{\rho}') \cdot \vec{J}_s(\bar{\rho}') \right] = -\hat{n} \times \vec{E}^{inc} \quad (1)$$

with $\bar{\rho}'$ and $\bar{\rho}$ being the source and observation points, respectively. \bar{G}^{EJ} represents the slab's electric dyadic Green's function, \vec{J}_s stands for the unknown surface current density on the MTS, and \vec{E}^{inc} is the incident electric field. The set of basis functions proposed for the solution of (1) can be written as

$$f_{n,\alpha,\beta}^p(\rho, \phi) = e^{-jn\phi} \Psi^n(\rho, \alpha, \beta) \quad (2)$$

where p is the direction of the current, for instance along x or along y , and the function Ψ^n is given by

$$\Psi_n(\rho, \alpha, \beta) = \frac{1}{2\alpha^2} e^{-\frac{\beta^2 + \rho^2}{4\alpha^2}} I_n\left(\frac{\beta\rho}{2\alpha^2}\right) \quad (3)$$

with $\rho = \sqrt{x^2 + y^2}$ being the space variable, and $I_n(\cdot)$ the modified Bessel function of the first kind and order n . It can be shown [16, 17] that the maximum's position will be approximately given by β , and the width of the function by α . One can also establish that the position of the maximum does not change with n as long as $\alpha < \beta/4$. Fig. 1 shows how the width of the basis function can be controlled by adjusting the value of α while β and n are kept constant. In Fig. 2 one can observe how the function's radial position can be fixed by changing β .

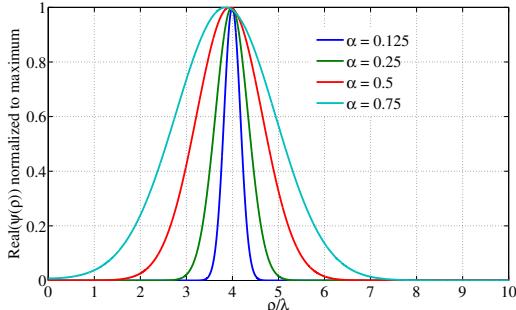


Fig. 1. Basis function in the space domain for $\beta = 4\lambda$, $n=0$ and different values of α .

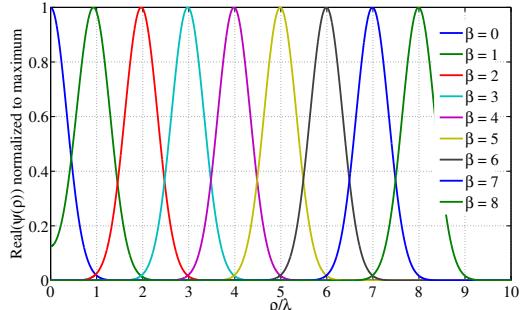


Fig. 2 Basis function in the space domain for $\alpha = 0.25$, $n=0$ and different values of β .

The spectral domain counterpart of (2) reads as

$$F_{n,\alpha,\beta}^p(k_\rho, \chi) = 2\pi j^n e^{-jn\chi} \Phi_n(k_\rho, \alpha, \beta) \quad (4)$$

with k_ρ being the radial variable in the spectral domain, whereas χ stands for the spectral angular variable. The function Φ_n in (4) is defined as

$$\Phi_n(k_\rho, \alpha, \beta) = e^{-\alpha^2 k_\rho^2} J_n(\beta k_\rho) \quad (5)$$

where $J_n(\cdot)$ is the Bessel function of the first kind and order n . Equation (4) can be exploited for the efficient evaluation of MoM reaction integrals in the spectral-domain. It has been shown in [17] that the use of the proposed basis for the analysis of a planar MTS antenna leads to a reduction in the number of unknowns by two orders of magnitude, with respect to the same problem discretized with Rao-Wilton-Glisson basis [18], while keeping the same level of accuracy.

3. Conclusion

An efficient approach has been discussed for the method of moments (MoM) analysis of metasurface (MTS) antennas. Instead of simulating the actual structure, made of several thousands of subwavelength patches, the presence of the MTS is accounted for in the integral equation using an impedance boundary condition. Then, when the MTS exhibits rotational symmetry, it can be solved using an efficient formulation for bodies of revolution (BoR). On the other hand, when analyzing planar MTS antennas which do not exhibit any particular symmetry, one may exploit a novel type of basis function with advantageous properties in both the space and spectral-domain. The closed-form spectrum of the introduced basis functions allows one to efficiently compute the MoM impedance matrix using the spectral-domain approach. More importantly, in both approaches, the global evolution of the surface current density is represented in an effective manner, which results in a reduction of the number of unknowns, if compared with subentire domain basis functions, like Rao-Wilton-Glisson functions defined on triangular domains.

4. References

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