A Macromodel-Based Algorithm for the Calculation of Lightning Radiated Electromagnetic Fields and Induced Voltages in Transmission Lines

Sina Mashayekhi[1] and *Behzad Kordi*[2]

[1] Department of Electrical and Computer Engineering, University of British Columbia, Vancouver, BC, Canada V6T 1Z4. Email: sina@ece.ubc.ca

[2] Department of Electrical and Computer Engineering, University of Manitoba, Winnipeg, MB, Canada R3T 5V6. Email: Behzad_Kordi@UManitoba.CA

Abstract

In this paper, we propose a fast and efficient algorithm for the calculation of lightning radiated electromagnetic (EM) fields in the space. These macromodel-based calculations will be done using a mixed time-frequency domain method. Vector Fitting algorithm is employed to trace the poles/residues position of the channel-base current-radiated electromagnetic fields system transfer function in different points of the space. The spatial representation of the poles and residues locations will result in efficient calculation of the induced voltage on the frequency dependant multiconductor transmission lines (MTL) over a lossy ground.

1 Introduction

Lightning has been known as an important natural phenomenon which has significant contributions in inducing transients in cables and transmission lines [1]. A typical lightning channel carries a large amount of transient current in the range of tens of kilo Amperes and is a significant source of electromagnetic radiation in the surrounding space. There have been many research studies on the modeling and measurement of lightning effects. The parameters that are studied in measurements are the channel base current, radiated electromagnetic fields, and induced voltages in transmission lines.

Transmission lines are critical components of a power system from the electromagnetic point of view due to their large dimensions and are susceptible to lightning radiated electromagnetic fields. These fields may induce voltages up to thousands of volts [2], which can travel on the line and harm the equipment connected to the network. Models used for the analysis of transmission lines, generally, fall into two groups, *Terminal-based* [3] and *Distributed* [4] models. The commonly-used terminal-based models are incapable of including field-to-transmission line coupling into consideration whereas the distributed models involve rigorous and time consuming calculations which cannot be easily employed for the simulation of large power systems. In this paper we will overcome these problems by introducing a macromodel-based algorithm.

An accurate knowledge of electromagnetic fields of external excitation is required to calculate induced voltages in the line by using field-to-line coupling equations. The problem of incident plane-wave electromagnetic coupling to MTL has already been addressed in the literature, for example in [5]. There are few papers that discuss the nonuniform incident fields, such as [6] where a hybrid FDTD and similarity transformation technique is used to calculate the additional voltage sources due to excitation of lossless transmission line. In this paper, we introduce a fast and efficient algorithm to calculate the radiated electromagnetic fields of lightning return stroke channel (RSC), and summarizing its effect at the terminals of the transmission line to obtain the induced voltages at those terminals. This enables frequency-domain software such as NEC, to work within the framework of time-domain simulators such as PSCAD/EMTDC.

2 Lightning RSC Transfer Function-Based Macromodel

We first obtain the *lightning return stroke-propagation medium* transfer function in the frequency domain for N distances from the RSC. Due to the smooth time-variation of lightning EM fields with space, we are able to approximate the transfer function by rational functions (using Vector Fitting algorithm) and trace the location of the poles and residues and map them into analytic functions. This will accelerate the calculation

978-1-4244-6051-9/11/$26.00 ©2011 IEEE

of lumped sources (forcing functions) which are placed at the terminal of the excited transmission line and represent the impact of external radiation. The forcing functions for an externally excited transmission line which lies in the z direction is given by [7],

$$V_{FT}(L) = \int_0^L \varphi_{11}(z)\left[E_z^{\mathrm{inc}}(h,z) - E_z^{\mathrm{inc}}(0,z)\right]dz + \int_0^h E_x^{\mathrm{inc}}(x,0)dx - \varphi_{11}(L)\int_0^h E_x^{\mathrm{inc}}(x,L)dx, \qquad (1)$$

$$I_{FT}(L) = -\int_0^L \varphi_{21}(z)\left[E_z^{\mathrm{inc}}(h,z) - E_z^{\mathrm{inc}}(0,z)\right]dz + \varphi_{21}(L)\int_0^h E_x^{\mathrm{inc}}(x,L)dx, \qquad (2)$$

where, φ_{ij} is the ij^{th} element of the chain parameter matrix, E_z^{inc} and E_x^{inc} are the horizontal and vertical components of the incident electric field along the line, and h and L are the line height and length, respectively. Equations (1) and (2) indicate that to obtain the forcing functions, one needs to have E field information at every point of the line. To demonstrate the implementation of the macromodel idea, let's consider the analytic formula for the horizontal electric field and azimuthal magnetic field over a PEC ground [8]. By using Engineering models of lightning RSC, and replacing the channel base current with the impulse function and applying the definition of Discrete Fourier Transform, the electric field formula at distance z from the channel base, and height x, in the frequency domain is calculated in a closed form. Using Cooray-Rubinestein formula [9], the electric field information (which indeed is the impulse response of the system) over the lossy ground is obtainable. Repeating these procedures for N points along the line will provide us with N sets of horizontal electric field frequency domain data. We use Vector Fitting algorithm to map each set to M poles/residues. M analytic functions are obtainable by tracing the same order poles/residues location in the complex plane. The simplest functions which we are using in this paper, are polynomial functions. We use 1^{st} order polynomial functions for the trace of poles and 2^{nd} order polynomials for the residues. For example, at any arbitrary point z, the horizontal component of the electric field at the location of the wire can be calculated using,

$$E_z^{\mathrm{inc}}(h,z) \cong \sum_{k=1}^{M} \frac{A_k z^2 + B_k z + C_k}{j\omega - (F_k z + G_k)}, \qquad (3)$$

where, z is distance variable, and A_k, B_k, C_k, F_k and G_k are coefficients for the trace of k^{th} pole/residue. Convolution of (3) and the channel-base current will give the electric field data in the time domain. Using this approach to obtain the electric field makes the calculations fast and efficient as the only required information is the electric field data at N points along the transmission line. Further, recursive convolution is possible using this macromodel to save time and required system memory space.

In (1), there are three integrals to be calculated. Our calculations shows, in lightning frequencies, vertical electric field is almost independent from the height from ground, x. So, the second and third integrals in (1) can be simply evaluated assuming a constant vertical electric field on the PEC ground. For calculation of the first integral which depends on the horizontal electric field along the line, we use (3). This allows us to find a closed form solution for (1) as,

$$\int_0^L \varphi_{11}(z)E_z^{\mathrm{inc}}(h,z)dz = \sum_{k=1}^{M}\int_0^L \cosh(\gamma_0 z)\{\frac{A_k}{D_k}z + (\frac{B_k}{D_k} - \frac{A_k E_k}{{D_k}^2}) + \frac{C_k - \frac{B_k E_k}{D_k} - \frac{A_k {E_k}^2}{{D_k}^2}}{D_k z + E_k}\}dz. \qquad (4)$$

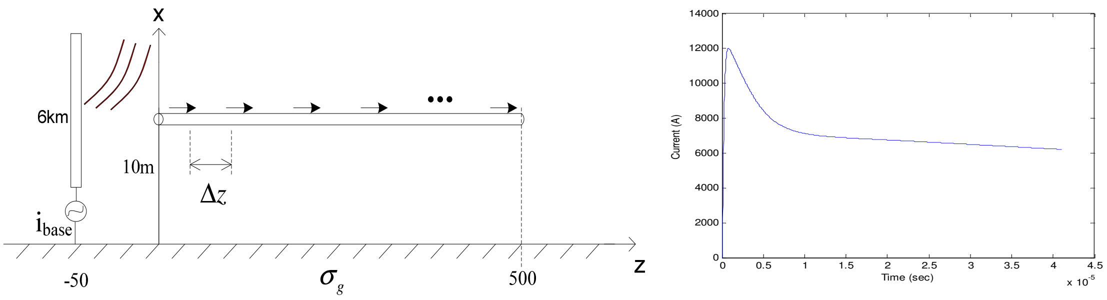
where, γ_0 is the propagation constant, $D_k = -F_k$, and $E_k = j\omega - G_k$. The calculation of the first and second terms of (4) in a closed form solution is straightforward. The last integral of this equation can be calculated as,

$$V_1{}' = \frac{A'{}_k}{D_k}\{\cosh(\frac{\gamma_0 E_k}{D_k})Chi(\frac{\gamma_0 E_k}{D_k} + \gamma_0 z) - \sinh(\frac{\gamma_0 E_k}{D_k})Shi(\frac{\gamma_0 E_k}{D_k} + \gamma_0 z)\}, \qquad (5)$$

where, $A'_k = C_k - B_k E_k/D_k - A_k E_k^2/D_k^2$ and $Chi(.)$ and $Shi(.)$ are *hyperbolic cosine integral* and *hyperbolic sine integral* functions, respectively,

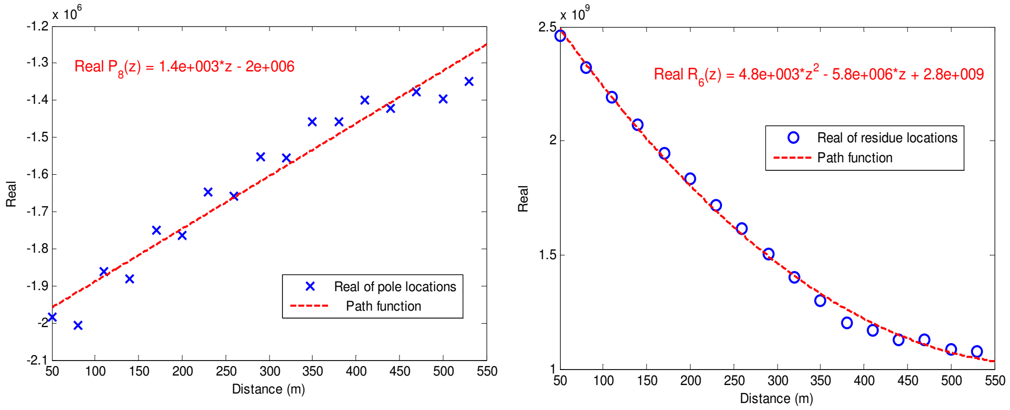
$$Chi(z) = \gamma + \ln(z) + \int_0^z \frac{\cosh(t) - 1}{t}dt, \quad Shi(z) = \int_0^z \frac{\sinh(t)}{t}dt, \qquad (6)$$

and, $\gamma = 0.57721$ is the *Euler-Mascheroni* constant. Using (5), calculation of (1) and similarly the current lumped source given by (2) in a closed form solution is possible.



(a) (b)

Figure 1: (a) Geometry of the case study. (b) Channel base current as an input of the macromodel.

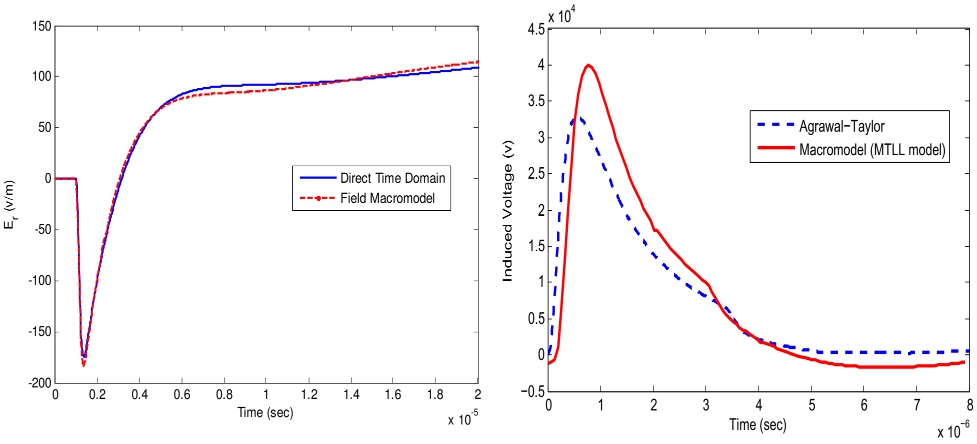


(a) (b)

Figure 2: (a) 8^{th} pole's real part locations in different distances. (b) 6^{th} residues's real part locations in different distances.

3 Simulation Results

In this section, we study an example adapted from [10]. A $6-km$ lightning flash occurs at a distance of $50m$ from a $500-m$ horizontal transmission line which is located at $10m$ above ground with the conductivity of $\sigma_g = 0.001S/m$. The geometry of this problem and the channel base current are shown in Figs. 1(a) and (b), respectively. We chose 17 equally-spaced points with a separation of $\Delta z = 30m$ on the line. For the lightning RSC, we used MTLL model with propagation speed along the channel equal to $1.5 \times 10^8 m/s$. 1^{st} and 2^{nd} order polynomial approximations is used for tracing the poles and residues of the horizontal electric field along the transmission line, respectively. As an example, Figs. 2(a) and (b) represent the tracing functions for the real parts of pole number 8 and residue number 6. Similar functions were obtained for the real and imaginary parts of other poles/residues. Figure 3(a) compares the horizontal electric field in an arbitrary distance from the RSC obtained by the proposed macromodel and direct time-domain method. Fig. 3(b) shows the induced voltage on the left terminal of the line obtained using the proposed closed-form solution and the results of [10]. Although very simple tracing functions is used, there is good agreement between the results.



(a)

(b)

Figure 3: (a) Horizontal electric field at z=310m and h=10m above the ground with $\sigma = 0.001 S/m$, using MTLL model. (b) Induced voltage at the left terminal of the transmission line of Fig. 1.

4 Conclusion

In this paper, we introduced a fast and efficient macromodel to calculate nonuniform electromagnetic fields induced by lightning return stroke channel. This is done by tracing the location of the poles and residues of the system transfer function. This leads to a fast technique for the analysis of excited transmission lines in time-domain simulators. This is done by efficient calculation of lumped sources at the terminals of the transmission line.

5 References

1. S. Rusck, "Protection of Distribution Systems" *Academic Press*, 1977, vol. 2, ch. 23.
2. C. Nucci, M. Paolone, and M. Bernardi, "Use of lightning location systems data in integrated systems for power quality monitoring," vol. 1, Oct. 2002, pp. 552 - 556.
3. B. Gustavsen, G. Irwin, R. Mangelrod, D. Brandt, and K. Kent, "Transmission line models for the simulation of interaction phenomena between parallel ac and dc overhead lines," in Int. Conf. on *Power Systems Transients* (IPST 99), Budapest, Hungary, June 1999, pp. 61-67.
4. B. Kordi, J. LoVetri, and G. Bridges, "Finite-difference analysis of dispersive transmission lines within a circuit simulator," *IEEE Trans. on Power Delivery*, vol. 21, no. 1, pp. 234 - 242, jan. 2006.
5. I. Erdin, A. Dounavis, R. Achar, and M. Nakhla, "Circuit simulation of incident field coupling to multiconductor transmission lines with frequency-dependent losses," *IEEE International Symposium on Electromagnetic Compatibility*, vol. 2, pp. 1084 - 1087 vol.2, 2001.
6. H. Xie, J. Wang, R. Fan, and Y. Liu, "A hybrid FDTD-spice method for transmission lines excited by a nonuniform incident wave," *IEEE Transactions on EMC*, vol. 51, no. 3, pp. 811 - 817, aug. 2009.
7. C. Paul, "Analysis of Multiconductor Transmission Lines," 2nd ed. IEEE Press, 2008.
8. B. Djebari, J. Hamelin, C. Leteinturier, and F. J., "Comparison between experimental measurements of the electromagnetic field emitted by lightning and different theoretical models. influence of the upward velocity of the return stroke,," in *4th Int. Symp. Tech. Exhib. Electromag. Compat*, 1981.
9. M. Rubinstein, "An approximate formula for the calculation of the horizontal electric field from lightning at close, intermediate, and long range," *IEEE Trans. EMC*, vol. 38, no. 3, pp. 531 - 535, aug. 1996.
10. C. Nucci, F. Rachidi, M. Ianoz, and C. Mazzetti, "Comparison of two coupling models for lightning-induced overvoltage calculations," *IEEE Trans. on Power Delivery*, vol. 10, no. 1, pp. 330 - 339, jan. 1995.