# New Expressions of Fringe Wave in EEC Consisting of Keller's Diffraction Coefficient <br> Maifuz Ali ${ }^{1}$ and Makoto Ando ${ }^{2}$ <br> ${ }^{1}$ Post-Doctoral Fellow, Dept. Electrical and Electronic Eng. Tokyo Institute of Technology, Tokyo, Japan <br> e-mail: maifuzali@lycos.com \& maifuzali@antenna.ee.titech.ac.jp <br> ${ }^{2}$ Professor, Dept. Electrical and Electronic Eng. <br> Tokyo Institute of Technology, Tokyo, Japan <br> e-mail: mando@antenna.ee.titech.ac.jp 


#### Abstract

Fringe wave current components of modified edge representation (MER) equivalent edge currents (EECs) are modified by Fresnel zone number. The dipole wave scattering from flat square plates are detailed with numerical examples and the accuracy of the fields predicted by this new fringe wave expressions is confirmed by comparing with MoM.


## 1 Introduction

The method of equivalent edge currents (MEC) is one of a high frequency approximation for diffractions. However equivalent edge currents (EECs) are clearly defined only at the diffraction points and ambiguities exist at the remaining part of the periphery. As a result the line integration of EECs along the complete periphery cannot be conducted in a straight forward manner. Derivation of EECs at general edge points has remained a problem for GTD-MEC [1]. Modified edge representation (MER) has been proposed to solve the problem in $[2,3]$. The EECs are determined for the fictitious edge segment at every point on the periphery in terms of classical Keller's diffraction coefficients for general incidence [4].

As in well known, total EECs (GTD-EECs) consist of the physical optics components (PO-EECs) and the fringe wave components (FW-EECs). MER with classical PO diffraction coefficient recover PO surface integration perfectively is demonstrated in [2]. On the other hand, FW-EECs account for the radiation from fringe waves peculiar to the edge. GTD-EECs as directly applied to MER actually lead to erroneous field when the observer is in the vicinity of the incident shadow and reflection shadow boundaries (ISB/RSB) for closely located source. Authors in [3] presented a simple approximation for for inclusion of FW-EECs in MER, where FW-EECs are weighted by a function of the angle between the modified edge and the real edge. This weighting function gives much better results than GTD-EECs but still there are some error at and near the boundaries (SBs). This error suggests the needs for new FW-EECs specially refined for MER. In this paper a function based on Fresnel zone number (FZN) distance [5] between the reflection point and the point of integration is introduced for the weighting of the FW-EECs. This function suppresses the contribution of FW-EECs for the observer in the vicinity of ISB/RSB and enhances the accuracy of MER even in grazing incidence, as is demonstrated by numerically comparing with MoM and PO.

## 2 Modified Edge Representation (MER) for the Definition of EEC

Considering an edge-fixed coordinate system, the position of the source and observer are defined by $\left(r_{i}, \beta_{i}, \phi_{i}\right)$ and ( $r_{o}, \beta_{o}, \phi_{o}$ ) respectively. The unit vector $\widehat{e}$ is along the edge and considered according to the counter clockwise direction and a fictitious edge $\widehat{\tau}$ (see Fig. 1(a)) at every point of integration is defined to satisfy the diffraction law for the given directions of incidence and observation $[2,3]$.


Figure 1: (a) Definition of the modified edge (b) A square plate illuminated by a dipole.

For soft and hard boundary conditions EECs along the edge of the scatterer $\hat{e}$ are given by [6, eq. (13-103)],

$$
\left[\begin{array}{c}
\overrightarrow{I_{s}}  \tag{1}\\
\overrightarrow{I_{h}}
\end{array}\right]=\frac{\jmath}{\eta k \sin \beta_{i}}\left[\begin{array}{cc}
D_{s} & D_{x h} \\
D_{x s} & D_{h}
\end{array}\right]\left[\begin{array}{c}
E_{\beta_{i}}^{i} \\
E_{\phi_{i}}^{i}
\end{array}\right] \hat{e}
$$

where $D_{s}, D_{x s}, D_{h}$ and $D_{x h}$ are the diffraction coefficients at the integration point $Q$ calculated with respect to a modified edge $\widehat{\tau}$ for a given source and observation as in Table. 1. $E_{\beta_{i}}^{i}(Q)$ and $E_{\phi_{i}}^{i}(Q)$ are parallel and perpendicular incident field components with respect to $\widehat{\tau}$ at the point $(Q)$. The far-zone electric field radiated by EECs is obtained by complete line integration along the real edge of scatterer $\widehat{e}$ as $[3]$, [ 6, eq. (13-104)];

$$
\begin{align*}
& \overrightarrow{E_{s}^{d}}=\frac{j \eta k}{4 \pi} \oint \hat{r_{o}} \times\left[\hat{r_{o}} \times \overrightarrow{I_{s}}\right] \frac{e^{-j k\left|r_{o}\right|}}{\left|r_{o}\right|} d l  \tag{2a}\\
& \overrightarrow{E_{h}^{d}}=\frac{j \eta k}{4 \pi} \oint\left[\hat{r_{o}} \times \overrightarrow{I_{h}}\right] \frac{e^{-j k\left|r_{o}\right|}}{\left|r_{o}\right|} d l \tag{2b}
\end{align*}
$$

Table 1: Diffraction coefficients of PO, FW and GTD

|  | $D_{s}$ | $D_{x s}$ | $D_{x h}$ | $D_{h}$ |
| :--- | ---: | ---: | ---: | :---: |
| PO | $\tan \left(\frac{\phi_{o}-\phi_{i}}{2}\right)-\tan \left(\frac{\phi_{o}+\phi_{i}}{2}\right)$ | 0 | $-2 \cos \beta_{i}$ | $\tan \left(\frac{\phi_{o}-\phi_{i}}{2}\right)+\tan \left(\frac{\phi_{o}+\phi_{i}}{2}\right)$ |
| FW | $\frac{1-\sin \left(\frac{\phi_{o}-\phi_{i}}{2}\right)}{\cos \left(\frac{\phi_{o}-\phi_{i}}{2}\right)}-\frac{1-\sin \left(\frac{\phi_{o}+\phi_{i}}{2}\right)}{\cos \left(\frac{\phi_{o}+\phi_{i}}{2}\right)}$ | 0 | $2 \cos \beta_{i}$ | $\frac{1-\sin \left(\frac{\phi_{o}-\phi_{i}}{2}\right)}{\cos \left(\frac{\phi_{o}-\phi_{i}}{2}\right)}+\frac{1-\sin \left(\frac{\phi_{o}+\phi_{i}}{2}\right)}{\cos \left(\frac{\phi_{o}+\phi_{i}}{2}\right)}$ |
| GTD(PO + FW) | $\frac{1}{\cos \left(\frac{\phi_{o}-\phi_{i}}{2}\right)}-\frac{1}{\cos \left(\frac{\phi_{o}+\phi_{i}}{2}\right)}$ | 0 | 0 | $\frac{1}{\cos \left(\frac{\phi_{o}-\phi_{i}}{2}\right)}+\frac{1}{\cos \left(\frac{\phi_{o}+\phi_{i}}{2}\right)}$ |

### 2.1 Diffraction from a Square Plate

A dipole along the x -axis is considered very near to a square PEC plate as shown in Fig. 1(b). The diffracted field and total field patterns in $\phi=33.75^{\circ}$ and $\phi=45^{\circ}$ plane are compared with Wipl-D simulated results shown in Figs. 2(a) and 2(b) respectively. For the observations at and near the RSB and ISB, GTDMER deviates faraway from Wipl-D simulated results while PO matches with it almost perfectly. At the


Figure 2: Total and diffracted field patterns (a) at $\phi=33.75^{\circ}$ plane (b) at $\phi=45^{\circ}$ plane.
same time, however, it is observed that at the angles far from SBs, the GTD-MER matches with the Wipl-D simulated results very well when compared with the PO. The above observation implies that the FW defined by the difference between GTD- and PO-MER should be modified especially for the observer near SBs for the grazing incidence. Actually, the FW in Table 1 have been used widely simply because of its finiteness in contrast with the infinities of GTD and asymptotic PO of Keller type. Here our claim is that finiteness is not enough for FW components and they should be revised to enhance the accuracy of GTD-MER.

## 3 Correction Factor for Fringe Wave using the Fresnel Zone Number (FZN)

A weighting function based on FZN of [5] is introduced to correct the FW at and near the SBs as follows.
Each integration point along the periphery of the scatterer is counted in terms of Fresnel zone number $n$ which is defined as $n=2 L / \lambda$, where $L$ is the path difference between two path; direct path from source $(\mathrm{S})$ to observer $(\mathrm{O})$ and diverted path from source $(\mathrm{S})$ to observer $(\mathrm{O})$ via the point on the scatterer $(\mathrm{P})$, and $\lambda$ is the wavelength. The distance between point of reflection and integration is defined by Fresnel zone number as $\left|n_{P}-n_{R}\right|=\Delta n^{R} \leq \Delta n_{B}^{R}$ (see Fig. 3(a)). Where, $n_{P}$ and $n_{R}$ are the Fresnel zone numbers at the point of integration on edge of the scatterer and the reflection point ( R ), respectively. The parameter $\Delta n_{B}^{R}$ is the number which determines limiting value of $\Delta n^{R}$.


Figure 3: Weighting function to modify fringe wave (a) Fresnel zone (b) Weighting Function $\overline{E Y E}\left(\frac{\Delta n^{R}}{\Delta n_{B}^{R}}\right)$.

A weighting function based on this FZN (see Fig. 3(b)) is proposed to improve FW components given by

$$
\begin{align*}
\overline{E Y E}\left(\frac{\Delta n^{R}}{\Delta n_{B}^{R}}\right) & =\frac{1}{2}\left\{1-\cos \left(\frac{\Delta n^{R}}{\Delta n_{B}^{R}} \pi\right)\right\} & & \text { for } \Delta n^{R} \leq \Delta n_{B}^{R} \\
& =1 & & \text { for } \Delta n^{R}>\Delta n_{B}^{R} \tag{3}
\end{align*}
$$

when integration point merges with the point of reflection $\Delta n^{R}=0, \overline{E Y E}\left(\frac{\Delta n^{R}}{\Delta n_{B}^{R}}\right)=0$. If the point of integration is far away from the point of reflection such that $\Delta n^{R}>\Delta n_{B}^{R}, \overline{E Y E}\left(\frac{\Delta n^{R}}{\Delta n_{B}^{R}}\right)=1$. The fringe wave (FW) diffraction coefficient is modified with the help of $\overline{E Y E}\left(\frac{\Delta n^{R}}{\Delta n_{B}^{R}}\right)$ and new GTD diffraction coefficients become as

$$
\left[\begin{array}{ll}
D_{s}^{\text {new }} & D_{x h}^{\text {new }}  \tag{4}\\
D_{x s}^{\text {new }} & D_{h}^{\text {new }}
\end{array}\right]=\left[\begin{array}{ll}
D_{s}^{P O} & D_{x h}^{P O} \\
D_{x s}^{P O} & D_{h}^{P O}
\end{array}\right]+\overline{E Y E}\left(\frac{\Delta n^{R}}{\Delta n_{B}^{R}}\right)\left[\begin{array}{ll}
D_{s}^{F W} & D_{x h}^{F W} \\
D_{x s}^{F W} & D_{h}^{F W}
\end{array}\right]
$$

Now, new $\overline{E Y E}$ augmented new GTD MER (PO MER $+\overline{E Y E} \times \mathrm{FW}$ MER) gives better agreement in all angle of observation including at and near BSs even for grazing incidence as shown in the Fig. 2.

## 4 Conclusion

In this work, the contribution of FW-EECs to total EECs are weighted by a function based on Fresnel zone number. Uniform fields are predicted everywhere including ISB and RSB using only classical diffraction coefficients and the numerical results for diffraction from flat plates demonstrate the potential of these EECs.

## Acknowledgment

This work was conducted in part as "the Research and Development for Expansion of Radio Wave Resources " under the contract of the Ministry of Internal Affairs and Communications, Japan.

## References

[1] M. Ando, T. Murasaki, and T. Kinoshita, "Elimination of false singularities in gtd equivalent edge currents," in Microwaves, Antennas and Propagat., IEE Proceedings H, vol. 138, no. 4, Aug 1991, pp. 289 - 296.
[2] T. Murasaki and M. Ando, "Equivalent edge currents by the modified edge representation: Physical optics components," Electronics, IEICE Transactions on, vol. E75-c, no. 5, pp. 617-624, May 1992.
[3] T. Murasaki, M. Sato, Y. Inasawa, and M. Ando, "Equivalent edge currents for modified edge representation of flat plates: Fringe wave components," Electronics, IEICE Transactions on, vol. E76-c, no. 9, pp. 1412-1419, Sep. 1993.
[4] J. B. Keller, "Geometrical Theory of diffraction," J.Opt. Soc. Amer., vol. 52, pp. 116-130, Feb. 1962.
[5] T. Kohama and M. Ando, "Localization of radiation integrals using fresnel zone number," IEICE Trans. Electron, vol. E95-C, no. 5, pp. 928-935, May 2012.
[6] Constantine A. Balanis, Advanced Engineering Electromagnetics. New York: John Wiley \& Sons Inc., 1989.
[7] P. Ya. Ufimtsev, "Method of edge wave in the physical theory of diffraction," Air Force System Command, Foreign Tech. Div, 1971.

