Analytical prediction of the polarized Doppler spectrum from nonlinear ocean surface at microwave frequency

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Abstract

We present the use of combined hydrodynamic and electromagnetic analytical models for the simulation of the polarized ocean Doppler spectrum at microwave frequencies. We consider linear and weakly nonlinear sea surfaces after the Choppy Wave Model and incorporate them in the Weighted Curvature Approximation surface scattering method. Statistical expressions are derived, for the Doppler spectrum as well as for its central frequency and width. Results compare favorably with rigorous numerical computations for one-dimensional surfaces published in the literature. The simplicity of the analytical models provide a valuable tool for the Doppler analysis of two-dimensional sea-surfaces.

1 Introduction

The Doppler analysis associated to the backscattered radar return from the sea surface is a very valuable tool in ocean remote sensing. The Doppler spectrum indeed carries much more information than a mere radar backscattering cross section under a given incidence. For HF radiowaves, it is easily interpretable in terms of wind and current conditions, and the experimental data can be accurately modelized by an analytical theory such as the perturbative approach of Barrick and Weber [1], which describes second-order interactions of both the electromagnetical and hydrodynamical processes. In the microwave regime, Doppler characteristics turn out to be more complex. The observed Doppler spectra are usually broader than in the HF regime, with a mean frequency sometimes much higher than the free Bragg frequency, depending of wind speed and radar incidence. This effect is more pronounced in horizontal (H) polarization and at large incidences. Today, however, no asymptotic theory is capable of fully explaining and reproducing these experimental observations. Recent results [2] have shown that modern analytical scattering models can account for some characteristic features of microwave Doppler spectra such as the non-trivial dependence of the central frequency upon the incidence angle and its sensitivity to polarization. This study was, however, restricted to the mean Doppler shift and did not discuss the impact of surface nonlinearities.

In the last decade, a series of papers presented numerical simulations of microwave sea Doppler spectra with rigorous scattering formulation and weakly nonlinear hydrodynamics after Creamer [3], and also West models. As no statistical formulation was available, Doppler spectra had to be gathered by averaging the backscattered field from a large number of time-evolving sample surfaces, a procedure which is highly time-consuming and limited, even with todays facilities. Therefore most of the studies have been limited to one-dimensional (1D) surfaces, with the exception of [4], which was restricted to L-band and small wind speeds, however.

We show that a statistical model for microwave sea Doppler spectrum can be obtained by combining two analytical theories, namely the Weighted Curvature Approximation (WCA) surface scattering model [5, 6], of recently improved formulation [7] and the Choppy Wave Model (CWM) [8] weakly nonlinear surfaces model. The numerical tests of this paper have been designed identical to the extensive simulations presented in [9, 10] with a rigorous scattering method: 1D linear and Creamer surfaces with Pierson-Moskowitz spectrum at L- and X-band for 5 and 7 m.s^{-1} wind speed.

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2 The microwave ocean Doppler spectrum

Denoting $\mathbb{S}(t)$ the backscattered amplitude from the surface at time t (we refer to [11] for the exact definition), the time correlation function of the backscattered field is the limit of the statistical average:

$$\mathcal{C}(t) = \frac{4\pi}{|A|}\left(\langle \mathbb{S}(t)\mathbb{S}^*(0)\rangle - |\langle \mathbb{S}(0)\rangle|^2\right) \qquad D(\omega) = \int_{\mathbb{R}} \mathrm{e}^{-\mathrm{i}\omega t}\mathcal{C}(t)dt \tag{2.1}$$

for an infinite illuminated area $|A|$. Note that $\mathcal{C}(0)$ is the classical definition of the Normalized Radar Cross Section (NRCS). The Doppler spectrum is the corresponding Fourier transform $D(\omega)$, with $f = \frac{\omega}{2\pi}$ is the Doppler frequency shift. Waves travelling away from the radar mainly contribute to negative Doppler shifts while waves travelling towards the radar mainly create positive shifts. Thus Doppler spectra usually contains two peaks with different amplitudes depending on the radar look direction. It is customary to artificially restrict the sea spectrum to waves travelling towards (or away from) the radar. In that case the Doppler spectrum is centered around some positive (or negative) peak frequency and the Doppler centroid f_c and width γ can be defined through the first two moments of the spectrum:

$$f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi}\frac{\int_{\mathbb{R}}\omega D(\omega)d\omega}{\int_{\mathbb{R}} D(\omega)d\omega} = \frac{-\mathrm{i}}{2\pi}\frac{\partial_t\mathcal{C}(0)}{\mathcal{C}(0)} \qquad \gamma^2 = \frac{1}{(2\pi)^2}\frac{\int_{\mathbb{R}}\omega^2 D(\omega)d\omega}{\int_{\mathbb{R}} D(\omega)d\omega} - f_c^2 = \frac{-1}{(2\pi)^2}\frac{\partial_{tt}\mathcal{C}(0)}{\mathcal{C}(0)} - f_c^2 \tag{2.2}$$

Single scattering theories generally show a polarization ratio S_H/S_V (V is for vertical polarization) that is independent of the roughness [11]. This is the case for the Kirchhoff-tangent plane approximation (KA) and first order small perturbation method (SPM1) and small slope approximation (SSA1). The direct consequence is that the normalized Doppler spectrum $D(\omega)/\mathcal{C}(0)$ predicted by those theories is polarization independent, with the same values of f_c and γ in V or H pol. Recently revisited by [7], scattering amplitude in the WCA writes as a correction to the SSA1 that is proportional to a squared slope. For 1D surfaces $z = \eta(x,t)$, it writes

$$\mathbb{S}(t) = \frac{\mathbb{B}}{Q_z}\int_{\mathbb{R}}\frac{dx}{2\pi}\mathrm{e}^{\mathrm{i}Q_H x}\mathrm{e}^{\mathrm{i}Q_z\eta(x,t)} + \frac{Q_z(\mathbb{K}-\mathbb{B})}{Q_H^2}\int_{\mathbb{R}}\frac{dx}{2\pi}(\eta'(x,t))^2\ \mathrm{e}^{\mathrm{i}Q_H x}\ \mathrm{e}^{\mathrm{i}Q_z\eta(x,t)}. \tag{2.3}$$

with $\mathbb{K}$ and $\mathbb{B}$ the Kirchoff's and Bragg's kernels [11], $Q_H = -2\frac{\omega}{c}\sin\theta$ and $Q_z = 2\frac{\omega}{c}\cos\theta$, c the speed of light and θ the monostatic angle. This formulation was found adapted to the statistical calculation of $\mathcal{C}(t)$ and its derivates for surfaces that are Gaussian processes as well as to the numerical evaluation of the scattering amplitude on deterministic surfaces.

Recently some of the authors [8] proposed a weakly nonlinear model termed "Choppy Wave Model" (CWM) based on a Lagrangian description of sea surface particles. The CWM can be constructed in a simple way by horizontal displacement of a linear surface $z = \eta(\mathbf{r},t)$, sum of independent waves satisfying the gravity-capillarity dispersion relationship $\omega = \sqrt{g|\mathbf{k}|(1+|\mathbf{k}|^2/k_M^2)}$ with $k_M = 363.2$ rad.m^{-1} the wave number with minimum phase speed. The CWM is obtained by performing the transformation $(\mathbf{r},\eta(\mathbf{r},t)) \to (\mathbf{r}+\mathbf{D}(\mathbf{r},t),\eta(\mathbf{r},t))$ with $\mathbf{D}$ the so-called Riesz transform (Hilbert transform in 1D) of the linear surface η. The corresponding nonlinear surface $\tilde{\eta}$, is implicitely defined by the relation $\tilde{\eta}(\mathbf{r}+\mathbf{D}(\mathbf{r},t),t) = \eta(\mathbf{r},t)$. For Monte-Carlo simulations, the model is numerically efficient as it can be entirely generated by Fast Fourier Transform. The CWM can be made compatible with the statistical calculation of $\mathcal{C}(t)$ since Kirchhoff-like integrals transform as

$$\int_{\mathbb{R}}\frac{d\mathbf{r}}{(2\pi)^2}\mathrm{e}^{\mathrm{i}\mathbf{Q_H}\cdot\mathbf{r}}\mathrm{e}^{\mathrm{i}Q_z\tilde{\eta}(\mathbf{r},t)} = \int_{\mathbb{R}}\frac{d\mathbf{r}}{(2\pi)^2}\mathrm{e}^{\mathrm{i}\mathbf{Q_H}\cdot(\mathbf{r}+\mathbf{D}(\mathbf{r},t))}\mathrm{e}^{\mathrm{i}Q_z\eta(\mathbf{r},t)}J(\mathbf{r},t) \tag{2.4}$$

where J is the Jacobian of the transformation $\mathbf{r} \to \mathbf{r}+\mathbf{D}(\mathbf{r},t)$. Gaussian processes techniques can thus be applied.

3 Numerical comparisons

Figure 1 shows the Doppler spectra in the KA and H-WCA for linear and CWM surfaces at 5 m.s^{-1} wind speed, for a PM spectrum at 50 degrees incidence angle in X band (electromagnetic wavelength $\lambda_e = 3$ cm). It has been obtained from the numerical Fourier transform of $\mathcal{C}(t)$ in a few minutes on an ordinary personal computer. We have superimposed the Monte-Carlo results over 100 linear and nonlinear surfaces, showing an excellent agreement with the statistical formulation. The Bragg frequency is given as reference.

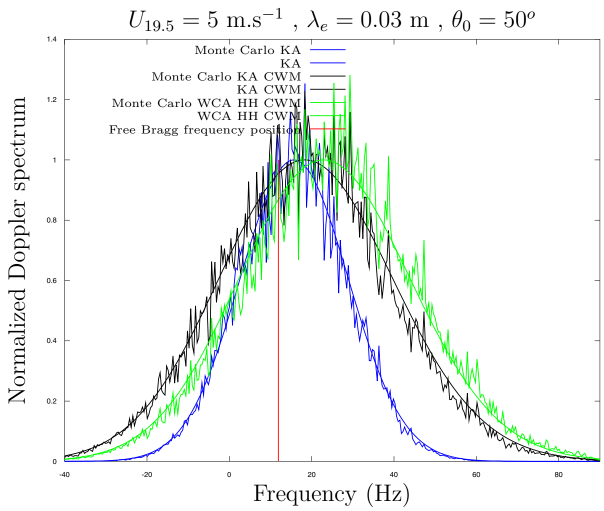

Figure 1: Comparison of statistical and Monte-Carlo KA, KA-CWM and H-WCA-CWM Doppler spectrum shape at 50 degree incidence in X band with $U_{19.5} = 5$ m.s^{-1}.

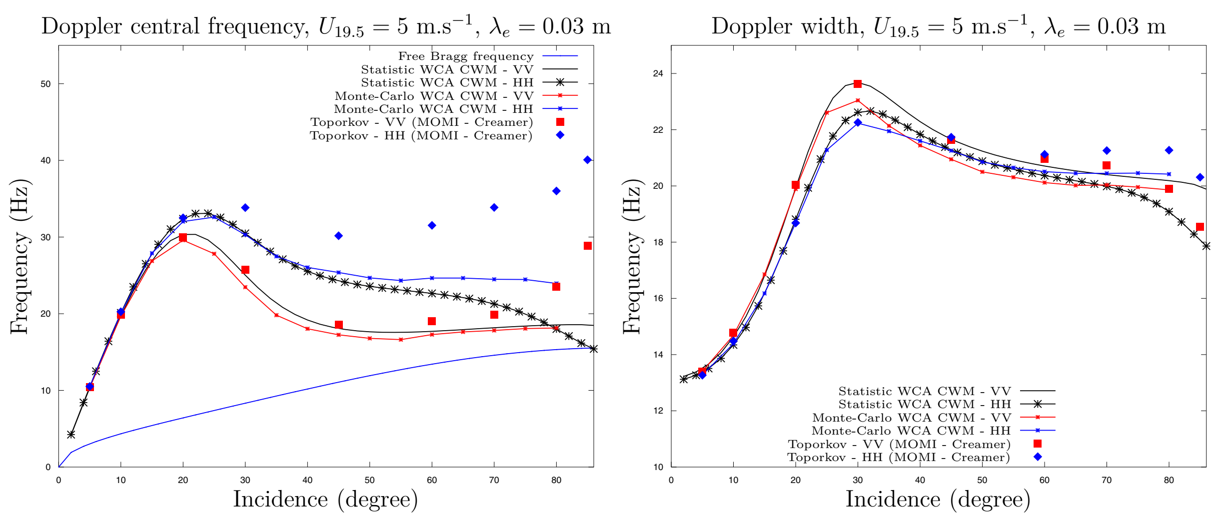

Figure 2: Comparison of Monte-Carlo and Statistical WCA-CWM Doppler central frequency and width in, X band with $U_{19.5} = 5$ m.s^{-1}.

Figure 2 shows the Doppler central frequency and width as a function of the incidence angle for the WCA-CWM in X band at 5 m.s^{-1} wind speed. The statistical formulation is in overal good agreement with the Monte-Carlo averaging and much less computationally time consuming. The noticeable differences are a measure of the accuracy of each kind of computations. The results presented in [9] also appear on figure 2. Our studies have shown that the Creamer model used in their sea surface generation is responsible of the observed discrepancies for non-grazing angles, say up to 70°.

4 Conclusion

We have investigated the characteristics of the microwave sea Doppler spectrum in the framework of analytical electromagnetical and hydrodynamical models. The model combining WCA and CWM is sensitive to both polarization and hydrodynamical modulation and still enjoys a statistical, numerically efficient, formulation. Doppler spectra in the microwave regime are quite different from those observed in HF. Their central frequency is higher than the free Bragg frequency, is polarization-dependent and has a non-monotonic behavior with incidence. The spectra are much broader, even at large angles, a property clearly determined by the nonlinear nature of the waves. Further work is in progress to employ these promising techniques for geophysical applications.

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