Using the Half Fourier Transform for SEM analysis of both Early and Late Time Responses In the Presence of Noise

***Tapan K Sarkar*[1], *and Magdalena Salazar Palma*[2]**

[1]Department of Electrical Engineering and Computer Science, Syracuse University, Syracuse, New York, USA, tksarkar@syr.edu

[2]Departmento de Teoria de la Senal y Communicaciones, Universidad Carlos III de Madrid, Leganes, Madrid, Spain m.salazar-palma@ieee.org

Abstract

A technique for estimating the SEM parameters of damped sinusoids utilizing both early and late time transient scattering data contaminated by noise is described using the Half Fourier Transform (HFT). The importance of this novel methodology is how to simultaneously exploit both early time and late time data as for a practical system it is difficult to separate them and still be able to identify the late time poles along with the early time specular type of returns.

1. Introduction

The singularity expansion method (SEM) proposed by Baum has been applied to quantify an electromagnetic response in an expansion of complex resonances of the system. It has been shown that the dominant complex natural resonances of a system are a minimal set of parameters that define the overall physical properties of the system. So, a transient scattering response is analyzed in terms of the damped oscillations corresponding to the complex resonant frequency of the scatterer or target. In general, the signal model of the observed late time of an electromagnetic-energy-scattered response from an object can be written as

$$y(t) = x(t) + n(t) \approx \sum_{m=1}^{M} R_m \exp(s_m t) + n(t); \quad 0 \le t \le T, \tag{1}$$

where $y(t)$ = observed time domain response, $n(t)$ = noise in the data,
$x(t)$ = signal, R_m = residues or complex amplitudes, $s_m = -\alpha_m + j\omega_m$,
α_m = damping factors, ω_m = angular frequencies ($\omega_m = 2\pi f_m$).

After sampling, the time variable, t is replaced by kT_s, where T_s is the sampling period. The sequence can be rewritten as

$$y(kT_s) = x(kT_s) + n(kT_s) \approx \sum_{m=1}^{M} R_m z_m^k + n(kT_s) \text{ for k=0,…,}N\text{-1}, \tag{2}$$

$$z_m = e^{s_i T_s} = \exp(-\alpha_m + j\omega_m)T_s \text{ for m=1,2,…,}M. \tag{3}$$

Since the resonances describe global wave fields that encompass the scattering object as a whole, the SEM series representation encounters convergence problem when applied to the early time response of the objects. Early time response is strongly dependent on the nature of the source, the location of the source, and the location of the observer. Usually the early time response shows impulse-like characteristics. Because of this difficulty, most previous techniques used just late time signals only. To excite the early time response a narrow Gaussian pulse is selected. A Gaussian pulse is an entire function and is quite adequate to describe pulse-like components in early time. Complex exponentials are used to describe the late time signals. The concept of a 'Turn-on time' is utilized to consider a time when the fully excited resonance can be used formally. So, the transient scattered field can be modeled a

$$Scattered\ field = Early\ Time + Late\ Time = Gaussian\ Pulses + Damped\ Sinusoids \times u(t-\tau)$$

where $u(t)$ is a unit step function and τ is 'Turn-on time'. But the boundary in between early time and late time is not clear and actually there is an intermediate zone in which the early time pulse-like component and the late time damped sinusoids are coupled together. So the 'Turn-on time' can be determined by an optimization routine.

Since the resonance describe global wave fields that encompass the scattering object as a whole, the SEM series representation encounters convergence difficulties at early times when portions of the objects are not yet excited. Because of this difficulty, most previous techniques used just late time signals only. The SEM representation does not account for the impulsive portion of the early time system responses behavior.

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Here, the Half Fourier Transform is used for parameter optimization, because the parameter ϕ is continuous and the mathematical representations of the Half Fourier Transform (HFT) of functions are clear and simple. The functional representation is almost the same except for the coefficient and the sign in the exponent. It means that their contributions in the Half Fourier Transform domain are almost the same. That is not true in the ordinary Fourier transform domain. Usually signals that we encounter in real life are causal, that is, $x(t) = 0$ if $t \le 0$. And the initial time as to when a resonant component starts is different from component to component. So it is possible to assume that each resonant component has a different 'Turn-on time'.

2. Calculation of the HFT of an arbitrary waveform

The generalized Fourier Transform operator F_ϕ by, $F_\phi\left\{ e^{-x^2/2} H_n(x) \right\} = e^{-jn\phi} e^{-x^2/2} H_n(x)$ where ϕ is a variable parameter, $-\pi \le \phi \le \pi$, with negative values of ϕ corresponding to the inverse transform and $H_n(t)$ is the n[th] order Hermite polynomial. Therefore, the usual Fourier Transform operator can be written as $F_{\pi/2}$ to denote the eigenvalue $e^{-jn\pi/2}$ associated with the orthogonal Hermite functions $e^{-x^2/2} H_n(x)$. The associate Hermite (AH) polynomials $h_n(t,\lambda)$ are defined in terms of the Hermite polynomials $H_n(t,\lambda)$ through $h_n(t,\lambda) = \frac{1}{\sqrt{2^n n! \sqrt{\pi}\,\lambda}} H_n\left(\frac{t}{\lambda}\right) \exp\left(-\frac{t^2}{2\lambda^2}\right),\ n \ge 0$; where λ is a scaling factor. The Hermite polynomials are generated recursively through $h_0(t) = \frac{e^{-\left(t^2/2\right)}}{\sqrt{\sqrt{\pi}}}$; $h_1(t) = \frac{2te^{-\left(t^2/2\right)}}{\sqrt{2\sqrt{\pi}}}$; $h_n(t) = \frac{1}{\sqrt{n}}\left[\sqrt{2}\, t h_{n-1}(t) - \sqrt{n-1}\, h_{n-2}(t)\right]$, for $n \ge 2$. The AH polynomials are orthonormal to each other and form a complete set of basis in the interval $\left[-\infty, \infty\right]$. If $x(t)$ is a piecewise smooth function defined on a finite interval $\left[-p, p\right]$ and $\int_{-\infty}^{\infty} e^{-t^2} x^2(t)\,dt < \infty$, then $x(t)$ can be expanded using the AH series as $x(t) = \sum_{n=0}^{\infty} a_n h_n(t)$, for $-\infty < t < \infty$; with $a_n = \int_{-\infty}^{\infty} x(t) h_n(t)\,dt$. Therefore, $F_\phi x(t)(u) = \; = \sum_{0}^{\infty} a_n e^{-jn\phi} h_n(u)$. The Half Fourier Transform (HFT) can be obtained by substituting α with $\pi/4$.

3. Optimization

Complex natural frequencies occur in complex conjugate pairs and they lie in the left half plane with a nonzero real part. To represent real signals we treat two conjugate poles together. So, the scattered field can be represented as

$$x(t) = \frac{1}{2}\sum_{m=1}^{M} c_m e^{-\alpha_m t} \left\{ e^{j(\omega_m t + \phi_m)} + e^{-j(\omega_m t + \phi_m)} \right\} u\left(t - \tau_m\right) + \sum_{n=1}^{N} A_n \exp\left\{ -C_n \frac{\left(t - B_n\right)^2}{2} \right\}$$

where $t \ge 0, \tau > 0$, $\alpha_m > 0$. c_m's and ϕ_m's are the amplitudes and the phases, respectively. α_m's and ω_m's are the damping factors and the angular frequencies. A_n's and B_n's are amplitudes and time shift of the Gaussian pulses. C_n's are coefficients which represent the pulse width. M is the number of damped sinusoidal signals and N is the number of Gaussian pulses.

To apply a parameter identification algorithm, the parameter set is defined by

$$\underline{p} = \left[\omega_1\ \phi_1\ c_1\ \alpha_1\ \tau_1 \cdots \omega_M\ \phi_M\ c_M\ \alpha_M\ \tau_M\ ;\ B_1\ A_1\ C_1 \cdots B_N\ A_N\ C_N\right]$$

while the residual vector is defined by

$$r = \frac{1}{2}\left\| \overline{G}(u,t) - \overline{G}_R(u,t) \right\|_2^2$$

where $\overline{G}(u,t) = [\overline{X}(u)\ ;\ \overline{x}(t)]$ and $G_R(u,t) = [\overline{X}_R(u)\ ;\ \overline{x}_R(t)]$. $\overline{X}(u)$ is the HFT of the measured signal $\overline{x}(t)$, and $\overline{G}_R(u,t)$ is the reconstructed half Fourier transform. Both the original signal and its HFT are used to compute the optimized parameters. $\| \bullet \|_2^2$ defines the squared $\mathcal{L}^2$ norms. $\overline{X}(u)$ is pre-calculated with the time domain scattered field. From the Half Fourier Transform of a shifted Gaussian pulse and a damped sinusoid with a *Turn-on time,* $\overline{X}_R(u)$ is constructed as

$$\overline{X}_R(u) = \frac{1}{2}\sum_{m=1}^{M} c_m \frac{\sqrt{1+j}}{2}\left\{ e^{j\phi_m} e^{\frac{j}{2}v_{m1}^2} \exp\left\{-\frac{j}{2}\left(u-\sqrt{2}v_{m1}\right)^2\right\}\left[1-\Phi\left(\gamma'_{m1}\sqrt{\beta}\right)\right]\right.$$
$$\left. + e^{-j\phi_m} e^{\frac{j}{2}v_{m2}^2} \exp\left\{-\frac{j}{2}\left(u-\sqrt{2}v_{m2}\right)^2\right\}\left[1-\Phi\left(\gamma'_{m2}\sqrt{\beta}\right)\right]\right\}$$
$$+\sum_{n=1}^{N} A_n \sqrt{\frac{1-j}{C_n - j}} \exp\left\{-\frac{C_n}{C_n^2+1}\left(u-\frac{B}{\sqrt{2}}\right)^2\right\}\exp\left\{\frac{j}{2}\left[\frac{C_n^2-1}{C_n^2+1}\left(u-\frac{B}{\sqrt{2}}\right)^2+\frac{B^2}{2}-\sqrt{2}uB\right]\right\}$$

where $v_{m1} = \omega + j\alpha$, $v_{m2} = -\omega + j\alpha$, $\gamma'_{m1} = j\left(\sqrt{2}u - v_{m1} - \tau_m\right)$, and $\gamma'_{m2} = j\left(\sqrt{2}u - v_{m2} - \tau_m\right)$.

4. Example

The example presented is a wire scatterer. The time domain transient electro-magnetic scattering responses from various objects have been calculated using an inverse Fourier transform of the frequency domain data using a numerical electromagnetics code called HOBBIES [2]. Noise is added to the time domain data. Generally speaking, noise is more dominant in later time rather than at early times. In this study, signal to noise ratio (SNR) is defined as, $SNR = 10\times\log_{10}\left[\sum_{i=t_1}^{t_2} s(t_i)^2 / \sum_{i=t_1}^{t_2} n(t_i)^2\right]$, where $s(t)$ is a signal and $n(t)$ is noise. Time domain scattered field is obtained using the same procedure with that of the previous section and then white Gaussian noise is added with a zero mean and with a finite SNR. The noise contaminated signal and the Half Fourier Transform of the noisy signal is used to identify the complex resonant frequency of the object. An average error is taken from twenty trials for each SNR.

The thin wire scatterer of length L and diameter d, which is excited by an incident pulse of electromagnetic radiation. As shown in Figure 1, the length of the wire scatterer is 50 mm and the aspect ratio (L/d) is 100. The incident field is coming from 45° from the wire axis and is polarized with respect to the theta direction. In this case 7 damped sinusoids and 5 Gaussian pulses are used to fit the time domain and the Half Fourier Transform domain data. Noise is added to the time domain data in the range between 10% and 40%. The order of the expansion for the associate Hermite basis functions to carry out the Half Fourier Transform is determined using the time-bandwidth product (2BT+1) rule, where B is the bandwidth in the frequency domain and T is the time duration of the signal. It means that to approximate a given waveform of duration T and practically band limited by B (one-sided bandwidth), using an orthonormal set of basis functions in the time domain, at least (2BT+1) pieces of basis are necessary from a mathematical point of view. In this example the frequency band B is 100 GHz and the total time duration T is 5 nsec. Therefore, to achieve this time-bandwidth product for the backward scattered field one needs approximately N = (2×100×5 + 1) = 1001 coefficients of the Hermite expansion. Figure 2 represents the backward scattered electromagnetic field without noise in the time domain. Figure 3(a) plots the time domain noisy signal with a SNR=10 dB and Figure 3(b) is the reconstructed signal after optimizations. Twenty trials are performed for each SNR and then an average estimate of the error is computed. The root mean square error defined by the $RMS\ Error = \sqrt{\sum_{i=1}^{N}\{y_o(t_i) - y_r(t_i)\}^2 / N}$ where y_o is the noisy time domain data, y_r is reconstructed data using optimized parameters and N is the total number of data samples.

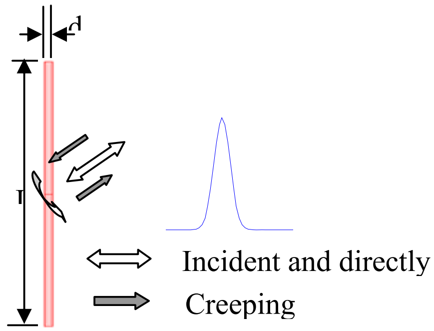


Figure 1. A wire scatterer model.

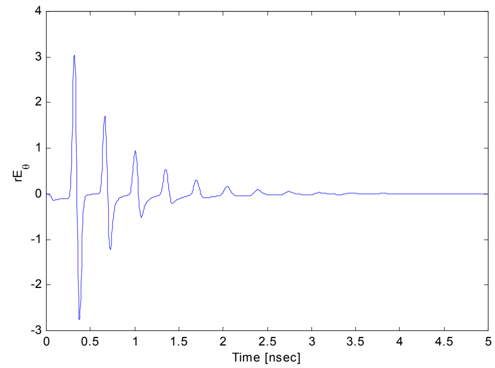
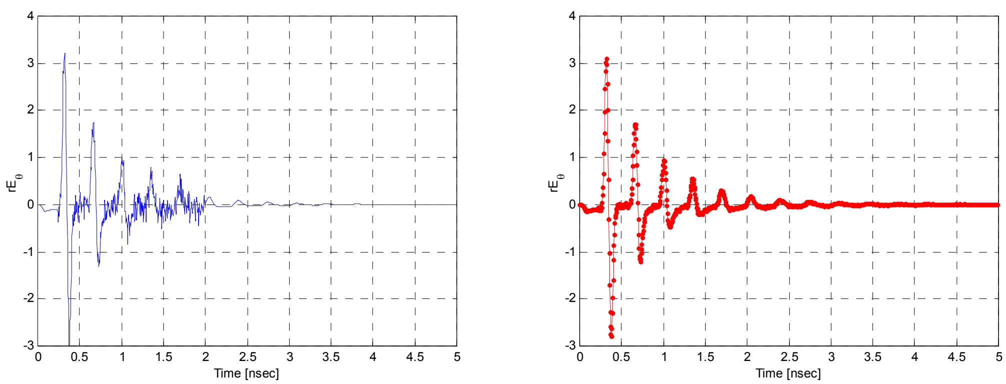


Figure 2. Time domain response of a wire scatterer.



(a) Time domain noise contaminated signal (b) Time domain reconstructed signal

Figure 3. Time domain noise contaminated and reconstructed signal for SNR = 10 dB for a wire scatterer.

5. Conclusion

An examples has been presented with noise-contaminated data. The example is a wire scatterer and a finite closed cylinder. Gaussian noise has been added to the time domain transient data. Root mean square error is computed after twenty trials for each signal to noise ratio. The errors in the reconstruction of the signals in the time domain and in the Half Fourier Transform domain decrease with higher signal to noise ratio (SNR), and so does the RMS errors for the complex poles. So if the SNR is moderately large, it is possible to obtain several of the dominant complex resonant frequencies with good accuracy.

For this example, the threshold in the noise level above which reliable estimates for the poles are observed is approximately 30dB of SNR. If the signal to noise ratio is lower than 30 dB the root mean square error in the time and in the Half Fourier Transform domains are not good enough to do identification. If we just consider a few dominant poles, the imaginary parts of the poles are in good agreement with the analytic values and the real part of the poles display small variations.

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6. Reference

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