On the Use of the Monte Carlo Method for Electromagnetic Field Simulation

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Abstract

The knowledge of electromagnetic interferences plays a significant role in today's research and development. However, the actual realization of large systems may vary or even be unknown. Hence, the electromagnetic field of the system has to be estimated statistically. Therefore this paper describes how a statistical field simulation of the electromagnetic field of the system can be accomplished by modeling the system with subsystems and varying the parameters of these subsystems.

1. Introduction

The electromagnetic field simulation is an important and often used method for the calculation or estimation of field profiles. Commonly used techniques like the Method of Moments (MoM) or the Finite Elements Method (FEM) are based on meshing the surfaces or volumes of the model and solving the resulting matrix equations. This is the most time consuming part of the MoM and the FEM. Unfortunately, for small changes in the model these matrices and therefore the calculations have to be done anew.

In this paper the use of the Monte Carlo Method for electromagnetic field simulations is presented. This simulation method is based on simulating a model with random parameters many times. Therefore an alternative simulation technique is used, based on multipole sources, where each of these multipole sources stands for one subsystem. The difference of the simulation with multipole sources is that it is not based on the solution of matrix equations. Instead, the resulting field strength is directly calculated by the multipole coefficients of the sources.

This makes it possible to easily apply variations to the simulation model, such as a change in phase, position or orientation of a source or even a variable number of sources. One could for example use this possibility to perform statistical field simulations. And as these variations are adopted very fast the overall simulation can be accomplished in an acceptable amount of time.

2. Description of the Sources

Throughout this work, multipole sources are used for the calculation of the electromagnetic fields. This means that a source is described by multipoles, each located at the point of origin and representing electrical and magnetic dipoles, quadrupoles, octopoles, etc. The characteristics of these multipoles, and thus the radiation pattern of the source, are expressed by multipole coefficients A, B, C and D. Where A and B represent electrical multipoles and C and D represent magnetic multipoles. Furthermore A is equivalent to C, and B is equivalent to D.

For example the coefficient $A_{1,0}$ stands for an electrical dipole oriented into the z-axis, $A_{2,0}$ for an electrical quadrupole of the subgrade 0 and the coefficient $A_{2,2}$ for an electrical quadrupole of the subgrade 2. With these multipole coefficients the electrical and the magnetic fields can be calculated like it is performed in [1-2].

3. Parameterization of the Sources

In order to use the Monte Carlo Method for a statistical field simulation, each unknown variable of a source is treated as a random parameter. This can be the phase of the source, the orientation and location in space, the number of sources or even more. The probabilities for these parameters can be based on any probability density function (PDF). In this chapter possible parameterizations of the sources are shown.

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To derive the probability distribution of the electromagnetic field, this is calculated many times. For each calculation, every single unknown or variable is treated as a random parameter and is chosen anew. This makes it possible to easily comprise a lot of unknowns, like the phase, orientations, the number and location of the sources, or even more.

3.1 Phase of the Sources

Multipole sources describe independent sources and the coefficients of these sources are determined independently from each other. This means that the phases of the varying sources to each other are undefined while using them in one simulation model. Thus the phases of the sources ψ_i are handled as random parameters between $-\pi$ and π.

$$A_{n,m,i} = A_{n,m} \cdot e^{j\psi_i} \qquad \text{with} \qquad \psi_i \in [-\pi, \pi] \tag{1}$$

3.2 Orientation of the Sources

One possibility to include a rotation of the sources is a transformation of the multipole coefficients like it is performed in [3]. The new spherical waves $\boldsymbol{F}_{m,n}(r, \vartheta, \varphi)$ in the original coordinate system (x, y, z) are calculated as functions of the original spherical waves $\boldsymbol{F'}_{m,n}(r', \vartheta', \varphi')$, given in the rotated coordinate system (x', y', z'):

$$\boldsymbol{F}_{m,n}(r, \vartheta, \varphi) = \sum_{\mu=-n}^{n} e^{jm\phi} \cdot d_{\mu m}^{n} \cdot e^{jm\chi} \cdot \boldsymbol{F'}_{\mu,n}(r', \vartheta', \varphi') \tag{2}$$

The drawback of this method is however, that the transformed spherical waves, and thus the multipole coefficients, are functions of all multipole coefficients of the same grade. For simple sources like dipoles this could mean, that while the original source consists of one coefficient only, the transformed source has up to three coefficients. As the Monte Carlo method is based on the repeated simulation with a lot of runs, this could cause a significant increase of simulation time. This effect becomes even more important with an increasing grade. Thus, a different approach for a variable orientation of a source is implemented in this paper.

Instead of transforming the multipole coefficients, the point, in which the electromagnetic field is calculated, is transformed. This method can be seen in the 2D-example in Fig. 1. The original source, a dipole on the y-axis, is rotated with the angle ϕ around the z-axis. Simultaneously the original coordinate system (x, y, z) is transformed in the same way, resulting in the coordinate system (x', y', z')

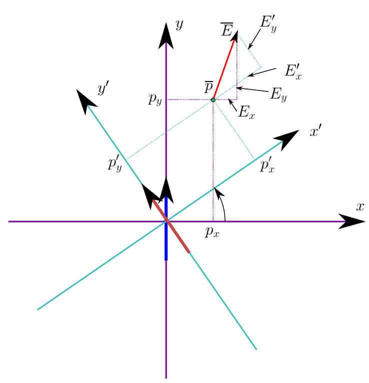

Fig. 1: Transformation of a dipole

The transformation of the source, and thus the transformation of the coordinate system, is described by $\boldsymbol{T}$. Then the coordinates of the observation point $\boldsymbol{p}$ in the transformed coordinate system are given by

$$\boldsymbol{p}' = \boldsymbol{T}^{-1} \cdot \boldsymbol{p} \tag{3}$$

As the multipole coefficients describing the rotated source maintain the same in the transformed coordinate system, the electromagnetic field can be easily calculated in the transformed coordinate system at the coordinates given by $\boldsymbol{p}'$. The resulting field vector $\boldsymbol{E}'$ of course has to be transformed back to the original coordinate system (x, y, z) by

$$\boldsymbol{E} = \boldsymbol{T} \cdot \boldsymbol{E}' \tag{4}$$

This of course is not bound to 2D. In this paper the matrix $\boldsymbol{T}$ describes three subsequent rotations:

$$\boldsymbol{T} = \boldsymbol{T}_{\boldsymbol{z_2}}(\chi) \cdot \boldsymbol{T}_{\boldsymbol{y_1}}(\theta) \cdot \boldsymbol{T}_{\boldsymbol{z}}(\phi) \tag{5}$$

The first rotation is around the z-axis with the angle ϕ, the second rotation is around the new y_1-axis with the angle θ and the third rotation is around the z_2-axis with the angle χ.

3.3 Number and Location of Sources

Not always the total number of sources and their position in space is known. Therefore these parameters can be treated as random variables as well. The number of sources n can for example be a uniformly distributed discrete random variable (eq. 6) or by using other distributions like a binomial distribution it is also possible to include probabilities for the usage of sources p (eq. 7).

$$p(n) = \frac{1}{n_{max}} \qquad \text{with} \qquad n \in [1, n_{max}] \tag{6}$$

$$B(k|p, n) = \binom{n}{k} \cdot p^k \cdot (1-p)^{n-k} \tag{7}$$

where n is the number of present sources, k is the number of active sources and p is the probability that a present source is active. Furthermore the source locations can be chosen arbitrary in space. But of course, there have to be probabilities, which can be based on geometric boundaries:

$$p(x_i) = \frac{1}{x_{max} - x_{min}} \qquad \text{with} \qquad x_i \in [x_{min}, x_{max}] \tag{8}$$

Alternatively an open-end distribution, like a Gaussian distribution, can be used to select the locations of the sources. This makes it also possible to include uncertainties for known locations, assumed, that the standard deviation is known.

4. Results

To keep the simulations as simple as possible, dipoles, oriented on the z-axis ($A_{1,0} = 1$) were used throughout these simulations.

4.1 Random phase

The influence of the phase on the superposition of the electromagnetic fields can be seen in Fig. 2. This figure shows the histogram of the electric field in the point $(0.5\text{m}, 0\text{m}, 0\text{m})$, which was generated by two dipoles, located at $(1\text{m}, 0\text{m}, 0\text{m})$ and $(-1\text{m}, 0\text{m}, 0\text{m})$. This result can be validated by an analytical calculation. In the xy-plane there is only a ϑ-component of the electrical field. The magnitude of the electrical field of the dipole at $(1\text{m}, 0\text{m}, 0\text{m})$ is $E_{\vartheta_1} = 0.1054 \frac{V}{m}$ and the magnitude of the electrical field of the dipole at $(-1\text{m}, 0\text{m}, 0\text{m})$ is $E_{\vartheta_2} = 0.0353 \frac{V}{m}$. The overall electrical field strength in the point $(0.5\text{m}, 0\text{m}, 0\text{m})$ is calculated in the complex plane

$$|E_\vartheta| = \sqrt{\left|E_{\vartheta_1}\right|^2 + \left|E_{\vartheta_2}\right|^2 - 2\left|E_{\vartheta_1}\right|\left|E_{\vartheta_2}\right| cos(\varphi)} \tag{9}$$

with a uniformly distributed phase difference φ between E_{ϑ_1} and E_{ϑ_2}. To allow an analytical calculation of the PDF of $|E_\vartheta|$ (eq. 9 has to be monotone), the phase is without loss of generality limited to the interval $[0, \pi]$. The result is shown in Fig. 2. It can be seen that the histogram and the PDF have the same characteristics.

4.2 Rotated dipole

Fig. 3 shows the simulation results for a rotated dipole, located at the origin. Since a dipole, oriented in the direction of the z-axis, was used, the rotation around the z-axis and the third rotation around the z_2-axis would be gratuitous, and thus the rotation angles ϕ_i and χ_i were set to zero. The second rotation angles θ_i of the dipoles were chosen between 0 and $\frac{\pi}{4}$. In Fig. 3 it can clearly be seen, that the minimum field value in the histogram equals the minimum emitted electric field of the dipole. Moreover, the behavior of the histogram can be validated with an analytical probability density function (cf. Fig. 3, red curve).

4.3 Variable number and location of the sources

In chapter 3.3 it was shown, that the number of sources and their locations can be chosen based on any distribution. The effect of the choice of the distribution can be seen in Fig. 4. The simulations results, where the location of the sources is based on a uniform distribution, are presented by the blue histogram. The red histogram shows the simulation results for a number of sources, based on a binomial distribution ($n = 200$, $p = 0.3$).

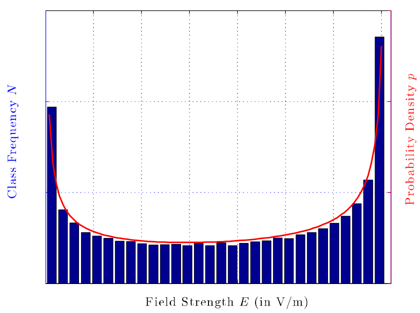

Fig. 2: Histogram and PDF for two dipoles with random phases

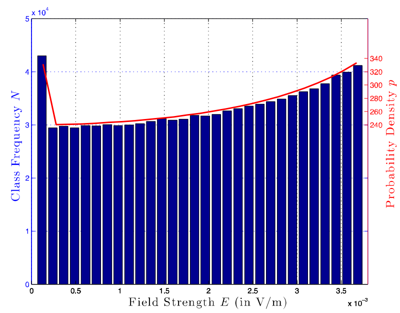

Fig. 3: Histogram and PDF for a rotated dipole

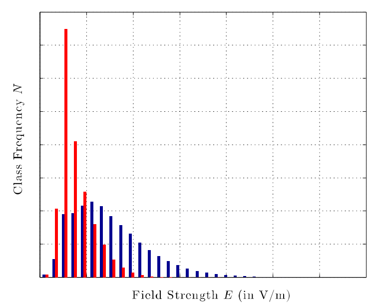

Fig. 4: Histograms for a variable number of dipoles, based on a uniform (blue) and a binomial (red) distribution

5. Conclusion

A new approach for the statistical simulation of electromagnetic fields is presented. It is based on the interpretation of the sources as multipole sources. This makes it possible to run a simulation with a lot of sources, including variations or uncertainties, like the phase, the orientation, the number and the location of the sources.

The results of these simulations are histograms of the electromagnetic field strength. From these, for example the expected value or the standard deviation of the electromagnetic field can be calculated. Furthermore, from the histograms a probability density function of the electromagnetic field could be derived for further calculations like the failure probability of systems, when exposed to the simulated sources.

6. References

[1] J.D. Jackson, *Klassische Elektrodynamik*, Berlin [u.a.]: de Gruyter, 2006.

[2] P. Kralicek, “Makromodellierung des Strahlungsverhaltens von Subsystemen zur Systemanalyse,” Shaker, 2006.

[3] J.E. Hansen, *Spherical Near-field Antenna Measurements*, London: Peregrinus, 1987.