Transient Analysis of Crosstalk in PCB Interconnects Including Process Variation

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Abstract

This paper addresses the generation of an enhanced transmission line model for the transient analysis of long bus-like interconnects with the inclusion of parameters uncertainties. The proposed approach is based on the expansion of the well known frequency-domain telegraph equations in terms of orthogonal polynomials and on the back conversion to time domain via Fourier superposition. An application example involving the crosstalk analysis of a PCB coupled-microstrip interconnect with uncertainties in the relative dielectric permittivity and trace separation concludes the paper.

1. Introduction

Nowadays, the availability of effective solutions for the prediction of the effects of parameters uncertainty on the response of electronic equipments is highly desirable. Manufacturing process unavoidably introduces sources of uncertainty that may cause significant differences between simulated and measured responses, like higher crosstalk levels, thus possibly causing violations of noise margins.

The typical resource allowing to collect quantitative information on the statistical behavior of the circuit response is based on the application of the brute-force Monte Carlo (MC) method, or possible complementary methods based on the optimal selection of the subset of model parameters in the whole design space. Such methods, however, are computationally expensive, and this fact prevents us from their application to the analysis of complex realistic structures.

Recently, an effective solution that overcomes the previous limitation has been proposed. This methodology is based on the polynomial chaos (PC) theory and on the representation of the stochastic solution of a dynamical circuit in terms of orthogonal polynomials. For a comprehensive and formal discussion of PC theory, the reader is referred to [1, 2] and references therein. PC technique enjoys applications in several domains of Physics; we limit ourselves to mention recent results on the extension of the classical modified nodal analysis (MNA) approach to the prediction of the stochastic behavior of circuits with uncertain parameters [3]. Also, the authors of this contribution have recently proposed an extension of PC theory to distributed structures described by transmission-line equations.

This paper extends the advocated PC-based approach to the stochastic analysis of the time-domain crosstalk of a PCB coupled microstrip structure.

2. Stochastic Transmission-Line Equations

This section discusses the modification of the classical transmission-line equations, as needed for incorporating the effects of the statistical variation of the per-unit-length (p.u.l.) parameters via the PC theory. The problem is addressed in frequency domain first, and then extended to time domain via Fourier superposition.

Classical Frequency-Domain Transmission-Line Model. For the sake of simplicity, the discussion is based on a lossless three-conductor line, as the coupled microstrip structure shown in Fig. 1, in presence of a single random

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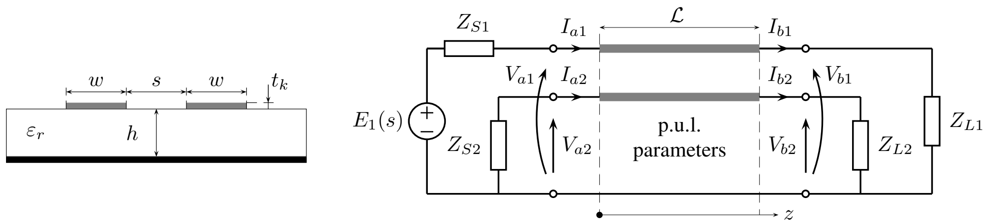

Figure 1: Test structure considered to demonstrate the proposed approach. Left panel: transmission-line cross-section; right panel: simulation test case.

parameter. The wave propagation on the structure is governed by the telegraphers equation in the Laplace domain [5]

$$\frac{d}{dz}\begin{bmatrix} \mathbf{V}(z,s) \\ \mathbf{I}(z,s) \end{bmatrix} = -s \begin{bmatrix} 0 & \mathbf{L} \\ \mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{V}(z,s) \\ \mathbf{I}(z,s) \end{bmatrix}. \tag{1}$$

In the above equation, s is the Laplace variable, $\mathbf{V} = [V_1(z,s), V_2(z,s)]^T$ and $\mathbf{I} = [I_1(z,s), I_2(z,s)]^T$ are vectors collecting the voltage and current variables along the multiconductor line (z coordinate) and $\mathbf{C}$ and $\mathbf{L}$ are the p.u.l. capacitance and inductance matrices, depending on the geometrical and material properties of the structure [5].

In order to account for the uncertainties affecting the guiding structure, we must consider the p.u.l. matrices as random quantities, with entries depending on the random variable ξ. Hence, (1) becomes a stochastic differential equation, leading to randomly-varying voltages and currents along the line.

Stochastic Frequency-Domain Transmission-Line Model. PC is a powerful tool allowing to solve in a clever way stochastic equations [2]. The idea behind this technique is the approximation of a random variable in terms of a truncated series of orthogonal polynomials that are functions of a predefined standard distribution. These polynomials play the same role as sinusoidal functions in the Fourier series expansion.

For the current application, both the voltage and current variables of (1) as well as the per-unit-length matrices are represented in terms of a truncated series, leading . For instance,

$$\mathbf{C} = \sum_{k=0}^{P} \mathbf{C}_k \phi_k(\xi), \quad \mathbf{L} = \sum_{k=0}^{P} \mathbf{L}_k \phi_k(\xi) \tag{2}$$

where P is the order of the expansion that generally lies within the range $2 \div 10$ for practical applications and the $\mathbf{C}_k, \mathbf{L}_k$ matrices are the expansion coefficients with respect to the orthogonal components ϕ_k computed according to [2]. Briefly speaking, the above expansion terms can be computed via the projection of (2) onto the orthogonal polynomials ϕ_k by means of a properly defined inner product. As an example, for the case of Gaussian random variables, the orthogonal basis functions are the Hermite polynomials ϕ_0=1, ϕ_1=ξ, ϕ_2=$(\xi^2 - 1)$,$\ldots$ Readers are referred to [2, 3] and references therein for a comprehensive and formal discussion of polynomial chaos.

The application of expansion (2) in terms of Hermite polynomials to the p.u.l parameters and to the unknown voltage and current variables yields a modified version of (1), whose second row becomes

$$\begin{aligned} \frac{d}{dz}(\mathbf{I}_0(z,s)\phi_0(\xi) + \mathbf{I}_1(z,s)\phi_1(\xi) + \mathbf{I}_2(z,s)\phi_2(\xi)) &= -s(\mathbf{C}_0\phi_0(\xi) + \mathbf{C}_1\phi_1(\xi) + \\ &+ \mathbf{C}_2\phi_2(\xi))(\mathbf{V}_0(z,s)\phi_0(\xi) + \mathbf{V}_1(z,s)\phi_1(\xi) + \mathbf{V}_2(z,s)\phi_2(\xi)), \end{aligned} \tag{3}$$

where a second-order expansion (i.e., $P = 2$) is assumed; the expansion coefficients of electrical variables and of p.u.l. parameters are readily identifiable in the above equation.

Projection of (3) and of the companion relation arising from the first row of (1) on the first three Hermite polynomials leads to the following augmented system, where the random variable ξ does not appear, due to the projection integral.

$$\frac{d}{dz}\left[\begin{array}{c}\tilde{\mathbf{V}}(z,s)\\ \tilde{\mathbf{I}}(z,s)\end{array}\right]=-s\left[\begin{array}{cc}0 & \tilde{\mathbf{L}}\\ \tilde{\mathbf{C}} & 0\end{array}\right]\left[\begin{array}{c}\tilde{\mathbf{V}}(z,s)\\ \tilde{\mathbf{I}}(z,s)\end{array}\right]. \tag{4}$$

In the previous equation, vectors $\tilde{\mathbf{V}}=[\mathbf{V}_0,\mathbf{V}_1,\mathbf{V}_2]^T$ and $\tilde{\mathbf{I}}=[\mathbf{I}_0,\mathbf{I}_1,\mathbf{I}_2]^T$ collect the different coefficents of the polynomial chaos expansion of the voltage and current variables. It is worth noting that (4) is analogous to (1) and plays the role of the set of equations of a multiconductor transmission line with a number of conductors that is $(P+1)$ times larger than those of the original line. It is ought to remark that the increment of the equation number is not detrimental for the method, since for small values of P (as typically occurs in practice), the additional overhead in handling the augmented equations is much less than the time required to run a large number of MC simulations.

Extension of the procedure to the general case of lossy transmission lines with multiple random parameters is straightforward, and amounts to including the resistance and conductance matrices in (1) and the corresponding augmented matrices in (4). Moreover, the univariate polynomials in (2) must be replaced by proper multivariate polynomials, built as the product combinations of the univariate ones. Similarly, the orthogonality relation needs to be suitably modified according to the furmulae summarized in [1].

Boundary conditions and simulation. The standard procedure for the solution of a loaded transmission line like the one of Fig. 1 amounts to combining the port electrical relations of the terminal elements defining the source and load with the transmission-line equation, and solving the system (cfr Ch.s 4 and 5 of [5]). Similarly, when the problem becomes stochastic, the augmented transmission-line equation (4) is used in place of (1) together with the projection of the source and the load equations on the first $P+1$ Hermite polynomials. It is worth noting that in this specific example, no variability is included in the terminations, hence the augmented characteristics of the source and load turn out to have a diagonal structure. Once the unknown voltages and currents are computed, the quantitative information on the spreading of circuit responses can be readily obtained from the analytical expression of the unknowns. As an example, the frequency-domain solution of the terminal voltage V_{b2} with $P=2$, is $V_{b2}(j\omega)=V_{b2,0}(j\omega)+V_{b2,1}(j\omega)\xi+V_{b2,2}(j\omega)(\xi^2-1)$, where the first numerical index denotes the conductor and the second one denotes the expansion term. The above relation can be used to compute the PDF of the output quantity (e.g., the magnitude $|V_{b2}(j\omega)|$) using the rules of random variable transformations given in [4].

Time-domain solution. The time-domain response can be readily obtained from the frequency-domain solution by considering a periodic input source and expressing it in terms of a truncated Fourier series Being the system of Fig. 1 linear, its time-domain behavior is in principle obtainable by the superposition of the analyses carried out for all signal harmonics. For the individual solution at frequency f_n, the voltage source of Fig. 1 is replaced by its n-th harmonic component, i.e., $E(s_n)=E(j2\pi f_n)$. The time-domain expression of the output voltage $v_{b2}(t)$ is then obtained as a linear superposition of harmonics. The linearity of Fourier decomposition assures that the PC structure is preserved also for the time-domain expression of the output, i.e., $v_{b2}(t)=v_{b2,0}(t)+v_{b2,1}(t)\xi+v_{b2,2}(t)(\xi^2-1)$ In the above analysis, the DC term is neglected due to the differential nature of crosstalk, though in general it can be obtained from a DC calculation. However, in our specific application, DC terms are not affected by variability, since the transmission line plays a negligible role at low frequencies.

3. Numerical Results

In this section, the proposed technique is applied to the time-domain analysis of a coupled microstrip, with an active line fed by a Gaussian voltage pulse. Referring to Fig. 1, the nominal parameters are $w=100\,\mu$m, $s=80\,\mu$m, $h=60\,\mu$m, $t_k=35\,\mu$m, $\varepsilon_r=3.7$ and $\mathcal{L}=5\,$cm. The source and load elements are defined by the impedances $Z_{S1}=Z_{S2}=50\,\Omega$ and $Z_{L1}=Z_{L2}=1/(sC_L+G_L)$, being $C_L=10\,$pF, $G_L=1/(10\,\mathrm{k}\Omega)$. The input pulse waveform has a peak of 1 V and a width of approximately $0.35\,$ns at half amplitude. To compute its Fourier series, a period of 6 ns and $N=30$ harmonics are considered. The variability is provided by the relative permittivity ε_r and the trace separation s, that are assumed to behave as two independent Gaussian random variables with 3.7 and 80 μm mean values, respectively, and indentical 10% relative standard deviations. The approximate relations given in [6] were used to compute the PC expansion of the p.u.l. parameters.

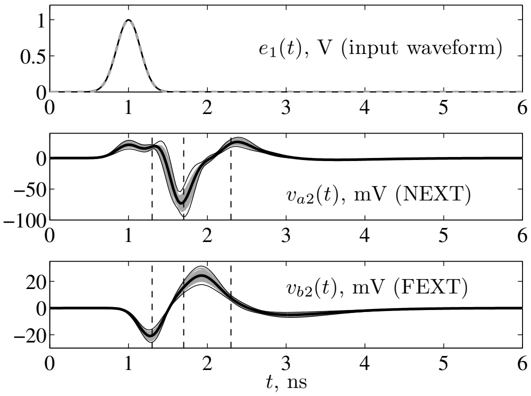

Figure 2: Top panel: input waveform $e_1(t)$; solid black line: analytical expression; dashed gray line: Fourier series approximation with 30 terms. Lower panels: near-end crosstalk $v_{a2}(t)$ (central panel) and far-end crosstalk $v_{b2}(t)$ (bottom panel); solid black thick line: deterministic response; solid black thin line: 3σ limits of the second-order PC expansion; gray lines: a sample of responses obtained by means of the MC method (limited to 100 curves, for graph readability).

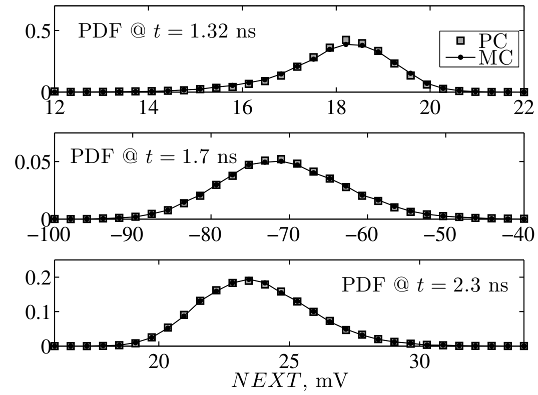

Figure 3: Probability density function of the near-end crosstalk $v_{a2}(t)$ computed at different times. Of the two distributions, the one marked MC refers to 20,000 MC simulations, while the one marked PC refers to the response obtained via a second-order PC expansion.

Figure 2 shows the time-domain voltage source as well as the near-end and far-end crosstalk on the quiet line. Referring to the lower panels, the black thick line represents the response of the structure for the nominal values of its parameters, while the thinner black lines indicate the limits of the 3σ bound, determined from the results of the proposed technique. Finally, a qualitative set of 100 MC simulations is plotted using gray lines. Clearly, the parameter variations lead to a spread in the crosstalk levels that is well predicted by the estimated 3σ limits. A better quantitative prediction is possible from the knowledge of the actual PDF of the network response. To this end, Figure 3 show the PDFs of $v_{a2}(t)$, obtained from the analytical PC expansions at different times, and compare them with the distributions generated by 20,000 MC simulations. The time points selected for this comparison correspond to the dashed lines shown in Fig. 2.

The good agreement between the PDFs obtained from the PC model and the corresponding set of MC simulations confirms the potential of the proposed method. It is also clear from this example that a PC expansion with a limited number of term (P=2 in the example of this study) is already accurate enough to capture the dominant statistical information of the system response, while leading to a speedup factor of $100 \div 600\times$, depending on the number of time points considered.

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