A Unified Microwave Radiative Transfer Model with Jacobian for General Planar Stratified Media

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1. INTRODUCTION

Currently, a major challenge in passive microwave remote sensing is the accurate and fast forward numerical modeling of the electromagnetic scattering and emission properties of any geophysical media consisting of soil, water, snow, ice, rain, cloud, fog, etc. Of importance in any such numerical model is accuracy, numerical stability, computational speed, applicability to both dense and tenuous scattering media, and the capability to produce a Jacobian for radiance assimilation purposes. To achieve these purposes Voronovich et al., [1] developed the discrete ordinate tangent linear radiative transfer (DOTLRT) model based on the analytical diagonalization and factorization of symmetric and positive definite transition matrices to provide inherent computational stability and high computational efficiency for all matrix operations required by discrete ordinate eigenanalysis method. By applying eigenvalue perturbation theory DOTLRT can rapidly compute the Jacobian for the observed antenna temperature with respect to any arbitrary scattering and absorbing layer parameter.

Although DOTLRT provides a comprehensive solution to the discrete radiative transfer equation (DRTE), it was originally derived only for sparse media (e.g., rain, fog, cloud, aerosols) using a Henyey-Greenstein (HG) phase matrix, and for non-refracting layers (i.e., no abrupt surfaces) with constant temperature profiles. These attributes greatly limit its application, especially for cases of dense media (e.g., snow, ice, soil) and layers with strong temperature gradients. In this paper, we present a new unified microwave radiative transfer (UMRT) model to extend DOTLRT in all of the above areas. This model can be applied to widely varying types of media for both forward transfer and radiance assimilation purposes. Within UMRT we partition media layers into two categories, which are treated distinctly as follows: 1) sparse medium layers, in which particles are loosely distributed and independent scattering is dominant, and 2) dense medium layers, in which particles occupy significant fractional volume and volumetric multiple scattering is dominant. For sparse medium layers, the cross-polarization is considered by using the full classical Mie phase matrix (4×4). Assuming independent scattering, UMRT sparse medium layers are parameterized by sets of particle size distribution functions for each of the different scatterer phases, for example, liquid sphere, ice spheres, etc. Calculations of the corresponding extinction, scattering and absorption coefficients, and phase matrices are performed for each of these phases. For dense medium layers,

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the dense media radiative transfer (DMRT) theory [2, 3] is applied within UMRT. The DMRT theory with the quasi-crystalline approximation (QCA) was developed by Tsang and colleagues beginning in the early 1980s. In UMRT, we employ the most recent (2008) version of the DMRT-QCA model by Tsang et al. [3]. This model uses a sticky particle assumption for moderately sized (i.e., Mie scale) spherical particles. In this model, the adhesion and aggregative formation of the sticky particles are simulated by using sticky pair distribution functions based on the Percus-Yevick (PY) approximation. The absorption and scattering coefficients are calculated under the DMRT framework.

Other nontrivial extensions in UMRT are: 3) the thermal radiation of a layer is allowed to be linear in height. This extension allows the application of a linearized temperature profile, which improves the accuracy of DOTLRT for sparsely vertically sampled profiles; 4) In UMRT, we solve for the radiation intensities at select Gaussian quadrature angles within every layer, but compensate for refraction at layer interfaces by applying Snell's law and a cubic spline interpolation matrix to the refracted radiation streams. 5) UMRT also inherits from DOTLRT the capability for rapid Jacobian calculation for a general medium model.

2. UMRT FRAMEWORK

UMRT assumes a planar stratified medium structure, and provides a solution for the brightness temperature $T_B(\theta, z)$ in upwelling (+) and downwelling (-) directions, accounting for polarization coupling caused by the reduced phase matrix. The differential radiative transfer equation is discretized over a set of quadrature angles θ_i, which are determined by the Gauss-Legendre nodes and Christoffel weights. Without presenting the details, we derive the following matrix form of the DRTEs consistent with DOTLRT [1]:

$$\frac{d}{dz}\begin{bmatrix}\bar{\boldsymbol{u}}_v\\ \bar{\boldsymbol{u}}_h\\ \bar{\boldsymbol{v}}_v\\ \bar{\boldsymbol{v}}_h\end{bmatrix} = \begin{bmatrix}-\bar{\bar{\boldsymbol{A}}}_0 & -\bar{\bar{\boldsymbol{C}}}_0 & -\bar{\bar{\boldsymbol{B}}}_0 & -\bar{\bar{\boldsymbol{D}}}_0\\ -\bar{\bar{\boldsymbol{E}}}_0 & -\bar{\bar{\boldsymbol{G}}}_0 & -\bar{\bar{\boldsymbol{F}}}_0 & -\bar{\bar{\boldsymbol{H}}}_0\\ \bar{\bar{\boldsymbol{B}}}_0 & \bar{\bar{\boldsymbol{D}}}_0 & \bar{\bar{\boldsymbol{A}}}_0 & \bar{\bar{\boldsymbol{C}}}_0\\ \bar{\bar{\boldsymbol{F}}}_0 & \bar{\bar{\boldsymbol{H}}}_0 & \bar{\bar{\boldsymbol{E}}}_0 & \bar{\bar{\boldsymbol{G}}}_0\end{bmatrix}\begin{bmatrix}\bar{\boldsymbol{u}}_v\\ \bar{\boldsymbol{u}}_h\\ \bar{\boldsymbol{v}}_v\\ \bar{\boldsymbol{v}}_h\end{bmatrix} + \begin{bmatrix}\bar{\boldsymbol{f}}\\ \bar{\boldsymbol{f}}\\ -\bar{\boldsymbol{f}}\\ -\bar{\boldsymbol{f}}\end{bmatrix} = \underbrace{\begin{bmatrix}-\bar{\bar{\boldsymbol{U}}} & -\bar{\bar{\boldsymbol{D}}}\\ \bar{\bar{\boldsymbol{D}}} & \bar{\bar{\boldsymbol{U}}}\end{bmatrix}}_{\text{DOTLRT}}\begin{bmatrix}\bar{\boldsymbol{u}}\\ \bar{\boldsymbol{v}}\end{bmatrix} + \begin{bmatrix}\bar{\boldsymbol{f}}\\ \bar{\boldsymbol{f}}\\ -\bar{\boldsymbol{f}}\\ -\bar{\boldsymbol{f}}\end{bmatrix} \tag{1}$$

where $\bar{\bar{\boldsymbol{U}}} = \begin{bmatrix}\bar{\bar{\boldsymbol{A}}}_0 & \bar{\bar{\boldsymbol{C}}}_0\\ \bar{\bar{\boldsymbol{E}}}_0 & \bar{\bar{\boldsymbol{G}}}_0\end{bmatrix}$, $\bar{\bar{\boldsymbol{D}}} = \begin{bmatrix}\bar{\bar{\boldsymbol{B}}}_0 & \bar{\bar{\boldsymbol{D}}}_0\\ \bar{\bar{\boldsymbol{F}}}_0 & \bar{\bar{\boldsymbol{H}}}_0\end{bmatrix}$, $\bar{\boldsymbol{u}} = \begin{bmatrix}\bar{\boldsymbol{u}}_v\\ \bar{\boldsymbol{u}}_h\end{bmatrix}$, and $\bar{v} = \begin{bmatrix}\bar{\boldsymbol{v}}_v\\ \bar{\boldsymbol{v}}_h\end{bmatrix}$.

Since all sub-matrix components in the matrices $\bar{\bar{\boldsymbol{U}}}$ and $\bar{\bar{\boldsymbol{D}}}$ from either the Mie or the DMRT-QCA theory are symmetric, both matrices $\bar{\bar{\boldsymbol{U}}}$ and $\bar{\bar{\boldsymbol{D}}}$ are symmetric, and therefore so are the matrices $\bar{\bar{\boldsymbol{U}}} + \bar{\bar{\boldsymbol{D}}}$ and $\bar{\bar{\boldsymbol{U}}} - \bar{\bar{\boldsymbol{D}}}$. Directly inheriting the procedure from DOTLRT [1, eqs. 24, 25] by applying the Gershgorin's circle theorem and noting that the inequality $k_e \geq k_s$ always holds in RT theory, we conclude that the matrices $\bar{\bar{\boldsymbol{U}}} + \bar{\bar{\boldsymbol{D}}}$ and $\bar{\bar{\boldsymbol{U}}} - \bar{\bar{\boldsymbol{D}}}$ are both positive semidefinite and applicable to the efficient inversion technique within DOTLRT.

Another fundamental difference between UMRT and DOTLRT is that UMRT solves the DRTE for a linear temperature profile. Owing to thermal emission, the layer generates radiation in up- (u) and down-welling (v) directions at its top and bottom surfaces, respectively. For the constant temperature profile in DOTLRT the self-radiation streams (denoted by subscript of *) a single layer are:

$$\bar{\boldsymbol{u}}_*(z=0)=\bar{\boldsymbol{v}}_*(z=0)=\bar{\boldsymbol{u}}_*(z=\Delta z)=\bar{\boldsymbol{v}}_*(z=\Delta z) \tag{2}$$

However, since a linear temperature profile is assumed in UMRT, we have

$$\bar{\boldsymbol{u}}_*(z=0)=\bar{\boldsymbol{v}}_*(z=0)\neq\bar{\boldsymbol{u}}_*(z=\Delta z)=\bar{\boldsymbol{v}}_*(z=\Delta z) \tag{3}$$

The above difference makes DOTLRT a layer-centric model while UMRT is a level-centric RT model. Applying block matrix inversion, the general solution of the inhomogeneous equation (1) in UMRT is

$$\begin{bmatrix}\bar{\boldsymbol{u}}_{inh}\\ \bar{\boldsymbol{v}}_{inh}\end{bmatrix}=\begin{bmatrix}\bar{\bar{\boldsymbol{A}}}^{-1}\left(\bar{\boldsymbol{f}}-\bar{\bar{\boldsymbol{A}}}^{-1}t_f\right)\\ \bar{\bar{\boldsymbol{A}}}^{-1}\left(\bar{\boldsymbol{f}}+\bar{\bar{\boldsymbol{A}}}^{-1}t_f\right)\end{bmatrix} \tag{4}$$

where t_f is the slope of the linear temperature source $\bar{\boldsymbol{f}}$ (Fig. 1). As depicted in Fig. 1, adding all of these contributions results in the following expression for the self-radiation field in terms of matrices $\bar{\bar{\boldsymbol{r}}}$ and $\bar{\bar{\boldsymbol{t}}}$.

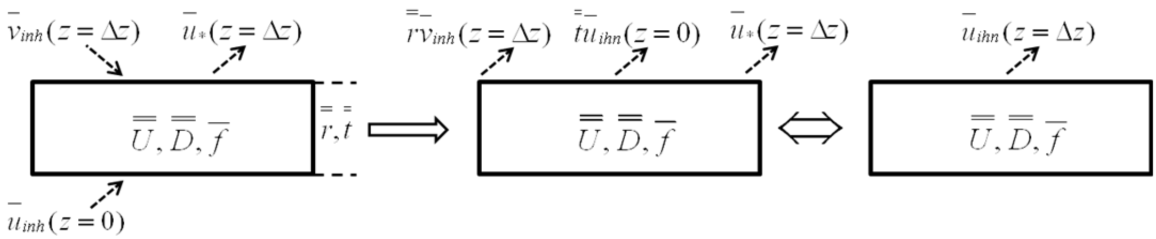

Figure 1. Calculation of the upwelling self-radiation of a single layer

$$\begin{aligned}\bar{\boldsymbol{u}}_*(z=\Delta z)&=\bar{\boldsymbol{u}}_{inh}(z=\Delta z)-\bar{\bar{\boldsymbol{r}}}\bar{\boldsymbol{v}}_{inh}(z=\Delta z)-\bar{\bar{\boldsymbol{t}}}\bar{\boldsymbol{u}}_{inh}(z=0)\\ \bar{\boldsymbol{v}}_*(z=0)&=\bar{\boldsymbol{v}}_{inh}(z=0)-\bar{\bar{\boldsymbol{r}}}\bar{\boldsymbol{u}}_{inh}(z=0)-\bar{\bar{\boldsymbol{t}}}\bar{\boldsymbol{v}}_{inh}(z=\Delta z)\end{aligned} \tag{5}$$

In UMRT, the recursive solutions for multilayer structure are solved at a fixed number M of Gaussian quadrature angles for every layer. Depending on the mean permittivity differences between two adjacent layers, the critical angle and Fresnel effects are also considered in the solution procedure. The calculation of the radiation Jacobian in UMRT follows the same procedure as in DOTLRT. Details on this calculation can be found in [1, sec. III].

3. NUMERICAL EXAMPLES

We use a single rain layer as an example by which to compare the brightness temperatures computed by UMRT under three reduced phase matrices: the Rayleigh, HG, and Mie phase matrices. Both constant and linear temperature profiles are used within this layer. The Marshall-Palmer (MP) size distribution [4] is assumed and the water permittivity is determined by Meissner and Wentz's double Debye expression [5] at a temperature $T = 0^{o}$C. For a single 1-km MP rain layer with a linear temperature profile ranging from 300 K at the bottom to 273 K at the top and 10 mm/hr rain rate, the computed brightness temperatures are plotted in Fig. 2.

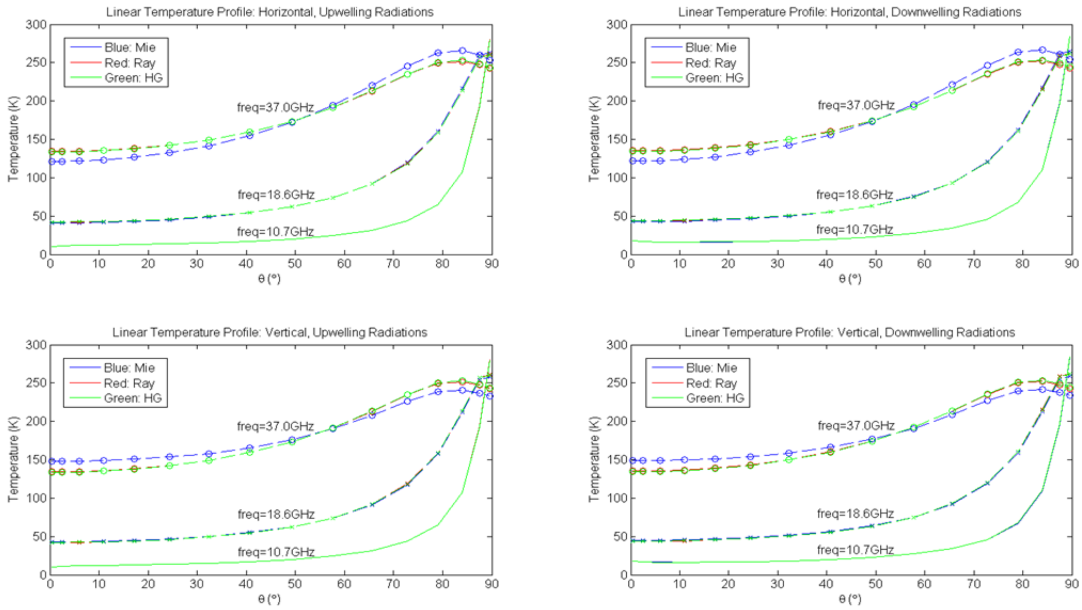

Figure 2. Brightness temperatures for a single MP rain layer with linear temperature profile. The brightness temperatures from top-left to bottom-right are: horizontal-upwelling, horizontal-downwelling, vertical-upwelling, and vertical-downwelling. Blue, red and green plots are made using the Mie, Rayleigh, and HG phase matrices, respectively.

4. SUMMARY

We present herein a new unified microwave radiative transfer (UMRT) model for accurate, fast and stable calculation of thermal radiation from any geophysical medium comprised of planar multilayer spherical scatterers of arbitrary electrical size. UMRT combines several a unique features from classical discrete ordinate radiative transfer theory and dense media radiative transfer theory under the DOTLRT framework for calculating the thermal radiation from either sparse or dense media. Other important features of UMRT include the inherent stability and high efficiency of matrix calculations for both the brightness temperatures and associated Jacobians, application of the reduced Mie and DMRT-QCA phase matrices, incorporation of linear radiation and temperature profiles within each layer, and use of the critical angle and Fresnel effects at layer interfaces.

5. REFERENCES

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