A Large domain complete basis functions for curved surfaces

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1. Introduction

The method of moments (MoM) with triangular surface discretization has become one of the most used techniques for solving electromagnetic scattering and radiation problems. This approach uses planar triangles to model the geometry and div-conforming basis functions to represent the surface current [1]. The planar triangle surface approximation, however, can lead to significant discretization errors when surfaces with high curvature are modelled. In near-field analysis the fidelity of the geometry modeling greatly impacts the accuracy of solutions so it is important to model curved features with curvilinear basis functions. We have recently introduced a complete analytical entire domain linear phase basis functions based on Shannon sampling theorem. These functions, called Linear-Phase Basis Functions (LPF) have been introduced in [2] for flat surfaces and are able to reconstruct the scattered field from a flat plate with a number of functions equal to the degrees of freedom of the scatterer. The GSF constitute a complete, discrete basis for any kind of equivalent currents on a flat finite surface. Their selection does not resort to any singular value decomposition (SVD) procedure, but they are selected in a non-redundant way by a Gram-Smith orthogonalization process. In this paper we present a generalization of the LPF introduced in [2] to curvilinear basis function able to represent any current on a curved surface.

2. Formulation

Let us consider the electromagnetic scattering problem constituted of a generic surface illuminated by an arbitrary
source (Fig. 1).

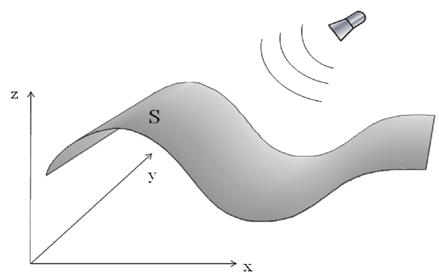

Fig.1: Electromagnetic scattering by a curved surface illuminated by an arbitrary source.

The surface S can be described using mathematical mapping. Every mapping is represented by γ_i that maps the parametric space $(u,v)\in U_i \subset \mathbb{R}^2$ into the real space $(x,y,z)\in S_i$ as depicted in Fig.2.

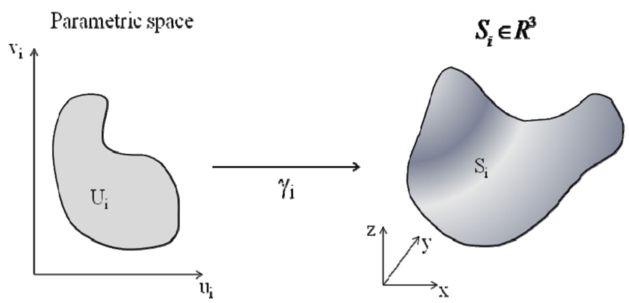

Fig.2: Mapping function from the

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If mappings are not known in a closed form, they can be obtained approximating S with a set of Non Uniform Rational B-Spline (NURBS) [3].
For the sake of simplicity, we will refer here to induced electric current $\mathbf{J}(x,y,z)$, understood that analogous steps can be followed for possibly additional magnetic currents in case of aperture or impedance surfaces.
Let us consider a generic mapping $\gamma_i : U_i \to S_i$. The induced current $\mathbf{J}_i(x,y,z)$ on S_i in the parametric space $\mathbf{J}_i(u,v)$ is a spatially limited function, that is

$$\mathbf{J}_i(u,v) = 0 \quad \forall |u| > \frac{D_u}{2}, \forall |v| > \frac{D_v}{2} \qquad (1)$$

As demonstrated in [3] the spatial limitation of $\mathbf{J}_i(u,v)$ implies the applicability of the usual sampling (Shannon) theorem to express the Fourier spectrum $\tilde{\mathbf{J}}_i(k_u,k_v)$ of $\mathbf{J}_i(u,v)$ as a function of an infinite set of spectral samples:

$$\mathbf{J}_i(u,v) = \frac{1}{D_u D_v} \sum_{p,q=-\infty}^{+\infty} \tilde{\mathbf{J}}_i\left(k_{up},k_{vq}\right) W_i(u,v) e^{-j\left(k_{up}u + k_{vq}v\right)} \quad (2)$$

were the spectral sampling points are given by

$$\left(k_{up},k_{yq}\right) = \left(p\Delta k_u, q\Delta k_v\right); \quad \Delta k_{u,v} = 2\pi / D_{u,v}. \qquad (3)$$

The representation in (2) implies that any current in the parametric space can be represented exactly in terms of a *discrete* set of *spatially windowed* (compactly supported) plane-wave basis functions that we call LPF according to [2].
A complete orthogonal set of basis functions for the current in the parametric space can be obtained by a Gram-Schmidt procedure using the usual L^2 inner product on the parametric space

$$\langle \mathbf{f},\mathbf{g} \rangle_{\mathbf{u}} = \iint_{U_i} \mathbf{f}^*(u,v) \cdot \mathbf{g}(u,v)\, du dv \qquad (4)$$

where $\mathbf{f},\mathbf{g} \in L^2(U_i)$.
The process starts with the GS function whose associated wavenumber is closest to the origin and proceeds on a spiral squared path by adding one more GS function at each step. Denoted by $\hat{\mathbf{J}}_1^{GS}$ the starting function, the orthogonal modes are constructed as

$$\mathbf{h}_1 = \hat{\mathbf{J}}_1^{GS} \;;\quad \mathbf{h}_n = \left\{ \hat{\mathbf{J}}_n^{GS} - \sum_{p=1}^{n-1} \frac{\left\langle \mathbf{h}_p, \hat{\mathbf{J}}_n^{GS} \right\rangle}{\left\langle \mathbf{h}_p, \mathbf{h}_p \right\rangle} \mathbf{h}_p \right\} \qquad (5)$$

normalized by using the norm $\|\mathbf{h}\| = \sqrt{\langle \mathbf{h},\mathbf{h} \rangle}$ induced by the scalar product i.e., $\mathbf{f}_n = \mathbf{h}_n / \|\mathbf{h}_n\|$.
We note that the coefficients in (5) are all expressed in a closed form [2].
The corresponding mapped functions $\overline{\mathbf{f}}_n(x,y,z) = \mathbf{f}_n \in L^2(S_i)$ in the real space forms a complete basis set for the current defined over the surface S_i [4].

3. Numerical Results

In order to show the completeness of the basis introduced above, a physical current distribution of a scattering problem obtained by a commercial Method of the Moment (FEKO) is projected on the new basis set. For the sake of simplicity we order the index doublet (*n,m*) into a single index *n* by starting from the origin of the sampling lattice (*n*=*m*=0) and moving in counterclockwise sense along a rectangular spirals; at each spectral step, the two polarization $\hat{\mathbf{p}} = \hat{\mathbf{u}}$ and $\hat{\mathbf{p}} = \hat{\mathbf{v}}$ are denoted by subsequent odd and even values of *n,* respectively. The scattering from a cylinder with radius $R = 1.5\lambda$ and height $h = 1.5\lambda$ illuminated by an electric dipole along *y*-direction at $(2\lambda, 0, 0)$ a cylinder illuminated by an electric dipole has been considered. The surface is described by the mapping:

$$\gamma_c = \begin{cases} x = R\cos u \\ y = R\sin u \\ z = v \end{cases} \qquad \begin{matrix} u \in [0, 2\pi) \\ v \in [0, h) \end{matrix} \qquad (8)$$

and shown in Fig.2.

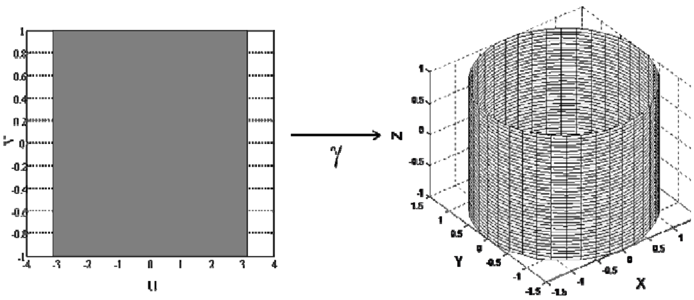

Fig.2: Mapping of a cylindrical surface.

The distribution of the energy associated to the terms of the expansion is depicted in Fig.3. It is apparent that most of the energy is contained in low-indexed terms, thus after a certain number of terms the series can be safely truncated.

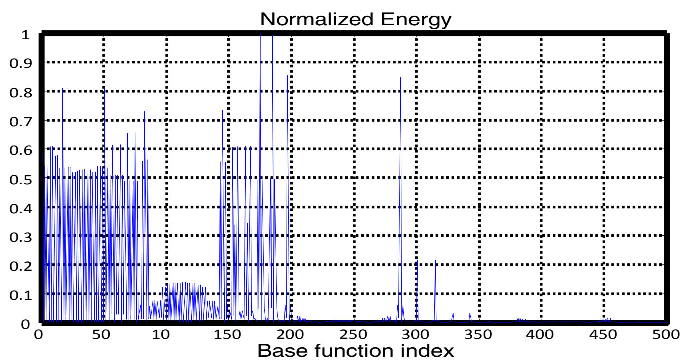

Fig.3: Histogram of the energy associated to the GSF spectra

Figure 4 presents successful comparative between the truncated current expansion using 350 terms and the reference solution provided by FEKO using 15878 RWG basis functions.

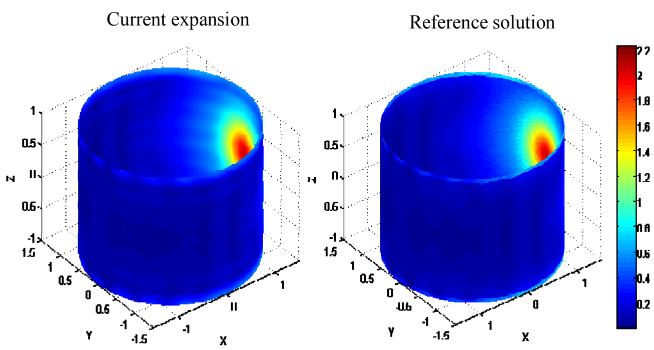

Fig.4: Comparison between a reference MoM solution and current expansion through GSF.

4. Conclusion

A new set of complete basis functions derived by the generalization of the Shannon theorem in the parametric space has been introduced, that is able to represent the field radiated by any current on a curved surface in a non-redundant way.

5. References

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