Multiple Targets Parameters Estimation for Bistatic MIMO Radar System

Zhiguo Liu[1], Yali Wang[2]

[1]No. 20 Research Institute of China Electronics Technology Group Corporation, Xi'an, 710068, China liu_zhiguo2013@163.com
[2]Faculty of Science, Air Force Engineering University, Xi'an 710051, China yayahope@163.com

Abstract

In this paper, we present a multiple targets localization and parameter estimation algorithm for a bistatic multiple-input multiple-output (MIMO) radar system. MUSIC estimator is directly employed to estimate the direction of arrivals (DOAs), the amplitude and phase estimator (APES) is used to derive closed-form solution of the direction of departures (DODs), then RCS can be obtained from solutions of DODs and DOAs via least squares method. The DODs and RCS of targets can be solved in close form, and all the parameters are paired automatically. Numerical examples are presented to verify the effectiveness of the proposed method.

1. Introduction

It has been demonstrated that MIMO radar can overcome performance degradations caused by the radar cross section (RCS) fluctuations [1-4], provide super-resolution spatial spectral estimates [3, 5-6], and significantly improve the parameter identifiability[7]. Several nonparametric target parameter estimation methods for MIMO radar with collocated antennas have been proposed in [8-9].

Based on the aforementioned approaches, we consider the signal model of multiple transmitting pulses as [10], and proposed a new method base on MUSIC and APES algorithm. In this paper, we focus on the narrowband bistatic MIMO radar. Similar to some of the aforementioned results, linearly independent waveforms are transmitted simultaneously via multiple transmitting antennas. We consider a combined method, which first estimates the direction of arrivals (DOAs) using MUSIC method and then using the APES estimator, closed-form solution of the direction of departures (DODs) is obtained by exploiting the estimates result of DOAs, finally obtains the RCS exploiting the result of DOAs and DODs estimation. The proposed method provides the joint estimation of DODs, DOAs and RCS where the parameters are paired automatically.

2. Signal model

Assume a MIMO radar system with two uniform linear arrays of N antennas at the transmitter and M antennas at receiver. d_t denotes the inter element space at the transmitter and d_r denotes the inter element space at the receiver. Assume that the number of targets in a particular range and Doppler bin is P. The system simultaneously transmits M linearly independent waveforms, denoted by, $\mathbf{s}_m = \left[s_m(1), \cdots, s_m(L)\right]$, $m = 1, 2, \cdots, N$. $\mathbf{s}_m\mathbf{s}_m^{\mathrm{H}} = L$, where $(\bullet)^{\mathrm{H}}$ indicates the Hermitian transpose and L being the snapshot number. Let φ_p and θ_p be the location parameter of the pth target, DOD and DOA, then let $\mathbf{b}(\varphi_p)$ and $\mathbf{a}(\theta_p)$ be the corresponding steering vector for the transmitting antenna array and the steering vector for the receiving antenna array. $\mathbf{b}(\varphi_p)$ and $\mathbf{a}(\theta_p)$ can be represented as:

$$\mathbf{b}(\varphi_p) = \left[1, e^{j2\pi d_t \sin\varphi_p/\lambda}, \cdots, e^{\left(j2\pi(N-1)d_t \sin\varphi_p/\lambda\right)}\right]^{\mathrm{T}} \tag{1}$$

$$\mathbf{a}(\theta_p) = \left[1, e^{j2\pi d_r \sin\theta_p/\lambda}, \cdots, e^{j2\pi(M-1)d_r \sin\theta_p/\lambda}\right]^{\mathrm{T}} \tag{2}$$

respectively.

We formulate the signal model as:

$$\mathbf{X} = \sum_{p=1}^{P} \mathbf{a}(\theta_p)\zeta_p \mathbf{b}^{\mathrm{T}}(\varphi_p)\mathbf{S} + \mathbf{V} = \mathbf{A}\boldsymbol{\Sigma}\mathbf{B}^{\mathrm{T}}\mathbf{S} + \mathbf{V} \tag{3}$$

978-1-4673-5225-3/14/$31.00 ©2014 IEEE

wher $\mathbf{A}=\left[\mathbf{a}\left(\theta_1\right),\cdots,\mathbf{a}\left(\theta_P\right)\right]$, $\mathbf{B}=\left[\mathbf{b}\left(\varphi_1\right),\ \cdots,\mathbf{b}\left(\varphi_P\right)\right]$, $\Sigma=diag\left(\zeta_1,\zeta_2,\ldots\zeta_P\right)$ and $\mathbf{S}=\left[\mathbf{s}_1^{\mathrm{T}},\mathbf{s}_2^{\mathrm{T}},\cdots,\mathbf{s}_M^{\mathrm{T}}\right]^{\mathrm{T}}$, with $(\bullet)^{\mathrm{T}}$ denoting the transpose. Where the columns of $\mathbf{X}\in\mathbb{C}^{N\times L}$ are the received data samples, $\zeta_p\in\mathbb{C}$ denotes the complex amplitude of the reflected signal from the pth target, which are proportional to the RCS of the target. Here we define ζ_p as the RCS of the pth target. Note that we assume RCS fluctuations are fixed during a scan, but vary independently from scan to scan. The matrix $\mathbf{V}\in\mathbb{C}^{N\times L}$ denotes noise matrix and rows of $\mathbf{V}$ are assumed to be independent, zero-mean complex Gaussian with unknown covariance matrix $\mathbf{Q}$.

Without loss of generality, it is assumed that the entries of the RCS matrix are independent of one another with identical distribution. It is assumed that the number of targets P is known a priori.

3. The proposed algorithm

For the signal model in (3), the spatial covariance matrix can be estimated with L snapshots by $\hat{\mathbf{R}}_{xx}=\mathbf{X}\mathbf{X}^{\mathrm{H}}/L$.Using eigen decomposition, $\hat{\mathbf{R}}_{xx}$ is denoted by $\hat{\mathbf{R}}_{xx}=\mathbf{E}_s\mathbf{D}_s\mathbf{E}_s^{\mathrm{H}}+\mathbf{E}_n\mathbf{D}_n\mathbf{E}_n^{\mathrm{H}}$,where $\mathbf{D}_s$ is a $P\times P$ diagonal matrix whose diagonal elements contain the P largest eigenvalues and $\mathbf{D}_n$ stands for a diagonal matrix whose diagonal entries contain the $N-P$ smallest eigenvalues. $\mathbf{E}_s$ is the matrix composed of the eigenvectors corresponding to the P largest eigenvalues of $\hat{\mathbf{R}}_{xx}$, while $\mathbf{E}_n$ represents the matrix including other eigenvectors. Note that $\mathbf{E}_s$ and $\mathbf{E}_n$ can be regarded as the signal subspace and the noise subspace, respectively.

We discuss MUSIC spatial spectral estimators for the proposed the MIMO radar system. We can estimate the DOAs of targets by searching for the peaks in the estimated MUSIC spectrum.

We construct the MUSIC spatial spectrum function

$$f\left(\theta\right)=1\Big/\left[\mathbf{a}^{\mathrm{H}}\left(\theta\right)\mathbf{E}_n\mathbf{E}_n^{\mathrm{H}}\mathbf{a}\left(\theta\right)\right] \tag{4}$$

Thus, the DOAs can be obtained by searching the peaks of the spatial spectrum function $f\left(\theta\right)$ over parameter space. The corresponding estimated DOAs are denoted as $\left(\hat{\theta}_1,\hat{\theta}_2,\cdots,\hat{\theta}_P\right)$, and the receive array manifold matrix $\hat{\mathbf{A}}=\left[\mathbf{a}\left(\hat{\theta}_1\right),\mathbf{a}\left(\hat{\theta}_2\right),\cdots,\mathbf{a}\left(\hat{\theta}_P\right)\right]$ can be obtained at the same time according to(2).

Now consider the DODs estimation. The objective function is designed as [11-12]

$$J\left(\mathbf{W},\mathbf{\Sigma}\right)=\left\|\mathbf{W}^{\mathrm{H}}\mathbf{X}-\mathbf{\Sigma}\mathbf{B}^{\mathrm{T}}\mathbf{S}\right\|^2 \tag{5}$$

Let $\mathbf{\Gamma}=\mathbf{\Sigma}\mathbf{B}^{\mathrm{T}}$ the objective function $J\left(\mathbf{W},\mathbf{\Gamma}\right)$ can be rewritten as

$$J\left(\mathbf{W},\mathbf{\Gamma}\right)=\mathrm{tr}\left(\left[\mathbf{W}^{\mathrm{H}}\mathbf{X}-\mathbf{\Gamma}\mathbf{S}\right]\left[\mathbf{W}^{\mathrm{H}}\mathbf{X}-\mathbf{\Gamma}\mathbf{S}\right]^{\mathrm{H}}\right) \tag{6}$$

The APES method can be formulated as

$$\min_{\mathbf{W},\mathbf{\Gamma}}J\left(\mathbf{W},\mathbf{\Gamma}\right)\ \text{ subject to }\mathbf{W}^{\mathrm{H}}\hat{\mathbf{A}}=\mathbf{I} \tag{7}$$

where the $\mathbf{I}$ is the identity matrix and $\hat{\mathbf{A}}$ is obtained by MUSIC estimator.

It can be shown that

$$J\left(\mathbf{W},\mathbf{\Gamma}\right)=\left[\mathbf{W}^{\mathrm{H}}\mathbf{X}-\mathbf{\Gamma}\mathbf{S}\right]\left[\mathbf{W}^{\mathrm{H}}\mathbf{X}-\mathbf{\Gamma}\mathbf{S}\right]^{\mathrm{H}}=\frac{1}{L}\left\{\left[\mathbf{\Gamma}-\mathbf{W}^{\mathrm{H}}\hat{\mathbf{R}}_{xs}\mathbf{R}_{ss}^{-1}\right]\hat{\mathbf{R}}_{ss}\left[\mathbf{\Gamma}-\mathbf{W}^{\mathrm{H}}\mathbf{R}_{xs}\hat{\mathbf{R}}_{ss}^{-1}\right]^{\mathrm{H}}\right\}+\mathbf{W}^{\mathrm{H}}\hat{\mathbf{R}}_{xx}\mathbf{W}-\mathbf{W}^{\mathrm{H}}\hat{\mathbf{R}}_{xs}\mathbf{R}_{ss}^{-1}\hat{\mathbf{R}}_{xs}^{\mathrm{H}}\mathbf{W} \tag{8}$$

where $\hat{\mathbf{R}}_{xs}$ are defined as $\hat{\mathbf{R}}_{xs}=\frac{1}{L}\mathbf{X}\mathbf{S}^{\mathrm{H}}$.

Hence, minimizing the objective function in (7) with respect to $\mathbf{\Gamma}$ yields $\mathbf{\Gamma}=\mathbf{W}^{\mathrm{H}}\hat{\mathbf{R}}_{xs}\hat{\mathbf{R}}_{ss}^{-1}$. Substituting $\mathbf{\Gamma}$ into(7), the optimization problem reduces to

$$\min_{\mathbf{W}}\mathbf{W}^{\mathrm{H}}\hat{\mathbf{Q}}\mathbf{W}\ \text{ subject to }\ \mathrm{Re}\left(\mathbf{W}^{\mathrm{H}}\hat{\mathbf{A}}\right)=\mathbf{I}, \tag{9}$$

where $\hat{\mathbf{Q}}=\hat{\mathbf{R}}_{xx}-\hat{\mathbf{R}}_{xs}\hat{\mathbf{R}}_{ss}^{-1}\hat{\mathbf{R}}_{xs}^{\mathrm{H}}$.

Since $\hat{\mathbf{Q}}$ is the estimate of the unknown noise covariance matrix $\mathbf{Q}$, the inverse of $\hat{\mathbf{Q}}$ exists. The solution of optimization problem of (9) is easily obtained

$$\hat{\mathbf{W}} = \hat{\mathbf{Q}}^{-1}\hat{\mathbf{A}}\left[\hat{\mathbf{A}}^{\mathrm{H}}\hat{\mathbf{Q}}^{-1}\hat{\mathbf{A}}\right]^{-1} \tag{10}$$

Therefore, inserting (10) in $\mathbf{\Gamma}$, the estimate of $\mathbf{\Gamma}$ is given by

$$\hat{\mathbf{\Gamma}} = \left[\hat{\mathbf{A}}^{\mathrm{H}}\hat{\mathbf{Q}}^{-1}\hat{\mathbf{A}}\right]^{-1}\hat{\mathbf{A}}^{\mathrm{H}}\hat{\mathbf{Q}}^{-1}\hat{\mathbf{R}}_{xs}\hat{\mathbf{R}}_{ss}^{-1} \tag{11}$$

Due to the fact that the RCS matrix $\mathbf{\Sigma}$ is diagonal, the rows of $\hat{\mathbf{\Gamma}}$ are the corresponding scaled columns of $\mathbf{B}$. Therefore, we normalize $\hat{\mathbf{\Gamma}}^{\mathrm{T}}$ such that the first row of normalized matrix is same to the first row of transmit array manifold matrix, i.e., $\mathbf{1}_{1\times P}$. $\mathbf{1}_{1\times P}$ is defined as $[1,1,\cdots,1]_{1\times P}$. Let $\bar{\hat{\mathbf{\Gamma}}}$ denote the normalized $\hat{\mathbf{\Gamma}}^{\mathrm{T}}$, $\boldsymbol{\gamma}_i = \xi_i\mathbf{b}(\varphi_i)$ to be the ith column of $\bar{\hat{\mathbf{\Gamma}}}$. Then, $\hat{\mathbf{b}}(\varphi_i) = \boldsymbol{\gamma}_i\xi_i^*/|\xi_i|^2$ has the same phases as the steering vector $\mathbf{b}(\varphi_i)$. Let $\mathbf{\Theta}_i = -angle\left(\hat{\mathbf{b}}(\varphi_i)\right)$, where $angle(\bullet)$ is to get the phase angles for each element of complex vector $\hat{\mathbf{b}}(\varphi_i)$. When $d_t \le \lambda/2$, we can unwrap the phase as follow

$$\begin{cases}\mathbf{\Theta}_i^u(1) = \mathbf{\Theta}_i(1) \\ \mathbf{\Theta}_i^u(n+1) = \mathbf{\Theta}_i^u(n) + \delta\phi(n)\end{cases}, \quad n = 1,\ldots,N-1 \tag{12}$$

where the superscript u, as in $\mathbf{\Theta}_i^u$, denotes the unwrapped phases, and

$$\delta\phi(n) = \begin{cases}\mathbf{\Theta}_i(n+1) - \mathbf{\Theta}_i(n), & |\mathbf{\Theta}_i(n+1) - \mathbf{\Theta}_i(n)| \leqslant \pi \\ \mathbf{\Theta}_i(n+1) - \mathbf{\Theta}_i(n) - 2\pi, & \mathbf{\Theta}_i(n+1) - \mathbf{\Theta}_i(n) > \pi \\ \mathbf{\Theta}_i(n+1) - \mathbf{\Theta}_i(n) + 2\pi, & \mathbf{\Theta}_i(n+1) - \mathbf{\Theta}_i(n) < -\pi\end{cases} \tag{13}$$

The unwrapped phases of transmit steering vector $\mathbf{b}(\varphi_i)$ is $\mathbf{\Theta}_i^u = [0, \frac{2\pi}{\lambda}d_t, \ldots, \frac{2\pi}{\lambda}(N-1)d_t]^T \sin(\theta_i)$.

Then the least squares (LS) principle is adopted to estimate transmit angle φ_i

$$\min_{\mathbf{c}_i}\left\|\mathbf{G}\mathbf{c}_i - \mathbf{\Theta}_i^u\right\| . \tag{14}$$

where $\mathbf{c}_i = [c_{i0}, c_{i1}]^{\mathrm{T}} \in \mathbb{R}^{2\times 1}$ is an unknown parameter vector, and $\mathbf{G} = [\mathbf{1}_{N\times 1}, \mathbf{\Pi}]$ where $\mathbf{1}_{N\times 1}$ is defined as $[1,1,\cdots,1]_{N\times 1}^{\mathrm{T}}$. The solution of (14) is $[\hat{c}_{i0}, \hat{c}_{i1}]^{\mathrm{T}} = \left(\mathbf{G}^{\mathrm{T}}\mathbf{G}\right)^{-1}\mathbf{G}^{\mathrm{T}}\mathbf{\Theta}_i^u$, then the LS estimate of the angle is $\hat{\varphi}_i = \arcsin(\hat{c}_{i1})$,and the transmit array manifold matrix can be obtained according to

$$\hat{\mathbf{B}} = \left[\mathbf{b}(\hat{\varphi}_1), \mathbf{b}(\hat{\varphi}_2), \cdots, \mathbf{b}(\hat{\varphi}_P)\right] \tag{15}$$

It is interesting to note that the DODs and DOAs of the targets are paired automatically.

Now we derive the estimate of RCS from solution of DODs and DOAs via least squares method. We recall that $\mathbf{\Gamma} = \mathbf{\Sigma}\mathbf{B}^{\mathrm{T}}$, the estimate of matrix $\mathbf{\Gamma}$ and $\mathbf{B}$ can be obtained by (11) and (15), respectively. By applying LS method to $\hat{\mathbf{\Gamma}} = \mathbf{\Sigma}\hat{\mathbf{B}}^{\mathrm{T}}$, the estimate of $\mathbf{\Sigma}$ follows:

$$\hat{\mathbf{\Sigma}} = \hat{\mathbf{\Gamma}}\hat{\mathbf{B}}^*\left(\hat{\mathbf{B}}^{\mathrm{T}}\hat{\mathbf{B}}^*\right)^{-1} \tag{16}$$

Then the RCS of the ith target can be obtained as $\hat{\zeta}_i = \left(\hat{\mathbf{\Sigma}}\right)_{ii}$, $i = 1, 2, \cdots, P$.

4. Simulations results

Consider a bistatic MIMO radar system where a uniform linear array with N=6 antennas at the transmitter, M=8 antennas at receiver and half-wavelength spacing between adjacent antennas for the two arrays. Assume that there three targets locate at $(\varphi_1, \theta_1) = (10^o, 15^o)$, $(\varphi_2, \theta_2) = (-8^o, 25^o)$, and $(\varphi_3, \theta_3) = (20^o, 35^o)$, respectively, and their RCS is generated from a normal distribution with mean 0 and standard deviation 1. The received signal has L snapshots and is corrupted by a additive noise is assumed to be the thermal noise only, which is spatially white, i.e., $\mathbf{Q} = \sigma^2\mathbf{I}$. In every example, 200 independent Monte Carlo simulations are conducted to assess the angle estimation performance of our algorithm. Define the root-mean-squared error (RMSE) as $\mathrm{RMSE}(\hat{x}) = \frac{1}{P}\sum_{p=1}^{P}\sqrt{\frac{1}{200}\sum_{n=1}^{200}\left(\hat{x}_{p,n} - x_p\right)^2}$, where $\hat{x}_{p,n}$ is the estimate of the true parameter x_p of the nth Monte Carlo trial.

We consider the RMSE of the target parameters estimated by the proposed method versus SNR. The RMSE and root-CRB of the DOAs, DODs, the real part of RCS and the imaginary part of RCS versus SNR are shown in Fig. 1, where

the number of snapshots is selected as L=128. Fig. 2 depicts the algorithmic performance with different L with SNR being 10 dB.

As shown in Fig.1-2, the performance of the proposed method improves steadily as the SNR and number of snapshots increases.

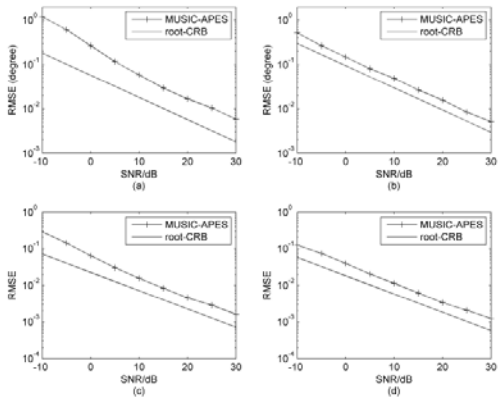

Fig. 1

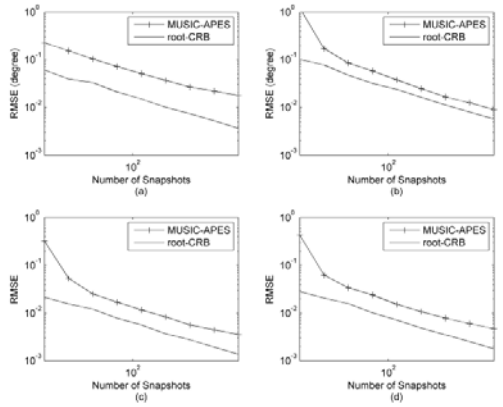

Fig. 2

Fig. 1. The RMSE and root-CRB of estimation for three targets versus SNR, L=128.
Fig. 2. The RMSE and root-CRB of estimation for three targets versus snapshots, SNR=10dB.
(a)DOA, (b) DOD, (c) the real part of RCS, (d) the imaginary part of RCS.

5. Conclusions

In this paper, a novel algorithm has been presented for the localization and estimation of multiple targets in the narrowband bistatic MIMO radar system. The MUSIC and APES estimators are considered to jointly estimate the DODs, DOAs and RCS. The solutions of parameters are paired automatically. Numerical examples are presented to verify its effectiveness. It is shown that the number of transmit antennas and receive antennas affect the performance of the DODs and DOAs estimation differently, while it does not significantly affect the performance of RCS estimation.

6. References

1. Fishler, E., et al. *MIMO radar: an idea whose time has come*. in *Radar Conference, Proceedings of the IEEE*. 2004. Philadelphia: IEEE.
2. Haimovich, A.M., R.S. Blum, and L.J. Cimini, *MIMO Radar with Widely Separated Antennas.* Signal Processing Magazine, IEEE, 2008. **25**(1): p. 116-129.
3. Fishler, E., et al., *Spatial diversity in radars-models and detection performance.* IEEE Transactions on Signal Processing, 2006. **54**(3): p. 823-838.
4. Li, J. and P. Stoica, *MIMO Radar with Colocated Antennas.* IEEE Signal Processing Magazine, 2007. **24**(5): p. 106-114.
5. Frazer, G.J., Y.I. Abramovich, and B.A. Johnson. *Spatially Waveform Diverse Radar: Perspectives for High Frequency OTHR*. in *Radar Conference, 2007 IEEE*. 2007.
6. Tabrikian, J. *Barankin Bounds for Target Localization by MIMO Radars*. in *Sensor Array and Multichannel Signal Processing, 2006. IEEE Workshop*. 2006.
7. Li, J., et al., *On Parameter Identifiability of MIMO Radar.* Signal Processing Letters, IEEE, 2007. **14**(12): p. 968-971.
8. Xu, L., J. Li, and P. Stoica. *Adaptive Techniques for MIMO Radar*. in *Sensor Array and Multichannel Processing, 2006. Fourth IEEE Workshop on*. 2006.
9. Li, J., P. Stoica, and X. Zheng, *Signal Synthesis and Receiver Design for MIMO Radar Imaging.* Signal Processing, IEEE Transactions on, 2008. **56**(8): p. 3959-3968.
10. Jin, M., G. Liao, and J. Li, *Joint DOD and DOA estimation for bistatic MIMO radar.* Signal Processing, 2009. **89**(2): p. 244-251.
11. Stoica, P., H. Li, and J. Li, *A new derivation of the APES filter.* IEEE Signal Processing Letters, 1999. **6**(8): p. 205-206.
12. Xu, L. and J. Li, *Iterative Generalized-Likelihood Ratio Test for MIMO Radar.* Signal Processing, IEEE Transactions on, 2007. **55**(6): p. 2375-2385.