A method of calculation of electron density profiles from h'(f) traces of vertical sounding

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Abstract

A necessity to develop a new method to calculate the electron density profiles has been vital due to inaccuracy when calculating the profiles through the widespread Huang-Reinisch method (including the one used at ISTP SB RAS) [1]. The algorithm we offer calculates electron density distribution in the middle ionosphere containing maximum in the E-layer (or without it) by height-frequency h'(f) traces of vertical sounding.

1. Introduction

In this paper we describe a method developed at ISTP SB RAS to calculate the electron density profile in the anisotropic ionosphere from vertical sounding. The input data in the problem provided are magnetic inclination and *h'(f)* traces for ordinary and extraordinary wave. The output data are the electron density profile (or the plasma frequency profile). The program implementing this algorithm has been tested for a great number of model profiles, and the profiles calculated provide a coincidence accurate enough with the original ones except a valley region discrepancy inevitable for any approximating method.

2. Determining the heights of the E and F layer peak

At the first stage, the program calculates the *E* and *F* layer peak heights. For this purpose, one can use the known empirical formula of the view:

$$h_m F2 = \frac{1490}{M(3000)F2 + \Delta M} - 176 \qquad (1)$$

which is known, at least, in its three versions:
the Shimazaki formula [2], for which $\Delta M = 0$;
the Bradley-Dudeney formula [3], where $\Delta M = \frac{0{,}18}{X_E - 1{,}4}$;
the Dudeney formula [4]: $\Delta M = \frac{0{,}253}{X_E - 1{,}215}$.
Here $X_E = f_0 F2 / f_0 E$.

The parameter $M(3000)F2 = \frac{MUF(3000)}{f_0 F2}$ is determined from the $h'(f)$ trace by the Smith method [5] in the following sequence: from dependence $h'(f)$ (for this instance, the $h'(f)$ trace in Figure 1b was simulated from the profile in Figure 1a with the F layer critical frequency $f_0 F2 = 4{,}58 MHz$ and peak height $h_m F2 = 206 km$) we obtain the dependence of the incidence angle on the frequency at oblique sounding $\varphi(f)$ [6]:

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$$\varphi(f) = arctg\left[\frac{\sin\left(\frac{D}{2R}\right)}{1+\frac{h'(f)}{R}-\cos\left(\frac{D}{2R}\right)}\right] \qquad (2)$$

where $D = 3000km$ is the path distance, R is the Earth radius. Further, we find the frequency at oblique sounding (Figure 1c):

$$f_{OS}(f) = \frac{k \cdot f}{\cos\varphi(f)} \qquad (3)$$

where $k = 1{,}116$ is the factor of the Earth sphericity for $D = 3000km$.

Finally, $MUF(3000)$-factor is equal to the maximum of function $f_{OS}(f)$ (Figure 1c).

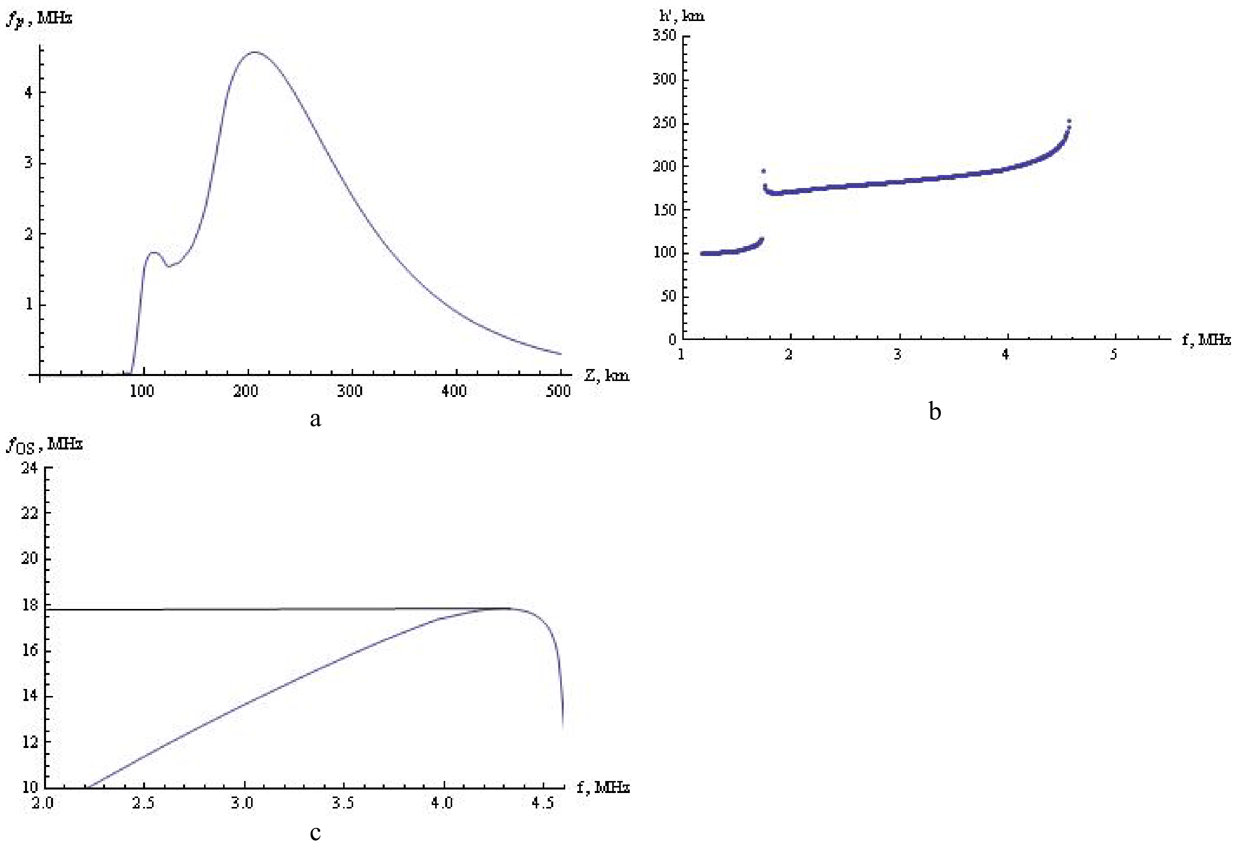

Figure 1. Determining the $MUF(3000)$-factor by the Smith method.

The listed ways to determine the layer peak height by using the $h'(f)$ function have been tested for a great number of the profiles taken from DPS-4, obtained by the IRI-2000 model or made by analytic functions. Analyzing the errors of the three formulas, the authors decided to use the Shimazaki formula to calculate the E and F layer peak heights.

3. Calculation of the electron density profile

The gyrofrequency is determined from the relation $f_{cx} - f_{co} \approx \frac{f_H}{2}$ [7].

To approximate the electron density profile below the E layer peak we use the function of the following view

$$f_p(z) = f_E \exp\left[-\left(\frac{h_m E - z}{l_E}\right)^{k_E}\right], \quad z \le h_m E \tag{4}$$

where f_E - is the E layer critical frequency determined by the O $h'(f)$ function, and the E layer peak height $h_m E$ is calculated by the method described above. l_E and k_E are found as follows: for each fixed degree k_E we carry out enumeration of l_E scale values within the 10 to 25 km range in 1 km increments whereas in the outer cycle k_E accepts values between 1,5 and 2,5 in 0,1 increments. We consider the ranges indicated cover the values l_E and k_E, necessary to approximate the overwhelming majority of the profiles below the E layer peak by a function of the view (4). Further, for each pair of values (l_{Ei}, k_{Ej}) the program simulates the O $h'(f)$ trace in $0{,}1 MHz$ increments over the range between the minimum frequency $f_{\min}$ recorded in an ionogram and the E layer critical frequency f_E. Afterwards, using the least square method, we search for the closest coincidence of the $h'(f)$ trace simulated with the original one, i.e. the minimum of the values is searched for

$$S(i,j) = \sum_f \left(h'_{\exp}(f) - h'^{i,j}_{sim}(f)\right)^2 \tag{5}$$

where $h'_{\exp}(f)$ - is the virtual height for the experimental $h'(f)$ trace at frequency *f*, $h'^{i,j}_{sim}(f)$ - is virtual height for the $h'(f)$ trace, simulated from the profile with parameters (l_{Ei}, k_{Ej}). The pair of (l_{Ei}, k_{Ej}), corresponding to the minimum $S(i,j)$, is the required one for substitution into Expression (4).

Further, to calculate the profile at heights $z \ge h_m E$ the approximating function of the following view is used:

$$f_p(z) = f_{c0} \exp\left[-\left(\frac{h_m F2 - z}{l_0}\right)^{k_0}\right] + (f_E - \Delta f)\exp\left[-\left(\frac{z - h_m E - \Delta z}{l_E}\right)^{k_E}\right] + f_z \exp\left[-\left|\frac{h_z - z}{l_z}\right|^{k_Z}\right] \tag{6}$$

In the first, basic summand (6) l_0 and k_0 are determined by the least square method as described above, with the only difference that the range of their possible values is extended to $45km \le l_0 \le 130km$ and $1{,}5 \le k_0 \le 3{,}5$, and comparison of group delays occurs then in the frequency interval $f_E < f < f_{co}$. The third summand (6) optimizes the approximation in the valley region: height h_z is the local minimum point of function $f_{co} \exp\left[-\left(\frac{h_m F2 - z}{l_0}\right)^{k_0}\right] + f_E \exp\left[-\left(\frac{z - h_m E}{l_E}\right)^{k_E}\right]$ in the interval $h_m E < z < h_m F2$. Small corrections Δf and Δz in the addend emerge because appearance of the first and third summand in Expression (6) results in violation of the indispensable conditions $f_p(h_m E) = f_E$ and $\frac{df_p(h_m E)}{dz} = 0$, typical of Relation (4).

In Figure 2, we present an example of the algorithm operation where the original profile (dashed line) was taken from an IRI-2000 model. Inevitable discrepancy in the valley region takes place as well as it does at small heights.

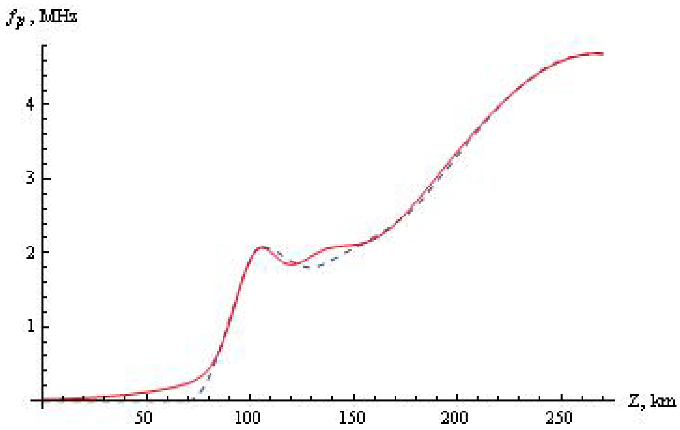

Figure 2. Example of the profile calculation: dotted line shows the original profile, solid line represents the calculated one.

4. References

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