Soliton Model For Broadband Electrostatic Noise

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Abstract

Generation of broadband electrostatic noise (BEN) in the plasma sheet boundary layer (PSBL) in terms of electron-acoustic solitons and double layers is proposed. The PSBL is treated as a multi-component magnetized plasma consisting of background electrons, counter-streaming electron beams and ions. The model is based on the multi-fluid equations and the Poisson equation, and uses the Sagdeev pseudo-potential technique. For the plasma parameters at the time of BEN in the PSBL observed by Cluster on 22 September 2004, the model predicts solitons/double layer with electric field $\sim$ (0.01-30) mV/m. It is proposed that the model can be a good candidate for explaining the generation of BEN in the PSBL.

1 Introduction

Matsumoto et al. [1] showed for the first time, from the analysis of the high time resolution of the plasma wave data from GEOTAIL, that the BEN in the PSBL actually consists of short electrostatic solitary waves (ESWs) whose Fourier spectrum give rise to the broadband nature of the noise. The electrostatic solitary structures can have either positive or negative potentials, and their electric field amplitude can vary from a fraction of a mV/m in the PSBL to a few 100 mV/m in the dayside auroral zone. The velocities of ESWs can vary from $\sim$ a few hundred to a few thousand km s^{-1}, and their parallel scale sizes from $\sim$ 100 m to a few 1000 m.

The most common interpretations for the ESWs observed in the PSBL are based on the nonlinear evolution of a bump-on-tail instability/electron two stream instability [2] leading to the formation of Bernstein-Greene-Kruskal (BGK) potential structures which reproduce well the observed electrostatic solitary waveforms. Other interpretations put forward for the generation of ESWs are based on electron-acoustic solitary waves [3,4,5]. Recently, the plasma measurements made on the Cluster spacecraft in the PSBL have shown that broadband ($\sim$ 2-6 kHz) electrostatic noise is associated with cold countersteaming electron beams flowing through the hot Maxwellian plasma [6,8]. The observed plasma parameters were (from Ref. [6]): total electron density, n_0=(0.27-0.32) cm^{-3}, core electron temperature, T_c =600-800 eV, ion temperature, T_i= 3.7-5.0 keV, electron beams drift speed, v_B = 7000-12000 km s^{-1}, electron beams temperature, T_B=10-100 eV, and magnetic field, B_0=8-12 nT. It is interesting to note these parameters are quite different than those employed in most theories for the PSBL ESWs based on two-stream instabilities with a bump-in-tail configuration, where the beam is hot and the back-ground electron population is cold [1]. Teste and Parks [6] suggested that electrostatic waves could be produced by the nonlinear evolution of the electron-acoustic and electron-ion instability driven by the electron beams as discussed by Schriver and Ashour-Abdalla [7]. Here, we propose an alternative generation mechanism for the PSBL electrostatic noise reported by Teste and Parks [6] in terms of an electron-acoustic soliton model developed for the magnetosheath ESWs [5].

2 Electron-acoustic Soliton/Double Layer Model

We model the PSBL plasma by a homogeneous, collisionless, and magnetized four component plasma consisting of hot core electrons (N_c, T_c, v_c), two cold electron beams (N_1, T_1, v_1) and N_2, T_2, v_2) propagating along the magnetic field, and hot ions (N_i, T_i, v_i), where N_j, T_j, v_j represent the equilibrium values of the density, temperature and beam velocity (along the direction of the ambient magnetic field) of the species

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j, and j=i, c, 1, and 2 for the ions, core electrons, beam 1 electrons and beam 2 electrons, respectively. We consider only the nonlinear electrostatic waves propagating parallel to the magnetic field, in which case the dynamics of the species is governed by the multi-fluid equations of continuity, momentum, and equation of state of each species, and the Poisson equation [5]. We transform these to a stationary frame moving with velocity V, the solitary wave velocity, i.e., $\xi = (x - Mt)$ where $M = V/C_i$ is the Mach number with respect to the ion thermal velocity. Then, solving for perturbed densities, putting these expressions in the Poisson equation, and assuming appropriate boundary conditions for the localized disturbances along with the conditions that ϕ=0, and $d\phi/d\xi$=0 at $\xi \to \pm, \infty$, we get the following energy integral [5],

$$\frac{1}{2}\left(\frac{\partial\phi}{\partial\xi}\right)^2 + \psi(\phi, M) = 0 \tag{1}$$

$$\begin{aligned}
\text{where, } \psi(\phi, M) \;&=\; \mu n_c^0 \left\{(M - v_c)^2 - \frac{(M - v_c)}{\sqrt{2}} B_c^{1/2}\right\} + n_c^0 T_c \left\{1 - 2\sqrt{2}(M - v_c)^3 B_c^{-3/2}\right\} \\
&+\; \mu n_1^0 \left\{(M - v_1)^2 - \frac{(M - v_1)}{\sqrt{2}} B_1^{1/2}\right\} + n_1^0 T_1 \left\{1 - 2\sqrt{2}(M - v_1)^3 B_1^{-3/2}\right\} \\
&+\; \mu n_2^0 \left\{(M - v_2)^2 - \frac{(M - v_2)}{\sqrt{2}} B_2^{1/2}\right\} + n_2^0 T_2 \left\{1 - 2\sqrt{2}(M - v_2)^3 B_2^{-3/2}\right\} \\
&+\; n_i^0 \left\{(M - v_i)^2 - \frac{(M - v_i)}{\sqrt{2}} B_i^{1/2}\right\} + n_i^0 \left\{1 - 2\sqrt{2}(M - v_i)^3 B_i^{-3/2}\right\}
\end{aligned} \tag{2}$$

is the pseudopotential, also known as the Sagdeev potential. Here,

$$\begin{aligned}
&B_c = A_c \pm \sqrt{A_c^2 - \frac{12T_c(M - v_c)^2}{\mu}}, \quad B_1 = A_1 \pm \sqrt{A_1^2 - \frac{12T_1(M - v_1)^2}{\mu}}, \\
&B_2 = A_2 \pm \sqrt{A_2^2 - \frac{12T_2(M - v_2)^2}{\mu}}, \quad B_i = A_i \pm \sqrt{A_i^2 - 12(M - v_i)^2}, \\
&A_c = (M - v_c)^2 + \frac{3T_c}{\mu} + \frac{2\phi}{\mu}, \quad A_1 = (M - v_1)^2 + \frac{3T_1}{\mu} + \frac{2\phi}{\mu}, \\
&A_2 = (M - v_2)^2 + \frac{3T_2}{\mu} + \frac{2\phi}{\mu}, \quad A_i = (M - v_i)^2 + 3 - 2\phi, \; \mu = \frac{m_e}{m_i}.
\end{aligned} \tag{3}$$

In Eq. (2), $n_j^0 = N_j/N_i$ such that $n_c^0 + n_1^0 + n_2^0 = n_i^0 = 1$, and the temperatures of the species are normalized with the ion temperature, velocities with the ion thermal velocity $C_i = (T_i/m_i)^{1/2}$, time with the inverse of ion plasma frequency, $\omega_{pi} = (4\pi N_i e^2/m_i)^{1/2}$, the lengths with the ion Debye length, $\lambda_{di} = (T_i/4\pi N_i e^2)^{1/2}$, the electrostatic potential ϕ by T_i/e, and the thermal pressures P_j with $N_i T_i$. Also, we have considered the same adiabatic index, i.e., γ =3, for all the species in the equation of state.

3 Soliton and Double Layer Solutions

Equation (1) will give a soliton solution when the pseudo-particle is reflected in the pseudo-potential field and returns to its initial state (zero potential drop). Therefore, for soliton solutions, the Sagdeev potential $\psi(\phi, M)$ must satisfy the following conditions: $\psi(\phi, M)$=0, $d\psi(\phi, M)/d\phi$ =0, $d^2\psi(\phi, M)/d\phi^2 < 0$ at ϕ=0, $\psi(\phi, M)$=0 at $\phi = \phi_0$, and $\psi(\phi, M) < 0$ for $0 < |\phi| < |\phi_0|$. The double layer solutions could also exist at the upper limit on the Mach number $M = M_{DL}$ provided one more additional condition, namely, $d\psi(\phi, M)/d\phi$ = 0 at $\phi = \phi_{DL}$ and $M = M_{DL}$, is satisfied. In such a case, the pseudo-particle is not reflected at $\phi = \phi_{DL}$ because of the vanishing pseudo-force and pseudo-velocities. Instead, it goes to another state producing an asymmetrical double layer (DL) with a net potential drop of ϕ_{DL}, where ϕ_{DL} is the amplitude of the double layer. From Eq. (2), it is seen that $\psi(\phi, M)$ and its first derivative with respect to ϕ vanish at ϕ =0.

4 Numerical results

We have numerically solved Eq. (2) for the Sagdeev potential, $\psi(\phi, M)$, as a function of ϕ for various values of Mach numbers for the case of counterstreaming electron beams having equal densities (i.e., N_1=N_2=N_B) and temperature (i.e., T_1=T_2=T_B), and equal and opposite streaming velocities (i.e., v_1=-v_2=v_B). Based on the observations provided in Teste and Parks [6], we consider the following normalized parameters for the numerical computations: $v_c = v_i$=0.0, T_c/T_i= 0.12- 0.22, T_B/T_i= 0.002-0.005, v_B/C_i=10.0-20.0, and N_c/n_0= 0.1-0.7 (correspondingly N_B/n_0= 0.45-0.15). Figure 1 shows the variation

Figure 1: Shows variations of Sagdeev potential, $\psi(\phi, M)$, versus electrostatic potential, ϕ, for electron-acoustic solitons/double layer for the identical counterstreaming electron beams with plasma parameters: $v_c = v_i$=0.0, $v_1 = -v_2 = v_B$=10.0, T_c=0.175, T_B=0.005, N_c/n_0=0.5, N_B/n_0=0.25 and for M = 25.10, 25.175, and 25.2161.

Figure 2: The profiles for electrostatic potential, ϕ (solid curves) and the electric field, E for the electron-acoustic solitons/double layer for the parameters of Figure 1. The curves 1, 2 and 3 are the Mach number M= 25.10, 25.175, and 25.2161.

of Sagdeev potential $\psi(\phi, M)$ versus the normalized electrostatic potential ϕ for various values of the Mach number for the case when the core electron density is equal to the density of both electron beams, i.e., N_c=2N_B. The curves for M= 25.10 and 25.175 correspond to the electron-acoustic soliton solution, and the curve M= 25.2161 to the double layer solution. There are no soliton or double layer solutions for the Mach numbers exceeding 25.2161. Figure 2 shows the profiles of normalized electrostatic potential, ϕ (solid curves) and electric field, E (dashed curves) obtained from the solution of Eq. (1), and corresponding to the parameters of Figure 1. It is interesting to note that the width (W) and amplitudes of both ϕ and E tend to increase as Mach number, M increases. The electric field has a bipolar structure for the solitons and a monopolar structure for the double layer. The properties of electron-acoustic solitons and double layers are given in Tables 1 for the parameters observed in the PSBL. For each case, the electron-acoustic solitons can exist over a range of V, W, τ, ϕ and E. However, the double layers have only one value of these parameters (the highest value of the range under each column).

5 Discussion

The model allows the existence of electron-acoustic solitons and double layers for the PSBL parameters reported by Teste and Parks [6]. Unfortunately, the Cluster WBD waveform data for this event discussed by Teste and Parks [6] are not available to confirm the presence of bipolar and monopolar pulses predicted by our analysis. However, we note that the pulse durations of $\sim$ (0.1-4.5) ms, typical for the PSBL as shown

Table 1: Variations of soliton velocity (V), electric field (E), soliton width (W) and pulse duration (τ), for various values of the core and beam electron temperatures for the equal density counterstreaming electron beams for the PSBL parameters: n_0=0.3 cm^{-3}, T_i= 4 keV, T_c/T_i=0.175, T_B/T_i=0.005, N_c/n_0= 0.5, N_B/n_0= 0.25, V_B/C_i= 10.0.

T$_c$/T$_i$	T$_b$/T$_i$=0.002				T$_b$/T$_i$=0.005			
	V (10^3 km s^{-1})	E (mV/m)	W (km)	τ (ms)	V (10^3 km s^{-1})	E (mV/m)	W (km)	τ (ms)
0.125	13.4-13.6	0.01-12.6	38.2-12.5	2.9-0.9	13.6-13.7	0.004- 9.8	45.2-11.6	3.3-0.8
0.150	14.4-14.6	0.01-12.6	39.0-12.8	2.7-0.9	14.6-14.7	0.01-10.3	46.8-14.3	3.2-1.0
0.175	15.3-15.5	0.01-12.9	52.2-16.0	3.4-1.0	15.5-15.6	0.01-10.8	58.3-12.9	3.77-0.8
0.200	16.2-16.4	0.01-13.3	73.5-16.1	4.5-1.0	16.4-16.5	0.01-11.4	59.2-20.7	3.6-1.3
0.225	17.0-17.2	0.01-13.8	77.9-18.5	4.58-1.0	17.1-17.3	0.01-12.1	97.8-19.6	5.7-1.1

in Figs 3 and 4 of Pickett et al. [8] from Cluster WBD measurements, could give rise to BEN ranging from $\sim$ 220 Hz to 10 kHz. Further, the amplitude of the intense electrostatic emissions ($\sim$ 0345 - 0346:50 UT on 22 September 2004, see Figure 2 of Teste and Parks, [6] is seen to be $\delta E \sim$ (0.05 - 0.1) mV/m. It is noticed that both the observed frequency range ($\sim$ 2- 6 kHz) and electric field amplitude are within the limit of predicted values by the electron-acoustic soliton model. In fact the predicted electric fields, corresponding to the lower range of soliton velocity ranges in Table 1, seem to be in excellent agreement with the observed δE. To summarize, the generation of BEN observed in the PSBL by Teste and Parks [8] may be explained by the electron-acoustic soliton/double layer model discussed here.

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7 References

1. H. Matsumoto, H. Kojima, T. Miyatake, Y. Omura, Y., M. Okada, I. Nagano, and M. Tsutui, "Electrostatic Solitary waves (ESW) in the Magnetotail: BEN wave forms observed by GEOTAIL," *Geophys. Res. Lett.*, 21, 1994, pp. 2915-2918.
2. Y. Omura, H. Matsumoto, T. Miyake, and H. Kojima, Electron beam instabilities as generation mechanism of electrostatic solitary waves in the magnetotail, *J. Geophys. Res.*, 101, 1996, pp. 2685-2697.
3. S. V. Singh, R. V. Reddy, and G. S. Lakhina, Broadband electrostatic noise due to nonlinear electron acoustic waves, *Adv. Space Res.*, 28, 2001, pp. 1643-1648.
4. A. P. Kakad, S. V. Singh, R. V., Reddy, G. S. Lakhina, S. G. Tagare, and F. Verheest, Generation mechanism for electron acoustic solitary waves, *Phys. Plasmas*, 14, 2007, 052305, doi:10.1063/1.2732176.
5. G. S. Lakhina, S. V. Singh, A. P. Kakad, M. L. Goldstein, A. F. Viñas, and J. S. Pickett, A mechanism for electrostatic solitary structures in the Earth's magnetosheath, *J. Geophys. Res.*, 114, 2009, A09212, doi:10.1029/2009JA014306.
6. A. Teste and G. K. Parks, Counterstreaming Beams and Flat-Top Electron Distributions Observed with Langmuir, Whistler, and Compressional Alfvén Waves in Earth's Magnetic Tail, *Phys.Rev. Letts.*, 102, 2009, 075003, DOI:10.1103/PhysRevLett.102.075003.
7. D. Schriver and M. Ashour-Abdalla, Broadband electrostatic noise due to field-aligned currents, *Geophys. Res. Lett.*, 16, 1989, 899-902.
8. J. S. Pickett, L.-J. Chen, O. Santolik, S. Grimald, B. Labraud, et al., Electrostatic solitary waves in current layers: from Cluster observations during a super-substorm to beam experiments at the LAPD, *Nonlin. Processes Geophys.*, 16, 2009, 431-442.