ELECTRON ACOUSTIC SOLITONS IN THE PRESENCE OF AN ELECTRON BEAM AND SUPERTHERMAL ELECTRONS

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Abstract

Existence of arbitrary amplitude electron acoustic solitons is studied in an unmagnetized plasma having cold electrons and ions, superthermal hot electrons and an electron beam. Using Sagdeev pseudo potential method, theoretical analysis is carried out by assuming superthermal hot electrons having kappa distribution. The results show that inclusion of beam alters the minimum value of spectral index and Mach number for which electron-acoustic solitons can exist and also changes their width and electric field amplitude. For the auroral region parameters, the maximum electric field amplitudes and soliton widths are found in the range ~ (100-400) mV/m and ~ (314-515) m, respectively.

1. Introduction

Electron acoustic mode is very useful in understanding the high frequency component of broadband electrostatic noise observed in different regions of the earth's magnetosphere. This mode described by two distinct temperature electron populations namely, 'cold' and 'hot' electrons. Electron-acoustic solitary waves have been observed in the Earth' magnetosphere by various satellites, e.g., Viking, FAST etc. [1-4]. Numerous theoretical studies on electron acoustic mode in the space plasmas have used multicomponent fluid models. For a long time researchers used Maxwellian or non-thermal distribution function as most probable distribution function for multicomponent species[5-8]. However, it is often observed that in space plasmas the distribution functions deviate from the Maxwellian due to the presence of superthermal particles having high energy tails [9]. These superthermal particles can be described by κ-distribution rather than Maxwellian or any other nonthermal distribution [10].

More recently, few studies have been done on electron-acoustic solitary waves using κ-distribution for hot electrons [11,12]. However, the study of electron-acoustic solitary waves involving κ-distribution have not taken into account the effect of electron/ion beams so far. Thus, the aim of this paper is to study the effect of electron beam on electron-acoustic solitary waves. Here, evolution of electron acoustic waves is studied in four-component plasma by adding intermediate electron beam to the three component model of Devanandhan et al, [13].

2. Theoretical Model

An infinite, homogeneous, collisionless, unmagnetized, four-component plasma is considered having fluid cold electrons, kappa distributed hot electrons, an electron beam and fluid ions. The normalized multi-fluid equations of continuity, momentum, and equation of state which governs the dynamics of cold electrons, beam electrons and ions are given by,

$$\frac{\partial n_j}{\partial t} + \nabla.\left(n_j v_j\right) = 0 \qquad (1)$$

$$\frac{\partial v_j}{\partial t} + v_j \nabla v_j + \frac{1}{\mu_j n_j}\nabla P_j - \frac{Z_j}{\mu_j}\nabla \phi = 0 \qquad (2)$$

$$\frac{\partial P_j}{\partial t} + v_j \nabla P_j + 3P_j \nabla v_j = 0 \qquad (3)$$

where the subscript j = c, b, i represents cold and beam electrons and ions respectively, $Z_j = \pm 1$ for electrons and ions respectively, and $\mu_j = m_j/m_e$. Here m_j, v_j, P_j, n_j and ϕ_j denote the mass, velocity, thermal pressure, number density and electrostatic potential for j^{th} species.

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The equations (1)-(3) are coupled through the Poisson equation which is givenas,

$$\frac{\partial^2 \phi}{\partial x^2} = n_h + n_c + n_b - n_i \tag{4}$$

The standard three dimensional isotropic kappa distribution function of hot electrons is given by (Thorne and Summers, 1991),

$$f_{0h}(v) = \frac{n_{0h}}{\pi^{\frac{3}{2}}\theta^3} \frac{\Gamma(\kappa)}{\sqrt{\kappa}\Gamma(\kappa - 1/2)} \left(1 + \frac{\text{v}^2}{\kappa\theta^2}\right)^{-(\kappa+1)} \tag{5}$$

Where $\Gamma(\kappa)$ is the gamma function and θ is a modified thermal speed given by $\theta^2 = (2 - 3/\kappa)\left(\frac{k_B T_h}{m_e}\right)$, κ is the spectral index, n_{0h}, T_h, m_e and v are the density, temperature, mass and velocity of the hot electrons, respectively. In order to have physically meaningful thermal speed one requires κ >3/2. The kappa distribution function reduces to Maxwellian distribution when $\kappa \to \infty$. Replacing $\frac{v^2}{\theta^2}$ by $\frac{v^2}{\theta^2} - \frac{2\phi e}{m\theta^2}$ in the distribution function and integrating over velocity space the number density for hot electrons can be obtained as

$$n_h = n_{0h}\left(1 - \frac{\phi}{(\kappa - 3/2)}\right)^{-(\kappa - 1/2)} \tag{6}$$

We have normalized the set of equations (1)-(4) for densities, velocities, lengths, temperature, time, potential, and thermal pressure by total electron density $n_0 = n_{0h} + n_{0c} + n_{0b} = n_{0i}$, thermal velocity of hot electrons $v_{th} = \sqrt{T_h / m_e}$, effective hot electron Debye length $\lambda_{dh} = \sqrt{T_h / 4\pi n_0 e^2}$, hot electron temperature T_h, inverse of electron plasma frequency $\omega_{pe}^{-1} = \sqrt{m_e / 4\pi n_0 e^2}$, electrostatic potential T_h/e and $n_o T_h$ respectively.

Using appropriate boundary conditions along with $\phi = 0$ and $d\phi / d\xi = 0$ at $\xi = \pm\infty$ Poisson equation can be integrated to yield the energy integral,

$$\frac{1}{2}\left(\frac{d\phi}{d\xi}\right)^2 + V(\phi, M) = 0 \tag{7}$$

where

$$\begin{aligned}
V(\phi) = {} & n_{0h}\left[1 - \left(1 - \frac{\phi}{\kappa - 3/2}\right)^{-(\kappa - 3/2)}\right] \\
& + n_{0c}\left[M^2 - \frac{M}{\sqrt{2}}\left\{M^2 + 3T_c + 2\phi + \sqrt{\left(M^2 + 3T_c + 2\phi\right)^2 - 12T_c M^2}\right\}^{1/2}\right] \\
& + n_{0c}T_c\left[1 - 2\sqrt{2}M^3\left\{M^2 + 3T_c + 2\phi + \sqrt{\left(M^2 + 3T_c + 2\phi\right)^2 - 12T_c M^2}\right\}^{-3/2}\right] \\
& + n_{0b}\left[(M - v_0)^2 - \frac{(M - v_0)}{\sqrt{2}}\left\{(M - v_0)^2 + 3T_c + 2\phi + \sqrt{\left((M - v_0)^2 + 3T_c + 2\phi\right)^2 - 12T_c (M - v_0)^2}\right\}^{1/2}\right]
\end{aligned}$$

$$+n_{0b}T_b\left[1-2\sqrt{2}\left(M-v_0\right)^3\left\{\left(M-v_0\right)^2+3T_c+2\phi+\sqrt{\left(\left(M-v_0\right)^2+3T_c+2\phi\right)^2-12T_c\left(M-v_0\right)^2}\right\}^{-3/2}\right]$$

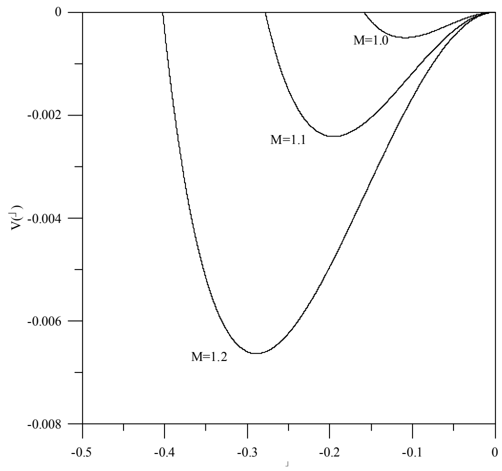
$$+\mu_i\left[M^2-\frac{M}{\sqrt{2}}\left\{M^2+\frac{3T_i}{\mu_i}-\frac{2\phi}{\mu_i}+\sqrt{\left(M^2+\frac{3T_i}{\mu_i}-\frac{2\phi}{\mu_i}\right)^2-\frac{12T_iM^2}{\mu_i}}\right\}^{1/2}\right]$$

$$+T_i\left[1-2\sqrt{2}M^3\left\{M^2+\frac{3T_i}{\mu_i}-\frac{2\phi}{\mu_i}+\sqrt{\left(M^2+\frac{3T_i}{\mu_i}-\frac{2\phi}{\mu_i}\right)^2-\frac{12T_iM^2}{\mu_i}}\right\}^{-3/2}\right] \tag{8}$$

Solitary wave solutions can be obtained from equation (8) when the Sagdeev potential $V(\phi,M)$ satisfies the following conditions: $V(\phi,M)=0$, $dV(\phi,M)/d\phi=0$, and $d^2V(\phi,M)/d\phi^2<0$ at $\phi=0$; $V(\phi,M)=0$ at $\phi=\phi_m$ and $V(\phi,M)<0$ for $0<|\phi|<|\phi_m|$, ϕ_m is the maximum potential. The condition $d^2V(\phi,M)/d\phi^2<0$ at $\phi=0$ is satisfied provided $M > M_0$, where M_0 is the critical Mach number.

3. Numerical Results

The auroral region parameters for the numerical computations have been taken from Dubouloz et al, [1] and are as follows: cold electron density N_{0c}=0.2 cm^{-3}, hot electron density N_{oh}=1.5 cm^{-3}, beam electron density N_{ob}=1.0 cm^{-3}, kappa κ=4, and normalized beam velocity V_0/V_{th} =0.1, cold to hot electron temperature ratio, ion to hot electron temperature ratio are $T_c/T_h=T_i/T_h$=0.001 and beam to hot electron temperature ratio T_b/T_h=0.01.



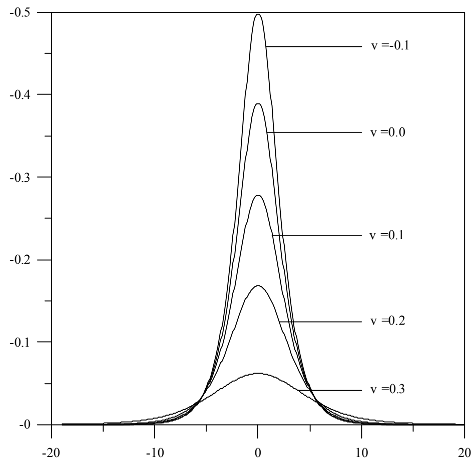


Figure 1 (left): Variation of Sagdeev potential V(ϕ) vs potential ϕ
Figure 2 (right): soliton profiles for different values of beam drift velocity.

Figure 1 shows the variation of sagdeev potential V(ϕ) with normalized potential ϕ for various values of M for the parameters mentioned above. Soliton solution exists for the Mach number ranging from 0.9 to 1.25. It is evident from the figure that increase in Mach number gives the higher amplitude solitary structures.

Keeping the value of Mach number M=1.1 and fixing other parameters as in Figure 1, we studied the effect of beam parameters such as beam temperature and drift velocity on the evolution of solitons. It is interesting to note that the soliton solution exists for both positive and negative values of V_0/V_{th} ranging from -0.1 to 0.3, i.e., soliton solutions can be obtained for both the cases for co- or counter streaming electron beams.

4. Conclusions and Discussions

We studied electron-acoustic solitons in four component plasma consisting of cold electrons and ions, hot superthermal electrons and an electron beam. It is found that the inclusion of electron beam significantly modifies the regime for the existence of solitons.

For the auroral region parameters mentioned in the previous section, the maximum electric fields are found to be 110mV/m, 242mV/m and 402mV/m for M=1.0, 1.1 and 1.2 respectively. The calculated soliton width and speed are in the range (486-329) m and (5960-8290) km/s respectively. Our results may be useful in understanding the electrostatic solitary waves in the auroral region of the Earth's magnetosphere where such electric field amplitudes for these structures are observed.

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6. References

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