Efficient Stochastic EMC/EMI Analysis using HDMR-generated Surrogate Models

***Abdulkadir C. Yücel*[1], *Hakan Bağcı*[2], *and Eric Michielssen*[1]**

[1]Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI 48109, USA, E-mails: {acyucel,emichiel}@umich.edu

[2]Division of Physical Sciences and Engineering, King Abdullah University of Science and Technology, Thuwal 23955-6900, KSA, E-mail: hakan.bagci@kaust.edu.sa

1. Introduction

Stochastic methods have been used extensively to quantify effects due to uncertainty in system parameters (e.g. material, geometrical, and electrical constants) and/or excitation on observables pertinent to electromagnetic compatibility and interference (EMC/EMI) analysis (e.g. voltages across mission-critical circuit elements) [1]. In recent years, stochastic collocation (SC) methods, especially those leveraging generalized polynomial chaos (gPC) expansions, have received significant attention [2, 3]. SC-gPC methods probe surrogate models (i.e. compact polynomial input-output representations) to statistically characterize observables. They are nonintrusive, that is they use existing deterministic simulators, and often cost only a fraction of direct Monte-Carlo (MC) methods. Unfortunately, SC-gPC-generated surrogate models often lack accuracy (i) when the number of uncertain/random system variables is large and/or (ii) when the observables exhibit rapid variations.

Here, an extension to SC-gPC methods that permits the accurate construction of surrogate models for EMI/EMC observables by addressing the above concerns is presented. The proposed method leverages so-called high-dimensional model representation (HDMR) expansions to express observables as finite sums of "component functions" that represent independent and combined contributions of uncertain/random variables to the observable [4]; this feature of the proposed method addresses concern (i) above. The HDMR expansion is built iteratively by including only the "most significant" component functions to minimize the computational cost of building the surrogate model [5]. The component functions that feature in the HDMR expansion are approximated via an *h*-adaptive stochastic collocation method [2]; this method effectively tailors the sampling points for the polynomial approximation to the component function in the uncertain/random (sub-) space, thereby addressing concern (ii) above. HDMR-generated surrogate models enable the efficient and accurate stochastic characterization of electronic systems subject to many more manufacturing uncertainties that can not be addressed using "classical" SC-gPC methods. The technique derives its efficiency from the fact that only low-order correlations between random variables that contribute significantly to an observable are incorporated in the expansion; the method automatically excludes less significant high-order contributions from the surrogate model, thereby dramatically reducing the computational cost associated with its generation and evaluation.

2. Formulation

Stochastic Model: Let $\mathbf{x} = (x_1, x_2, \ldots, x_{N_{\text{dof}}})$ be an N_{dof} - dimensional vector defined over the random space Ω. The elements of $\mathbf{x}$, x_i, $i = 1, \ldots, N_{\text{dof}}$, are random variables that represent uncertain/random system parameters (e.g., material properties, geometrical features, and make-up of circuits) and/or excitations (e.g., pulse amplitude, phase, duration, and angle of arrival). Let $V(\mathbf{x})$ represent a system observable (e.g., a voltage across a mission-critical circuit element or a reflected field amplitude); $V(\mathbf{x})$ typically is a complicated function of $\mathbf{x}$, which can only be evaluated using full wave electromagnetic simulators, especially when analyzing real-world EMC/EMI problems involving large-scale complex structures [6, 7]. For such problems, the stochastic characterization of $V(\mathbf{x})$, (i.e., the

978-1-4244-6051-9/11/$26.00 ©2011 IEEE

computation of its mean, variance, and probability density function (pdf)) is almost impossible by direct application of the MC method, which would require many evaluations of $V(\mathbf{x})$ using the computationally expensive electromagnetic simulator. On the other hand, one can apply the MC method to an approximation of $V(\mathbf{x})$, namely its surrogate model, a compact polynomial representation of $V(\mathbf{x})$. In this case, the difficulty lies in obtaining an accurate but cheap to evaluate surrogate model of $V(\mathbf{x})$. This can be achieved using the iterative HDMR method proposed in [5]. In what follows, a cut-HDMR technique for surrogate model generation is described, and its integration with an iterative scheme to dramatically reduce the computational cost of the surrogate model generation is detailed.

Surrogate Model Generation via HDMR: The HDMR expansion enables to approximate $V(\mathbf{x})$ in terms of component functions as [4]

$$V(\mathbf{x}) = V_0 + \sum_{\mathbf{u} \subseteq D} V_{\mathbf{u}}(\mathbf{x}_{\mathbf{u}}), \tag{1}$$

where $D = \{1, \ldots, N_{\text{dof}}\}$ is the general set of random variable indices, $\mathbf{u} \subseteq D$, and $\mathbf{x}_{\mathbf{u}}$ is a $|\mathbf{u}|$-dimensional random vector, $V_{\mathbf{u}}(\mathbf{x}_{\mathbf{u}})$ represents the component functions defined over Ω, and V_0 is the zeroth-order component function. For example, for $\mathbf{u} = \{1\}$, $V_{\mathbf{u}}(\mathbf{x}_{\mathbf{u}}) = V_1(x_1)$ is the first-order component function that represents the individual contribution of x_1 to $V(\mathbf{x})$, and for $\mathbf{u} = \{1,2,4\}$, $V_{\mathbf{u}}(\mathbf{x}_{\mathbf{u}}) = V_{124}(x_1, x_2, x_4)$ is the third-order component function that reveals the combined contribution of x_1, x_2, and x_4.

Representation (1) can be constructed using the cut-HDMR method [5], which expresses the component functions in terms of observable values on lines, planes, and hyperplanes (i.e. cuts) passing through a reference point $\bar{\mathbf{x}} = (\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_{N_{\text{dof}}})$, which is typically set to the coordinates of the mass center of Ω. The Cut-HDMR method constructs component functions of a given order S hierarchically by starting from the zeroth-order function computed as $V_0 = V(\bar{\mathbf{x}})$, summing all component functions up to order $S-1$, and subtracting the sum from $V^{\mathbf{u}_S}(\mathbf{x}_{\mathbf{u}_S})$. Here, $V^{\mathbf{u}_S}(\mathbf{x}_{\mathbf{u}_S})$ represents the observable $V(\mathbf{x})$ that is the function of random variables whose indices are elements of the set $\mathbf{u}_S$,i.e. $i \in \mathbf{u}_S$, while all remaining random variables whose indices are not elements of the set $\mathbf{u}_S$,i.e. $i \in D$ but $i \notin \mathbf{u}_S$, are set to their corresponding mean values (see [4, 5] for details). In this scheme, $V^{\mathbf{u}_S}(\mathbf{x}_{\mathbf{u}_S})$ can be approximated efficiently using the error controllable adaptive SC-gPC scheme described in [2]. Even when used together with an adaptive scheme for evaluating $V^{\mathbf{u}_S}(\mathbf{x}_{\mathbf{u}_S})$, the computational cost of the cut-HDMR can be quite high: for a given order S, the number of component functions that should be computed scales with $\sum_{j=0}^{s} N_{\text{dof}}!/(j!(N_{\text{dof}} - j)!)$ [5]. This limits the direct application of cut-HDMR in realistic large-scale EMC/EMI problems for large N_{dof}. This high cost can be reduced considerably by integrating an iterative scheme to the hierarchical cut-HDMR method, which automatically selects random variables that significantly contribute to $V(\mathbf{x})$ and iteratively includes these variables' higher-order component functions in the cut-HDMR expansion [5].

Iterative cut-HDMR Construction: The iterative cut-HDMR scheme [5] first constructs the first-order component functions by setting $S = 1$. Then, it computes

$$\beta_{\mathbf{u}} = \langle V_{\mathbf{u}}(x_{\mathbf{u}}) \rangle / V_0 \; ; \quad |\mathbf{u}| = 1, \tag{2}$$

where $\langle V_{\mathbf{u}}(x_{\mathbf{u}}) \rangle = \int V_{\mathbf{u}}(x_{\mathbf{u}})\, dx_{\mathbf{u}}$. Weights $\beta_{\mathbf{u}}$ are measures of the contributions of first-order component functions' means to the overall mean computed via constant zeroth-order component function. If $\beta_{\mathbf{u}}$ is larger than a prescribed tolerance ε_1, then the pertinent random variable is assumed to contribute significantly to $V(\mathbf{x})$. The second-order component functions involving these "important" random variables are marked as "candidates" for constructing the cut-HDMR expansion at the second level; they are only added to the expansion if their weights are larger than ε_1. This scheme is repeated in an iterative manner for all levels. For example, assume that the indices of the important random variables are found to be $\{1,2,4\}$ at the first level. Then, the second-order component functions with indices

$\{1,2\}$, $\{1,4\}$, and $\{2,4\}$ are considered to be included in the cut-HDMR expansion at the second level. Note that at the higher levels, the weights of the component functions $V_{\mathbf{u}}(x_{\mathbf{u}})$, $|\mathbf{u}| \geq 2$ are computed by

$$\beta_{\mathbf{u}} = \left| \left\langle V_{\mathbf{u}}(\mathbf{x}_{\mathbf{u}}) \right\rangle \right| \Big/ \left| \sum_{|\mathbf{v}|<|\mathbf{u}|-1} \left\langle V_{\mathbf{v}}(\mathbf{x}_{\mathbf{v}}) \right\rangle \right|. \tag{3}$$

To provide an additional stopping criterion, the decay rate of relative difference between observable means computed at two consecutive levels is defined as

$$\kappa = \left| \sum_{|\mathbf{v}|<|\mathbf{u}|} \left\langle V_{\mathbf{v}}(\mathbf{x}_{\mathbf{v}}) \right\rangle - \sum_{|\mathbf{v}|<|\mathbf{u}|-1} \left\langle V_{\mathbf{v}}(\mathbf{x}_{\mathbf{v}}) \right\rangle \right| \Big/ \left| \sum_{|\mathbf{v}|<|\mathbf{u}|-1} \left\langle V_{\mathbf{v}}(\mathbf{x}_{\mathbf{v}}) \right\rangle \right|. \tag{4}$$

If κ is smaller than a prescribed tolerance ε_2, then the cut-HDMR expansion is assumed converged.

3. Numerical Results

The iterative cut-HDMR method is used to statistically characterize the voltage coupled to the output terminal of a cascaded multiconductor transmission line network, constructed by connecting 40 blocks , each consisting of serially connected three-conductor transmission line networks [Fig.2(a-b)]. Each of these conductors resides on a substrate with thickness of 103.33 μm [Fig.2(c)]. The cascaded network is excited by a sinusoidal voltage source with frequency 2.4GHz [Fig. 2(a)]; 240 parameters characterize the uncertainty in the network ($N_{\text{dof}} = 240$). The widths of the all conductors are assumed to be normally distributed random variables with mean 230.9 μm and standard deviation 3.3 μm . The observable is the amplitude of the voltage at the output terminal of the network, V_{OUT} [Fig. 2(a)]; $|V_{\text{OUT}}|$'s pdf is obtained by running an MC simulation on the "exact" value of the observable and the surrogate models generated using iterative cut-HDMR with $\varepsilon_1 = 10^{-2}$ and $\varepsilon_1 = 10^{-4}$ [Fig 2(d)]. Fig. 2(d) shows that the pdfs obtained using the surrogate model are in good agreement with exact observable's pdf. To construct the surrogate models, the iterative cut-HDMR method required 720 and 855 deterministic simulations for $\varepsilon_1 = 10^{-2}$ and $\varepsilon_1 = 10^{-4}$, respectively. Note that 20000 deterministic simulations were needed to extract the pdf from the exact observable. For all executions of the iterative cut-HDMR, $\varepsilon_2 = 10^{-16}$.

4. References

[1] H. Bagci, A. C. Yucel, J. S. Hesthaven, and E. Michielssen, "A fast Stroud-based collocation method for statistically characterizing EMI/EMC phenomena on complex platforms," *IEEE Trans. Electromagn. Compat.,* vol. 51, pp. 301-311, 2009.

[2] A. C. Yucel, H. Bagci, and E. Michielssen, "An h-adaptive stochastic collocation method for stochastic EMC/EMI analysis," in *IEEE AP-S. Int. Symp.*, 2010, pp. 1-4.

[3] A. C. Yucel, H. Bagci, and E. Michielssen, "Fast probability density function estimation for statistical EMC/EMI characterization," in *USNC/URSI National Radio Sci. Meet.*, 2009.

[4] H. Rabitz, Ö. F. Alis, J. Shorter, and K. Shim, "Efficient input-output model representations," *Computer Physics Communications,* vol. 117, pp. 11-20, 1999.

[5] X. Ma and N. Zabaras, "An adaptive high-dimensional stochastic model representation technique for the solution of stochastic partial differential equations," *Journal of Computational Physics,* vol. 229, pp. 3884-3915, 2010.

[6] H. Bagci, A. E. Yilmaz, J. M. Jin, and E. Michielssen, "Fast and rigourous analysis of EMC/EMI phenomena on electrically large and complex cable-loaded structures," *IEEE Trans. Electromagn. Compat.,* vol. 49, pp. 361-381, May 2007.

[7] H. Bagci, A. E. Yilmaz, and E. Michielssen, "An FFT-accelerated time-domain multiconductor transmission line simulator," *IEEE Trans. Electromagn. Compat.,* vol. 52, pp. 199-214, 2010.

5. Figures

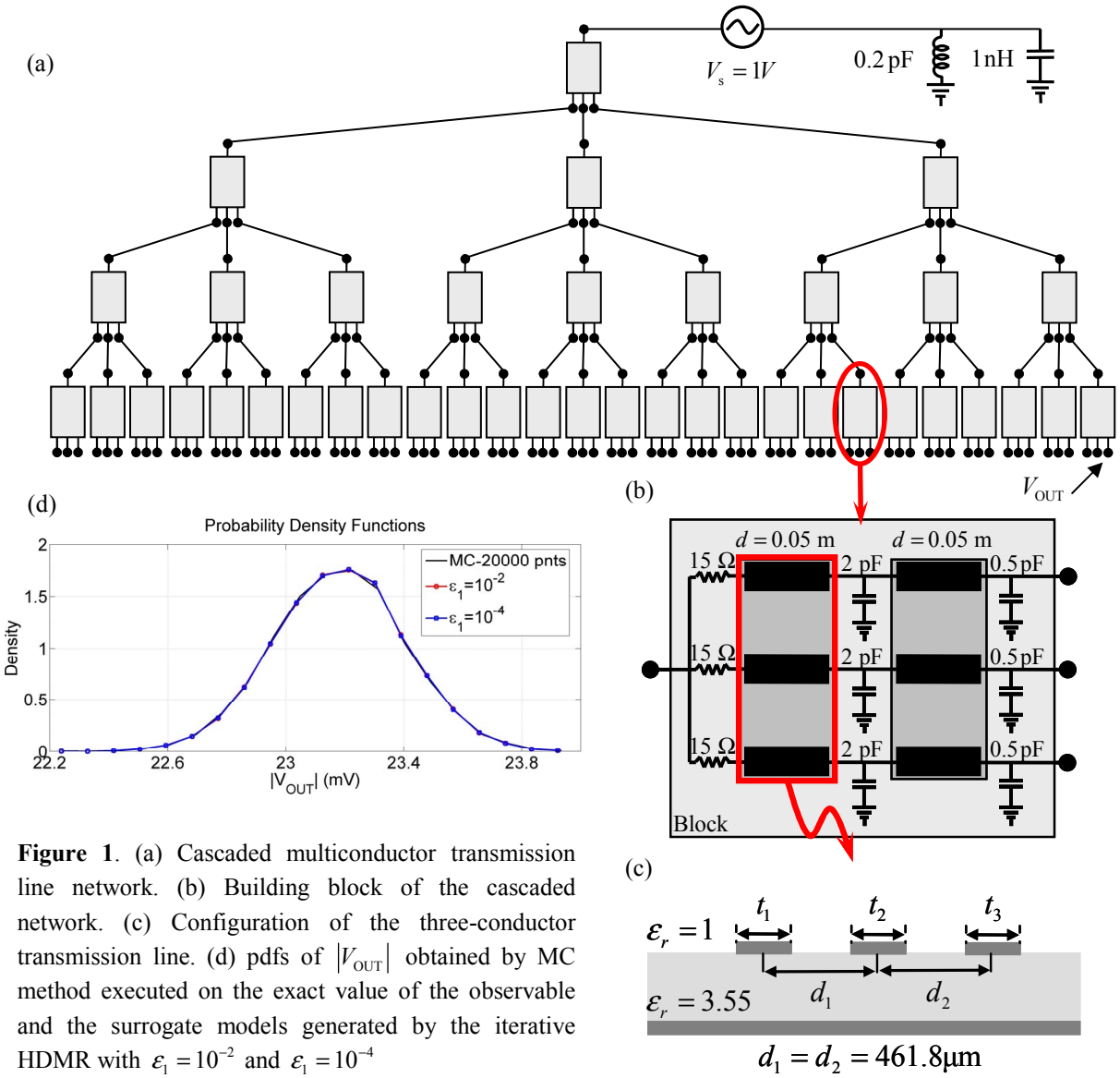

Figure 1. (a) Cascaded multiconductor transmission line network. (b) Building block of the cascaded network. (c) Configuration of the three-conductor transmission line. (d) pdfs of $|V_{\text{OUT}}|$ obtained by MC method executed on the exact value of the observable and the surrogate models generated by the iterative HDMR with $\varepsilon_1 = 10^{-2}$ and $\varepsilon_1 = 10^{-4}$