EISCAT Aperture Synthesis Imaging (EASI_3D) for the EISCAT_3D Project

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1. Introduction

Aperture Synthesis Imaging Radar (ASIR) is the technology, code-named EASI_3D, adopted by the EISCAT_3D project, that will give the new radar system imaging capabilities in 3-dimensions including sub-beam resolution in the plane across the transmitter antenna beam. When complemented by pulse compression techniques, it will provide 3-dimensional images of certain types of incoherent scatter radar targets with resolution of the order of 100 metres at 100 km range in any direction illuminated by the transmitter beam. The cross-beam resolution will vary as the inverse of the range squared. This ability will open new research opportunities to map small structures associated with non-homogeneous, non-steady, unstable processes such as aurora, summer and winter polar radar echoes (PMSE and PMWE), Natural Enhanced Ion Acoustic Lines (NEIALs), structures excited by HF ionospheric heating, meteors, space debris, and possibly others.

2. Aperture Synthesis Imaging Radar

Aperture Synthesis Imaging Radar (ASIR) [1, 2] is closer to the technology used by radio astronomers (VLBI, Very Long Baseline Interferometry) to image stellar objects [3] than to the SAR (Synthetic Aperture Radar) technique used onboard airplanes and satellites to map the Earths surface and other planetary surfaces. In the radio astronomy case the source itself spontaneously emits radiation that is collected by a number of passive antennas. In the former, the radar transmitter—acting like a camera flash— illuminates the target (the ionosphere or atmosphere) and a number of antennas collect the scattered radiation—exactly as in the radio astronomy case. From this point on, the two cases are essentially identical (although Earth's motion in the radioastronomy case is an important difference).

Under the assumption of small angle distribution of the source as seen from the measurement plane, which is the normal case, the radiation scattered by the sources projects a distribution of varying electric fields on planes transverse to the propagation direction (plane wave approximation) as the radiation propagates, that is, a diffraction pattern. All the information on the image is contained in this field distribution. It is straightforward to show that the spatial correlation function of the electric field distribution, under the very important assumption that the source distribution is completely incoherent, contains all the information to extract the image via a two dimensional Fourier transform. This is the essence of the van Cittert-Zernike theorem of optics [4]. In the parlance of imaging, the field distribution correlation function is called the Visibility Function $V(u,v)$, and its Fourier transform, that is, the image, the Brightness Distribution $B(l,m)$, where the uv-coordinates, expressed in units of wavelength, are on the plane of measurement and the lm-coordinates are the angular coordinates of the image expressed as directional cosines subtended from the plane of measurement. That is, the image is the angular spectrum of the source's intensity as projected on the plane of measurement:

$$B(l,m) = \iint V(u,v) e^{2\pi i(ul+vm)} du\, dv. \tag{1}$$

A minor factor, introduced by the third angular coordinate, has been omitted in this expression. This factor is constant within the assumption of small angle source distribution, and is easily corrected. ASIR is carried out by providing a means to measure the spatial autocorrelation function of the field distribution on the measurement plane, that is, the visibility function. This is carried out by distributing a number of receiving antennas to sample the electric (or potential, i.e. voltage) field distribution on the uv-measurement plane. The pairwise sets of antennas are called the baselines. Due to sampling, and other minor measurement dependent distortions, the equivalent discrete application of (1) produces unsatisfactory images, usually

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with insufficient quality to be of any use. Carefully crafted inversion algorithms are necessary to restore the raw images that are produced by a simplistic Fourier transform.

3. Image Inversion

In practical applications, the measured visibility function is discrete uneven and truncated and often also sparse. Under this conditions Eq. (1) has to be modified [5]:

$$B_\omega^D(l,m) = \iint V_\omega(u,v)S(u,v)e^{2\pi i(ul+vm)}du\,dv \tag{2}$$

where $S(u,v)$ is the *sampling function* and $B_\omega^D(l,m)$ is the *dirty* image. A subscript ω has been added, since the measurements are usually done for various small intervals of frequency, that is images can be obtained at once for different 'colour' components. The restoration of the image involves two operations, namely deconvolution and Fourier transformation. Using the convolution theorem:

$$B_\omega^D(l,m) = B_\omega \star G \tag{3}$$

where the star operator denotes convolution and the point spread function (also called synthesised beam, or impulse response, or Green's function) $G(l,m)$ is the Fourier transform of the sampling function:

$$G(l,m) = \iint S(u,v)e^{2\pi i(ul+vm)}du\,dv. \tag{4}$$

There are two other effects that distort the measured brightness that are usually small and will be ignored in this treatment for the sake of clarity. These are the finite width of the antenna beam elements and the finite bandwidth of the receivers. The correction for the former is unproblematic while the effects of the latter can be made negligible by using sufficiently narrow bandwidths.

The problem of image restoration is how to deconvolve Eq. (3) to obtain the *clean* brightness distribution. The deconvolution as such cannot reproduce the true visibility, and cannot be even performed, because the problem is highly singular: $V_\omega = V_\omega^D/S$, where V_ω^D is the measured visibility. The sampling function S is a very spiky function (δ-functions) with regions of the uv-plane in-between where no measurements were made. Thus, the division by S is undefined over these regions because S is zero in those regions. This implies, in strict terms, that the actual visibility is unrecoverable, and so is the brightness distribution. In mathematical terminology, the problem is the inversion of an operator equation that is non-homogeneous and has a non-empty null or kernel space, that is, the homogenous solution is non-empty. The latter are the solutions that map to the value zero resulting from the discrete sampling (the non measured points, which is a dense set). These are called the *invisible distributions* [5] which the sampling function maps to zero. Briefly, if B^D is a solution of the non-homogeneous equation (the dirty image), so is $B^D + \alpha I$, where I represents solutions of the homogeneous equation (the invisible distributions) and α an arbitrary number. In real applications, the kernel space, to which I belongs, is a large space of functions. The art of image inversion is to design algorithms that make a judicious choice among the invisible distributions to obtain a *restored* image. This task cannot be carried out with linear operations. Thus image inversion is intrinsically a non-linear problem. Ultimately the measure of success of image restoration amounts to adopting a proper choice strategy to pick out one member of the invisible distributions. This amounts invariably to extrapolation.

A common misconception is that pure incoherent scattering signals, such as the ones obtained from electron fluctuations of the ionospheric plasma, are not apt to interferometric imaging techniques, as is the case of ASIR. In fact, a *necessary* condition for interferometric imaging is that the source be completely incoherent, as has been mentioned above. In a plasma, the coherence length is approximately equal to the Debye length, which for the ionosphere is typically of the order of some centimetres at most, much smaller than the wavelengths employed which run in the metre scale. This property is in fact the one that makes possible multiple pulses and pulse coding techniques to work effectively for the improvement of range resolution. Sources of coherent radiation (light) are rare in nature. Lasers and masers are some of the exceptions.

4. The Maximum Entropy Method (MEM) Applied to Image Inversion

The unavoidable presence of noise forces a degree of ambiguity in any case, even if the visibility is sampled to perfection. The task is now to construct a numerical algorithm that narrows the choice in a systematic manner employing sensible choice criteria (constraints) and using a priori information whenever available. The expectation is that among a certain class of solutions there will be one or a few solutions that represent the true image in a satisfactory manner. Fortunately, this turns out to be the case. One such methodology is the CLEAN algorithm [6, 7] which is a heuristic iterative procedure based on the assumption that the image is composed of a set of point sources (targets). Another approach that has a more developed mathematical foundation is the Maximum Entropy Method (MEM).

The MEM makes a choice among the invisible distributions by maximising the entropy among all the accessible images that belong to the solution null space. This is tantamount to making a choice of the image among those with the maximum multiplicity, that is, those that probabilistically will occur most often. The procedure is similar to Boltzman's procedure to find the distribution of the most probable states of a gas in thermal equilibrium, that is, Boltzman's H-function of statistical mechanics. In contrast to statistical mechanics where the entropy function is uniquely determined by mechanical dynamics, it is by no means obvious of what form the entropy function should be in general optimisation problems such as in image inversion. One discrete form, among the many that can be found in the literature [6], is the following:

$$\mathcal{H} = -\sum_k B_k \ln \frac{B_k}{eM_k} \tag{5}$$

where B_k are samples of the brightness, ln is the natural logarithm, $e = 2.71\ldots$ its base, and M_k is a default image that allows the introduction of a priori information. Scientists at the Jicamarca Radio Observatory have used a more elaborated and improved version of the MEM to obtain images of ionospheric plasma turbulence with considerable success [8]. Briefly, the numerical problem is to find an extremum of the following functional, using Hysell's notation and the Einstein summation convention:

$$E[f(e_j, \lambda_j, \Lambda, L)] = S + \lambda_j(g_j + e_j - f_i h_{ij}) + \Lambda(e_j^2 \sigma_j^{-2} - \Sigma) + L(I_i f_i - F) \tag{6}$$

where f is the sought after brightness distribution, $S = -f_i \ln(f_i/F)$ is the entropy, I_i is a vector of ones, $F = I_i f_i$ is the integrated (total) brightness, g_j is the measured visibility, h_{ij} is the point spread function that contains the Fourier kernel, e_j are the random errors, σ_j^2 are the (theoretical) expected error variances, and Σ parametrizes the error norm, effectively constraining it. The remaining quantities are Lagrange multipliers as follows. The λ_j define the fundamental constraint on the entropy functional by relating the measured visibility (including the random errors) to the sought after brightness that makes the entropy functional an extremum. The other Lagrange multipliers put additional constraints that typically would ensure an improvement of the quality of the final solution: Λ puts a bound on the error norm equal to a preset value equal to Σ; and L constrains the total brightness effectively ensuring that the solution will be positive semi-definite (non-negative). An implementation of the algorithm for EASI_3D has been tested on simulated data and on real world data taken with the Jicamarca Radar.

5. Some Implications of Imaging as a Fourier Transform

Since, in principle, the relationship between the brightness distribution and the visibility function is a Fourier transform, there are several general consequences that are very useful to investigate in order to define a viable and good experimental design. The sampling theorem states that the sampled function must be band limited in order to avoid aliasing. Indeed, the measurement procedure in a natural manner ensures the compliance with this condition since the finite width of the antenna beams plays the role of an anti-aliasing filter. Furthermore, the tapered nature of the beam pattern plays the useful role of windowing that reduces sidelobes introduced by the truncation of the field of view (at the expense of losing some angular resolution). The farthest baselines determine the angular resolution along the baseline direction, while the shortest baselines determine the angular extent of the image. To attain the expected across-beam resolution for EISCAT_3D, it will be necessary to deploy smaller antenna modules outside from the main core antenna.

6. Applications of EASI_3D to some Incoherent Scatter Targets

Numerous dynamic phenomena at high latitudes are characterised by small scale structures that are not resolved by conventional radar techniques. Provided that the radar signals produced by these irregularities have a combination of sufficient signal-to-noise-ratio (SNR) and stationarity time, the 3-D imaging technique can provide important information for the investigation of the phenomena. A case in point at high latitudes is the small structure of electron density that is produced by auroral precipitation. Filamented structures with scales of tens of metres have been resolved by optical means. The possibility of radar imaging these filaments is helped by the large enhancements of electron density occurring during aurora, although time variability may limit the sharpness of the measured images. Other examples are polar mesospheric summer and winter echoes (PMSE and PMWE), upper tropospheric and lower stratospheric radar scatter produced by atmospheric turbulence, numerous types of small scale structures induced by artificial RF heating of the ionosphere, Natural enhanced ion acoustic lines (NEIALs), space debris, meteors, and possibly others.

7. References

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