Estimating Ambiguity Number of Radial Velocity for Ground Moving Targets from a Single SAR Sensor

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Abstract — **This paper addresses an ambiguity number estimation approach of cross-track velocity for ground moving targets from a single synthetic aperture radar (SAR) sensor. We first transform the target signatures into range frequency domain and then compress it for each range frequency. The resulting compressed envelope exhibits a straight line with its slope just proportional to the ambiguity number of the induced Doppler centroid. Then we can estimate its slope and accurately obtain this number by linear features detection techniques. And thus the true radial velocity can be completely retrieved. For dim moving targets, an improved estimation strategy is introduced. The effectiveness of this approach is demonstrated by theoretical analysis and real measured SAR data.**

Index Terms — **Ground moving targets indication (GMTI); Velocity estimation; Range frequency domain; Synthetic aperture radar (SAR); Ambiguity resolution**

I. INTRODUCTION

Synthetic aperture radar/ground moving targets indication (SAR/GMTI) system has been successfully and widely used in civilian and military areas to detect moving targets and relocate them in SAR imagery. The most important procedure of target relocation is the cross-track velocity estimation, since the phase induced by the motion of target leads to the image of the moving target mis-located in azimuth and the range walk by cross-track velocity causes smeared (defocusing) image. This velocity can be estimated according to the Doppler centroid of target spectrum band in a single SAR sensor [1-2]. The classical method is space-time-frequency processing [3-4].

Due to temporal sampling, the cross-track velocity estimation suffers the limitation of pulse repetition frequency (PRF). If the induced Doppler shift of target signature exceeds PRF/2, the ambiguity phenomenon of velocity estimation will occur. Classical solutions to this problem are to relocate the equipment with an increasing PRF [5] or a nonuniform PRF [6] for a single SAR sensor. Increasing PRF shortens the unambiguous range swath and simultaneously increases the memory requirements. The experiment with a nonuniform PRF needs more complex equipment and special image reconstruction algorithms. Other methods such as linear antenna array velocity SAR (VSAR) [7-8], dual-speed SAR [9] and multifrequency antenna array SAR [10-11] can be well exploited for velocity ambiguity resolution. However, the system complexity will increase and the equipment becomes more intricate.

Different from these ambiguity resolution methods above, some approaches utilizing additional information have been proposed for the available system with a single PRF [12, 13, and 14]. In [12], an approach has been proposed to estimate the cross-track velocity of fast moving targets together with the range walk induced by target motion, which can enlarge the unambiguous scope of velocity estimation to a certain extent. A. C. M. Paulo has proposed a velocity estimation method for fast moving target utilizing the skew of the two-dimensional (2-D) spectral signature of target [14]. The velocity estimation accuracy may be better than 3% provided that the moving target has a high signal-to-clutter-plus-noise ratio (SCNR) (digitally spotlighted out from the background and ensure that the SCNR is greater than 14 dB).

Unlike the traditional ambiguity resolving approaches, in this paper, we propose a method which can directly estimate the ambiguity number of cross-track velocity. The envelope in azimuth-compressed time and range frequency domain behaves as a straight line with its slope proportional to the ambiguity number. For moving targets with relatively low signal-to-noise ratio (SNR), we extract two looks data in the range direction and focus each look as well as possible. The ambiguity number can be estimated by the number of azimuth cell displacement between two focused images. The baseband velocity can be estimated by conventional algorithms [3, 4, 15 and 16]. The final accurate velocity is the combination of the baseband velocity and velocity ambiguity.

II. SIGNAL MODELING

This section describes a deterministic model for echoes backscattered from a point moving target and the received by the platform for sidelooking radar. The geometry relationship between the flying platform and target is shown in Fig. 1. v_a and v_c denote the along-track and cross-track velocities (projection on imaging plane), respectively. t_m is the slow

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time in azimuth. R_B and $R_s(t_m)$ are the nearest and instantaneous slant ranges between the platform and target, respectively. v is platform velocity. During the slow time interval, the target moves from Position a to b.

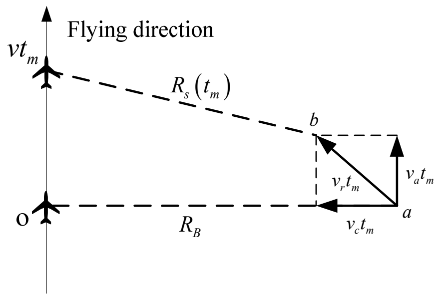


Fig. 1. Top-down view of the platform and a moving target geometry.

The range-compressed signal of target is expressed by

$$s(t,t_m)=\sigma_s Bw(t_m)\operatorname{sinc}\left(\Delta B\left(t-\frac{2R_s(t_m)}{c}\right)\right)\exp\left(-j\frac{4\pi}{\lambda}R_s(t_m)\right) \quad (1)$$

where t is the fast time, σ_s the complex reflectivity of target, B the range compression gain, $w(t_m)$ the azimuth envelope, ΔB the spectrum bandwidth of transmitted signal, c the speed of light and λ the wavelength. The instantaneous slant range $R_s(t_m)$ can be given by

$$R_s(t_m)\simeq R_B-v_c t_m+\frac{(v-v_a)^2}{2R_B}t_m^2 \quad (2)$$

For a SAR system, the cross-track velocity can be estimated by the Doppler centroid of moving target due to the fact that the induced phase is a linear term [see (2)]. However, estimation of the absolute Doppler centroid from target data in azimuth is difficult, since the pulsed nature of SAR gives rise to a periodic replication of the Doppler spectrum. Like the division of Doppler centroid estimation for SAR systems [14], we divide the true cross-track velocity into two parts according to the mission PRF: the baseband velocity and velocity ambiguity. The baseband velocity v_{base} is confined as

$$-\frac{\text{PRF}\lambda}{4}\le v_{base}\le\frac{\text{PRF}\lambda}{4} \quad (3)$$

The true cross-track velocity, however, is the sum of the baseband velocity and velocity ambiguity. It can be represented as

$$v_c=M\frac{\text{PRF}\lambda}{2}+v_{base} \quad (4)$$

where M denotes the ambiguity number with the value being integer (i.e., $M=\cdots-2,-1,0,1,2\cdots$). From (4), it is observed that if one can obtain the ambiguity number accurately, the true cross-track velocity can also be retrieved since the baseband velocity can be precisely estimated by conventional algorithms.

III. AMBIGUITY NUMBER ESTIMATION ALGORITHM

3.1 Proposed Algorithm Description

We begin by presenting the algorithm when there is no Doppler ambiguity. According to the method for the spectrum calculation of large time-bandwidth product and phase modulation signal (by application of the principle of stationary phase, [18, 19]), the range-compressed signal of target transformed into range and azimuth 2-D frequency domain can be formulated as

$$\begin{aligned}S(f,f_a)=&\sigma_s P(f)W\left(-\frac{cR_B}{2(f+f_c)(v-v_a)^2}\left(f_a-\frac{2v_c(f+f_c)}{c}\right)\right)\\&\exp\left(j\frac{\pi R_B}{2(v-v_a)^2}\left(\frac{cf_a^2}{f+f_c}-4v_c f_a\right)\right)\exp\left(-j\frac{4\pi R_B}{c}\left(1-\frac{v_c^2}{2(v-v_a)^2}\right)f\right)\end{aligned} \quad (5)$$

where $P(f)$ is the Fourier transform (FT) of the range envelope with f the range frequency, $W(f_a)$ is the FT of $w(t_m)$, f_a is the azimuth frequency with $-\text{PRF}/2\le f_a\le\text{PRF}/2$, and f_c is carrier frequency. Note that the constant phase has been ignored in arriving at the final form in (5), which will not influence the parameter estimation. In general, from (5), it can be seen that the azimuth frequency is coupled with range frequency, and the cross-track velocity exists in the linear term with respect to azimuth frequency. Therefore, we construct the reference function in 2-D frequency domain as follows:

$$\text{H}(f,f_a)=W_r(f,f_a)\exp\left(-j\frac{\pi cR_B f_a^2}{2(v-v_a)^2(f+f_c)}\right) \quad (6)$$

where $W_r(f,f_a)$ is the window function in 2-D frequency domain. According to (5) and (6), the signal (5) multiplied by the reference function (6) is written as

$$\begin{aligned}\widehat{S}(f,f_a)=&\sigma_s P(f)W\left(-\frac{cR_B}{2(f+f_c)(v-v_a)^2}\left(f_a-\frac{2v_c(f+f_c)}{c}\right)\right)\\&\exp\left(-j\frac{2\pi f_a v_c R_B}{(v-v_a)^2}\right)\exp\left(-j\frac{4\pi R_B}{c}\left(1-\frac{v_c^2}{2(v-v_a)^2}\right)f\right)\end{aligned} \quad (7)$$

By performing inverse Fourier transform (IFT) on (7) in the azimuth direction, the signature in range frequency and azimuth-compressed time domain is expressed by

$$\hat{s}(f,t_m)=A\sigma_s P(f)\operatorname{sinc}\left(B_D\left(t_m-\frac{R_B v_c}{(v-v_a)^2}\right)\right)\exp\left(-j\frac{4\pi R_B}{c}\left(1-\frac{v_c^2}{2(v-v_a)^2}\right)f\right) \quad (8)$$

where A is the azimuth compression gain, B_D the Doppler band of the moving target. From (8), it is found that when the inducing Doppler centroid is unambiguous, the envelope of the moving target is independent of range frequency.

In the situation where the ambiguity exists, the Doppler band of the moving target will be overlapped by the PRF. The baseband signature of target in 2-D frequency domain should be rewritten as

$$\begin{aligned}S(f,f_a)=&W\left(-\frac{cR_B}{2(f+f_c)(v-v_a)^2}\left(f_a+M\,\text{PRF}-\frac{2v_c(f+f_c)}{c}\right)\right)\\&\sigma_s P(f)\exp\left(j\frac{\pi R_B}{2(v-v_a)^2}\left(\frac{c(f_a+M\,\text{PRF})^2}{f+f_c}-4v_c(f_a+M\,\text{PRF})\right)\right)\\&\exp\left(-j\frac{4\pi R_B}{c}\left(1-\frac{v_c^2}{2(v-v_a)^2}\right)f\right)\end{aligned} \quad (9)$$

In this case, we utilize the approximate expression $\frac{1}{f+f_c} \approx \frac{1}{f_c} - \frac{f}{f_c^2} \left(f \ll f_c \right)$ and perform IFT on (9) in the azimuth direction. The resulting output can be represented by

$$\hat{s}(f,t_m) \approx \mathrm{sinc}\left(B_D \left(t_m - \frac{cM\mathrm{PRF}R_B}{2f_c^2\left(v-v_a\right)^2} f + \frac{R_B}{2\left(v-v_a\right)^2} \left(\frac{cM\mathrm{PRF}}{f_c} - 2v_c \right) \right) \right)$$
$$A\sigma_s P(f) \exp\left(-j \left(\frac{\pi c R_B M^2 \mathrm{PRF}^2}{2\left(v-v_a\right)^2 f_c^2} f + \frac{4\pi R_B}{c} \left(1 - \frac{v_c^2}{2\left(v-v_a\right)^2} \right) f \right) \right) \quad (10)$$

It can be observed that the slope of the compressed envelope in range frequency domain is

$$k = \frac{cM\mathrm{PRF}R_B}{2f_c^2\left(v-v_a\right)^2} \quad (11)$$

From (11), it is interestingly found that the ambiguity number of cross-track velocity can be directly estimated by the slope formulated above, which makes this method different from other approaches [12, 13, and 14]. The final velocity is the combination of the baseband velocity and ambiguity number estimated by the proposed methodology.

3.2 Estimation Strategy for Targets with Low SCNR

For weak moving targets with low SNR, we extract two range looks data and accumulate the target signal for each range look data along the range direction to improve the SNR. The upper and lower parts of available range bandwidth of the returned target signal are extracted to form two different looks with different wavelengths, the concept of which was suggested for the Doppler centroid estimation. We shift the signal of Look one with $\Delta f_r / 2$ (Δf_r is the range spectrum separation between two range looks) in order to make the range spectrum symmetric in range frequency domain. The signal of Look one after shifting is shown by

$$s_1\left(t,t_m\right) = \sigma_s B_1 w\left(t_m\right) \mathrm{sinc}\left(\Delta B \left(t - \frac{2R_s\left(t_m\right)}{c} \right) \right) \exp\left(-j \frac{4\pi}{c} \left(f_c - \frac{\Delta f_r}{2} \right) R_s\left(t_m\right) \right) \quad (12)$$

where B_1 is the range compression gain of Look one. Similarly, the received signal of Look two is shown by

$$s_2\left(t,t_m\right) = \sigma_s B_2 w\left(t_m\right) \mathrm{sinc}\left(\Delta B \left(t - \frac{2R_s\left(t_m\right)}{c} \right) \right) \exp\left(-j \frac{4\pi}{c} \left(f_c + \frac{\Delta f_r}{2} \right) R_s\left(t_m\right) \right) \quad (13)$$

where B_2 is the range compression gain of Look two. After the azimuth cell walk correction, the imaging amplitudes of the target signal for both two range looks are given as follows

$$\left|\hat{s}_1\left(t,t_m\right)\right| = \left| AB_1\sigma_s \,\mathrm{sinc}\left(B_D \left(t_m + \frac{R_B}{2\left(v-v_a\right)^2} \left(\frac{cM\mathrm{PRF}}{f_c - \Delta f_r/2} - 2v_c \right) \right) \right) \right.$$
$$\left. \mathrm{sinc}\left(\Delta B \left(t - \frac{2R_B}{c} \left(1 - \frac{v_c^2}{2\left(v-v_a\right)^2} \right) + \frac{cR_B M^2 \mathrm{PRF}^2}{4\left(v-v_a\right)^2 \left(f_c - \Delta f_r/2\right)^2} \right) \right) \right| \quad (14)$$

$$\left|\hat{s}_2\left(t,t_m\right)\right| = \left| AB_2\sigma_s \,\mathrm{sinc}\left(B_D \left(t_m + \frac{R_B}{2\left(v-v_a\right)^2} \left(\frac{cM\mathrm{PRF}}{f_c + \Delta f_r/2} - 2v_c \right) \right) \right) \right.$$
$$\left. \mathrm{sinc}\left(\Delta B \left(t - \frac{2R_B}{c} \left(1 - \frac{v_c^2}{2\left(v-v_a\right)^2} \right) + \frac{cR_B M^2 \mathrm{PRF}^2}{4\left(v-v_a\right)^2 \left(f_c + \Delta f_r/2\right)^2} \right) \right) \right| \quad (15)$$

where A is the azimuth compression gain. Since the envelope in range frequency after azimuth compression is not a straight line vertical to the azimuth axis in the presence of velocity ambiguity, the azimuth cell walk should be corrected as well as possible to finely focus the image of target for each range look data.

From (14) and (15), it is found that the number of azimuth cell displacement between two focused images is also proportional to ambiguity number and can be expressed by

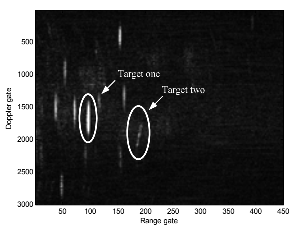
$$M = -\frac{2N_a\left(v-v_a\right)^2 \left(f_c^2 - \Delta f_r^2/4\right)}{cR_B \,\mathrm{PRF}^2 \,\Delta f_r} \quad (16)$$

where N_a is the number of azimuth cell displacement between Look one and two (with positive or negative sign). Due to the fact that v_a is far smaller that v, the along-track velocity will not influence the ambiguity number estimation. From the aforementioned derivations (14) and (15), it can be seen that not only the azimuth cell but also the range cell have displacement between two images due to the cross-track velocity ambiguity. The number of range cell displacement N_R is given by $N_R = \frac{cR_B f_c f_s \Delta f_r M^2 \,\mathrm{PRF}^2}{2\left(v-v_a\right)^2 \left(f_c^2 - \Delta f_r^2/4\right)^2}$. Fortunately, it is found that the number of range cell displacement is small and far less than a range cell. The cross-track velocity ambiguity can be well calculated by the number of azimuth cell displacement between two target images which are located at the same range gate.

IV. EXPERIMENTS WITH REAL MEASURED DATA

In this section, we investigate the performance of proposed method by real measured data. To verify the ambiguity resolution performance, we use a set of GMTI radar data which is recorded by an airborne X-band radar system.

Fig. 2 shows the residual clutter after the clutter rejection in azimuth Doppler domain, from which moving targets can be well seen. These include moving targets with high SCNR (Target one) and dim moving targets (Target two) with relatively low SCNR. As expected, other moving targets appear misplaced due to different cross-track velocities.



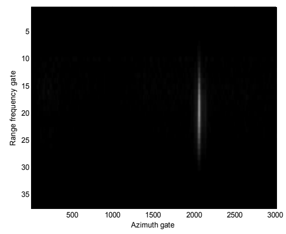
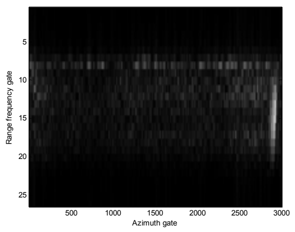


Fig. 2. Scene echo in Doppler frequency domain after clutter rejection.

Fig. 3. Envelope of Target one after azimuth compression.

We extract the data involving the information about Target one from the range gates 76 to 112, and transform the data into range frequency domain. The azimuth compression procedure is then performed. The envelope of Target one after azimuth compression in range frequency domain is shown in Fig. 3. It is seen that the envelope is vertical to the azimuth axis, which indicates that the ambiguity number of the estimated radial velocity is zero.



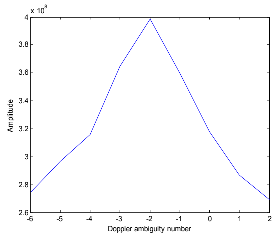
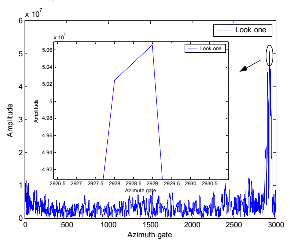


Fig. 4. Envelope of Target two after azimuth compression.

Fig. 5. Doppler ambiguity number estimation for Target two.

Secondly, the ambiguity number estimation approach is applied for fast moving targets (Target two labeled in Fig. 2). Similarly, the data of Target two is extracted from the range gates 175 to 200. Again, the extracted data is transformed into range frequency domain and compressed in the azimuth direction. The envelope after azimuth compression is shown in Fig. 4, from which it is clearly seen that the corresponding slope is not vertical to the azimuth axis. In this situation, the ambiguity number can be estimated by the slope of envelope with the curve plotted in Fig. 5. It is found that the ambiguity number of Target two is -2 (the negative denotes that the moving direction is reverse).



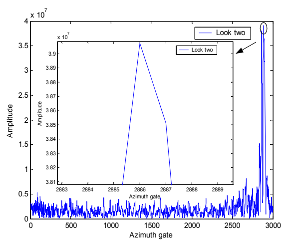


Fig. 6. Output of Look one and its amplified curve for Target two.

Fig. 7. Output of Look two and its amplified curve for Target two.

In the following example, we use (16) to estimate the ambiguity number of cross-track velocity for the dim moving target (Target two). Here we extract two range looks data and focus the target as well as possible for each look data. The corresponding outputs are plotted in Figs. 6 and 7 together with their amplified maps. From these figures, we can determine the number of azimuth cell displacement is 43 and the corresponding ambiguity number is about -2 (true value -1.96 and the slant range of Target two is about 8250m). Note that this ambiguity number is integer. Therefore, this number can be accurately determined.

V. CONCLUSIONS

This paper is primarily describing a new approach to resolve the cross-track velocity ambiguity. We divide cross-track velocity into two parts: the baseband velocity and velocity ambiguity. The proposed methodology exploits the fact that, in range frequency domain, the envelope of target signal, when compressed in the azimuth direction, exhibits a straight line with its slope proportional to the ambiguity number. For moving targets with relatively low SCNR, we extract two range look data and accumulate the energy for each look data to improve the SNR of target. The ambiguity number can be well determined by the number of azimuth cell displacement between two target images. Real measured SAR data illustrates the effectiveness of the ambiguity number estimation approach.

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