Research on parameter estimation of MPSK signals based on the generalized second-order cyclic spectrum

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Abstract-In order to solve the problem that the parameter estimation algorithm based on second-order cyclic statistics will significantly degrade in alpha stable distribution noise environment, a parameter estimation algorithm for M-ary phase shift keying (MPSK) signals based on the generalized second-order cyclic spectrum was proposed. The generalized second-order cyclic spectrum of MPSK signals was studied. For phase shift keying (PSK) signals with different *M* values, the relationship between the parameters of the signals (carrier frequency and symbol rate) and the parameters (spectrum frequency and cyclic frequency) in the corresponding generalized second-order cyclic spectrum were analyzed. And thus, the estimation algorithms for carrier frequency and symbol rate suitable for all PSK signals were given. The experimental results show that in alpha stable distribution noise environment, the algorithm based on generalized second-order cyclic spectrum is obviously superior to that based on traditional second-order cyclic spectrum, and the robustness of the algorithm is good.

Keywords-parameter estimation; alpha stable distribution; generalized second-order cyclic spectrum; carrier frequency; symbol rate

1. Introduction

Parameter estimation is one of the core issues in the field of communication signal processing. Previous researches on parameter estimation employ Gaussian noise as the background noise, for Gaussian distribution are easy to be analyzed and calculated. However, recent studies have found that the noise in wireless communications often shows a significant characteristic of short-time impulse with great amplitude, so background noise cannot be simply described with Gaussian distribution. More and more researchers believe that alpha stable distribution is the best model to describe the non-Gaussian impulse noise, and do a lot of studies in alpha stable distribution noise [1]-[5].

MPSK signals in communication are typical cyclostationary signals. Second-order cyclic statistics, including cyclic autocorrelation function and cyclic spectral density function, are effective tools for studying cyclostationary signals. Reference [6] proposes a parameter estimation algorithm for MPSK signals, the algorithm achieve a valid estimation for carrier frequency and symbol rate by searching special peaks of second-order cyclic spectrum in Gaussian noise. This is a representative program for parameter estimation algorithms based on second-order cyclic spectrum. Such algorithms have good performance in Gaussian noise, but are not suitable for alpha stable distribution noise. Alpha stable distribution noise does not have greater than or equal to α-order statistics(α stands for characteristic exponent, $0<\alpha<2$), so when the background noise is alpha stable distribution noise, the second-order cyclic spectrum of communication signals will fail, and the corresponding parameter estimation algorithm will lose efficacy. Fig. 1 shows the second-order cyclic spectrum of BPSK signal in alpha stable distribution noise ($\alpha=1.5$, without loss of generality, noise employs standard symmetric Alpha stable distribution-SαS as model). For the SαS noise do not have finite second moment that leads to the noise variance losing meaning, so a mixed signal to noise ratio-MSNR is employed, and MSNR=0dB is selected in Fig. 1. From Fig. 1, we can see that the second-order cyclic spectrum of BPSK signal is destroyed by Alpha stable distribution noise. To solve this problem, Reference [7] proposes a parameter estimation algorithm based on fractional lower-order cyclic spectrum. A deficiency of this method is that the index value of fractional lower-order cyclic statistic needs to be set according to the characteristic exponent value of alpha stable distribution noise. However, the characteristic exponent value is difficult to be estimated accurately. This paper proposes a parameter estimation algorithm based on generalized second-order cyclic spectrum, it can keep the spectral

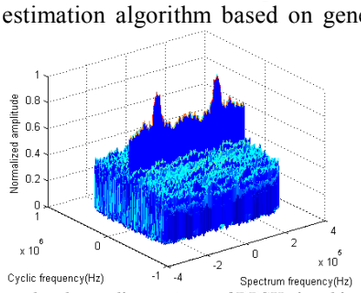

Fig.1. Second-order cyclic spectrum of BPSK signal in Alpha stable distribution noise

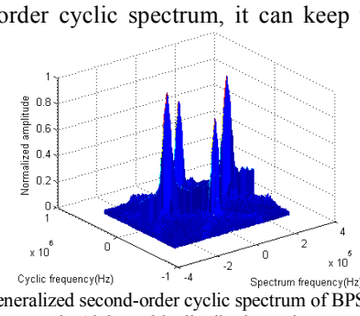

Fig.2. Generalized second-order cyclic spectrum of BPSK signal in Alpha stable distribution noise

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characteristics of cyclic spectrum, but also to avoid the estimation program of the characteristic exponent, so the algorithm have good estimation results in alpha stable distribution noise.

2. Generalized Second-Order Cyclic Statistics

Generalized second-order cyclic statistics are generalized extensions to traditional second-order cyclic statistics, including generalized second-order cyclic autocorrelation function and generalized second-order cyclic spectrum density function.

Definition 1: Generalized second-order cyclic autocorrelation function can be expressed as[8]

$$GR_{\mathrm{X}}^{\varepsilon}(\tau)=\frac{1}{T}\int_{-T_0/2}^{T_0/2} f\left(x\left(t+\frac{\tau}{2}\right)\right) f^{*}\left(x\left(t-\frac{\tau}{2}\right)\right)\exp(-\mathrm{j}2\pi\varepsilon t)\,dt \tag{1}$$

Where ε stands for cyclic frequency, $f(x)=\dfrac{\arctan(|x+\mathrm{j}\mathrm{H}(x)|)}{|x+\mathrm{j}\mathrm{H}(x)|}x$, $\mathrm{H}(\cdot)$ stands for Hilbert transform.

Definition 2: Fourier transform of generalized second-order cyclic autocorrelation function is called generalized second-order cyclic spectrum density function, referred to as generalized second-order cyclic spectrum, whose expression is as follows[8]:

$$GS_{\mathrm{X}}^{\varepsilon}(f)=\int_{-\infty}^{\infty} GR_{\mathrm{X}}^{\varepsilon}(\tau)\exp(-\mathrm{j}2\pi f\tau)d\tau \tag{2}$$

Where f is the spectrum frequency of signal.

Nonlinear transformation $f(\cdot)$ is also known as generalized transformation its role has been described in reference [8]. Any communication signal can be represented in the form of $x=r\cos\theta$, here r represents the amplitude, θ represents phase, so $f(x)=\arctan r\cdot\cos\theta$. It can be seen that generalized transformation maps the amplitude of the signal to a finite interval. For signals with noise, the amplitude of noise is also mapped to the finite interval, but the cycle and phase of signal are not changed, it is particularly important to maintain the phase of MPSK signals.

Generalized second-order cyclic statistics can solve the problem that alpha stable noise leads to the second-order cyclic statistics losing efficacy. Fig. 2 shows the generalized second-order cyclic spectrum of the BPSK that is in the same condition with Fig. 1, it can be seen that spectral features are improved greatly.

3. Parameter Estimation Based On Generalized Second-Order Cyclic Spectrum

MPSK is commonly used as a modulated signal in communication, and is a special class of non-stationary signals. Statistical properties of such signals show some cyclical changes. Cyclostationarity can be used to the estimation of carrier frequency and symbol rate.

3.1 Generalized Second-Order Cyclic Spectrum of MPSK Signals

Complex analytic form of MPSK signal can be expressed as

$$x(t)=\sqrt{E}V(t)\exp\{\mathrm{j}(2\pi f_c t+\phi_0)\} \tag{3}$$

Where E stands for the average power of the signal, $V(t)=\sum_n \varphi_n q(t-nT)$, $\varphi_n\in\left\{e^{\mathrm{j}\frac{\pi}{M}(2m-1)}\right\}$, $M=2^k$, $m=1,2,\cdots\cdots,M$. $q(\cdot)$ represents a rectangular pulse, T is symbol period, f_c is carrier frequency, ϕ_0 is initial phase.

According to equation (1) and equation (2), generalized second-order cyclic spectrums of MPSK signals can be obtained as follows:

When $M=2$ (BPSK),

$$GS_{\mathrm{BPSK}}^{\varepsilon}(f)=\begin{cases}\dfrac{\arctan^2\sqrt{E}}{4T}\left[Q\left(f-f_c+\dfrac{\varepsilon}{2}\right)Q^{*}\left(f-f_c-\dfrac{\varepsilon}{2}\right)+Q\left(f+f_c+\dfrac{\varepsilon}{2}\right)Q^{*}\left(f+f_c-\dfrac{\varepsilon}{2}\right)\right],\varepsilon=\dfrac{n}{T}\\ \dfrac{\arctan^2\sqrt{E}}{4T}\left[\mathrm{e}^{\mathrm{j}2\varphi_0}Q\left(f-f_c+\dfrac{\varepsilon}{2}\right)Q^{*}\left(f+f_c-\dfrac{\varepsilon}{2}\right)+\mathrm{e}^{-\mathrm{j}2\varphi_0}Q\left(f+f_c+\dfrac{\varepsilon}{2}\right)Q^{*}\left(f-f_c-\dfrac{\varepsilon}{2}\right)\right],\varepsilon=\pm 2f_c+\dfrac{n}{T}\end{cases} \tag{4}$$

When $M\geq 4$

$$GS_{\mathrm{MPSK}}^{\varepsilon}(f)=\frac{\arctan^2\sqrt{E}}{4T}\left[Q\left(f-f_c+\frac{\varepsilon}{2}\right)Q^*\left(f-f_c-\frac{\varepsilon}{2}\right)+Q\left(f+f_c+\frac{\varepsilon}{2}\right)Q^*\left(f+f_c-\frac{\varepsilon}{2}\right)\right],\varepsilon=\frac{n}{T} \tag{5}$$

Where, $Q(f)=\frac{\sin(\pi f t)}{\pi f}$, n represents integer, $\frac{n}{T}$ represents integer multiple of the symbol rate.

3.2 Description of Parameter Estimation Algorithm

From the generalized second-order cyclic spectrums of MPSK signals we can know that signals have cyclic frequencies n/T and $\pm 2f_c+n/T$ when M=2, so $\varepsilon=\pm 2f_c+n/T$ can be used to estimate the parameters(carrier frequency and symbol rate) of BPSK. Signals only have cyclic frequencies n/T when $M\geq 4$. Therefore, $\varepsilon=\pm 2f_c+n/T$ is not available in the case where M is not known. In this paper, the method applies to all MPSK signals, so $\varepsilon=n/T$ are used to estimate the signal parameters.

For the generalized second-order cyclic spectrum formulas of MPSK signals when $\varepsilon=n/T$, taking the envelope at $\varepsilon=0$, the following formula can be obtained

$$\left|GS_{\mathrm{MPSK}}^{0}(f)\right|=\left|\frac{\arctan^2\sqrt{E}}{4T}\left[Q^2(f-f_c)+Q^2(f+f_c)\right]\right| \tag{6}$$

From the nature of the function $Q(f)$, equation (6) obtains the maximum value at $f=\pm f_c$. Therefore, the carrier frequency can be estimated by searching the maximum value of the zero cyclic frequency section in generalized second-order cyclic spectrum.

Similarly, For the generalized second-order cyclic spectrum formulas of MPSK signals when $\varepsilon=n/T$, taking the envelope at $f=f_c$ or $f=-f_c$, the following formula can be obtained

$$\left|GS_{\mathrm{MPSK}}^{n/T}(\pm f_c)\right|=\left|\frac{\arctan^2\sqrt{E}}{4T}\left[Q^2\left(\frac{\varepsilon}{2}\right)+Q\left(2f_c+\frac{\varepsilon}{2}\right)Q^*\left(2f_c-\frac{\varepsilon}{2}\right)\right]\right| \tag{7}$$

From the nature of the function $Q(f)$, $Q(2f_c+\varepsilon/2)Q^*(2f_c-\varepsilon/2)\approx 0$, equation (7) obtains the maximum value at $f=\pm f_c$ and the second largest value at $\varepsilon=\pm 1/T$ .Therefore, the symbol rate can be estimated by searching the second largest value of $f=\pm f_c$ section in generalized second-order cyclic spectrum.

For MPSK signals mixed with SαS noise, based on the above analysis, the steps of estimating the carrier frequency and symbol rate may be obtained as follows:

Step 1: Generalized second-order cyclic spectrum can be calculated according to equation (1) and equation (2). It has the same symmetry with the traditional second-order cyclic spectrum, so in order to reduce the amount of computation, only part ($0\leq\varepsilon<\infty$, $0\leq f<\infty$) of spectrum be calculated.

Step 2: Search the maximum value at $\varepsilon=0$ section, and note the corresponding coordinate f_1, the estimated value of the carrier frequency is $\hat{f}_c=f_1$.

Step 3: On the basis of the carrier frequency has been estimated in step 2, search the maximum value at $f=\hat{f}_c$ section (except $\varepsilon=0$), and note the corresponding coordinate ε_1, the estimated value of the symbol rate is $\hat{r}_b=\varepsilon_1$.

4. Simulation and Results Analysis

In order to demonstrate the performance of the proposed algorithm, we use MATLAB software to carry out the related simulations. Signal parameters are: carrier frequency $f_c=150000$ Hz, sampling frequency $f_s=1200000$ Hz, symbol rate $r_b=50000$ B, average power E=1. MPSK signals have the same generalized second-order cyclic spectrum feature, so simulation employs QPSK and BPSK signal representing all MPSK signals.

Simulation 1: Evaluating algorithm performance in alpha stable distribution noise. In the background of additive SαS noise ($\alpha=1.5$), estimation methods based on generalized second-order cyclic spectrum and traditional second-order cyclic spectrum are tested, 100 tests for each MSNR. Normalized root mean square error is used to evaluate the performance of algorithms. The simulation results are shown in Fig.5.

As can be seen from the estimated results of symbol rate in Fig.5, the estimation method based on traditional second-order cyclic spectrum severely degraded within the range of the entire MSNR, but the proposed method is valid when MSNR>-2 dB. For the carrier frequency, the estimation performance of the traditional method is poor

at low MSNR, and the performance of two methods is quite at high MSNR. This shows that the generalized second-order cyclic spectrum can suppress alpha stable distribution noise effectively, and the method based on generalized second-order cyclic spectrum has good estimation performance. Overall, for BPSK and QPSK, the parameter estimation performance of the proposed method is quite. Carrier frequencies and symbol rates of the two signals can be estimated effectively when MSNR is greater than a certain value. This shows that the proposed method is suitable for MPSK signals.

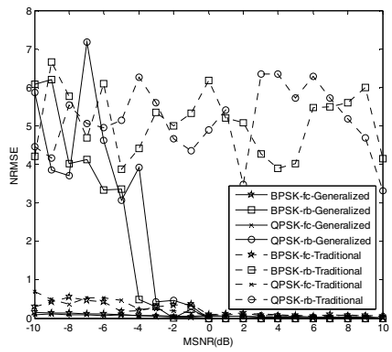

Fig.5. Performance evaluation figure of algorithm in standard SαS noise

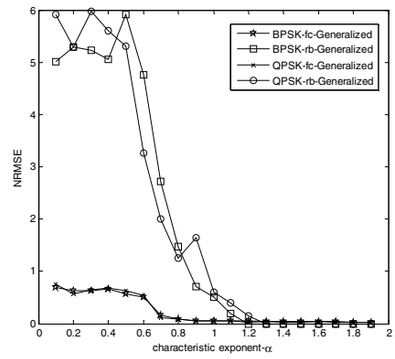

Fig.6. Estimation results in different values of α

Simulation 2: In addition to the MSNR, α value also is a key factor affecting the performance of the algorithm. This simulation evaluates algorithm performance in different α values. For SαS noise, MSNR=0dB, α changes with the step 0.1 in the interval [0.1, 1.9]. 100 tests for each α value, the estimation results are shown in Fig.6.

Fig. 6 shows that, carrier frequency estimation results are relatively stable in the whole α interval. As can be seen simultaneously, the smaller the α value and the greater error of symbol rate estimation. The reason is that the smaller α value and the more obvious peak characteristics of alpha stable distribution, the feature point value of the symbol rate estimation is smaller than the feature point value of the carrier frequency estimation, and it is susceptible to this peak characteristic. Symbol rate can achieve an accurate estimate when $\alpha \geq 1.2$. The α value of alpha stable distribution noise in communication channel mainly concentrate in the interval (1, 2), so the proposed estimation algorithm has good application in practice.

5. Conclusion

This paper proposes a parameter estimation algorithm for MPSK signals based on generalized second-order cyclic spectrum. Using the properties that generalized second-order cyclic spectrum can suppress alpha stable distribution noise and represent the cycle characteristics of the signal, this paper analyzes the generalized second-order cyclic spectrum of MPSK signals, on this basis, achieves a reliable estimate of the signal parameters. Simulation results show that the algorithm have good estimation performance and high robustness.

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