

# Dealing with Complexity in EMC Modelling

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## Abstract

The paper addresses features of EMC which make it a particularly challenging modeling applications. These features include the very broadband nature of the interactions taking place thus encompassing regions of non-linearity and of uncertain parameter values; the inherent uncertainties in the geometrical configurations and dimensions of the multitude of components; and the ubiquitous presence of multi-scale features. The challenge thus is to address complexity without resorting to extremely lengthy computations which require enormous computational resources and tedious and time consuming problem definition and input data preparation. The emphasis of this paper is on the embedding of macromodels to describe complex materials and the treatment of uncertainties.

## 1. Introduction

The particular aspects of EMC which make it a particular challenge for modelers are brought about by the very wide band of frequencies covered by EMC regulations and electromagnetic interoperability. For practical purposes this range start from dc and exceeds 6 GHz, although in many military applications the upper limit can extend into the THz range [1]. Taking a LF example of an electronic drive which is controlled by switching in the kHz range, conducted emission calculations extend to 30MHz but as regards radiated emissions the range extends to several GHz. Admittedly, the amount of power emitted at such high frequencies is limited, but nevertheless designers have to characterize and test designs at least over the frequency range specified in regulations. For the example of an electronic drive, the requirements for normal operations of this drive are determined by power frequencies (50 or 60 Hz) and few harmonics-a much restricted frequency range compared to the EMC range of interest.

The inevitable consequence of this is that design and placement of components at high frequencies is not well controlled and that many electrical material parameters are not accurately known and vary considerable with frequency. The modeler is thus confronted with two difficulties. First, material models must be broadband and allow for significant variations with frequency. Allowance must also be made for the several artificial and composite materials now becoming available. Second, many of the material and geometrical details, e.g. exact location of cable, are not accurately known. Instead, some parameters are problem variables which are random with a known mean and standard deviation. The problem then is to obtain the EMC behaviour of the system subject to random input parameters. In this paper we summarize some of the work under these two headings. Other important problems in EMC modeling such as the treatment of multi-scale problems are dealt with in another paper.

## 2. Material Modelling in the Time-Domain

Due to the broadband nature of EMC studies and the prevalence of electrical pulses in digital design many models for EMC are operated in the time-domain. This makes it possible to obtain from one simulation (impulse response) the frequency response over a wide frequency range through a Fourier Transform. Several powerful numerical models in the time-domain are available, such as FDTD and TLM [2, 3]. Bulk material electrical properties when viewed over a wide range of frequency exhibit significant variations which must be included in models if accurate EMC predictions are to be made. A particular example where this type of calculation is important is the calculation of the shielding effectiveness (SE) of cabinets. In cases where the wall materials is not a perfect conductor the penetration of EM energy is a frequency dependent phenomenon which can be described in terms of a transfer function defined in the s-domain. A simple example is given below of such a functions (e.g. reflection coefficient) for a case where the frequency-response is approximated by an expression with three zeros and three poles.

$$F(s) = \frac{b(s - s_{z0})(s - s_{z1})(s - s_{z2})}{(s - s_{p0})(s - s_{p1})(s - s_{p2})} \quad (1)$$

By applying the bilinear transformation [4] this expression is transformed in to the z-domain in the following form,

$$F(z) = B_0 + \frac{\sum_{i=1}^3 (B_i - B_0 A_i) z^{-i}}{1 + \sum_{i=1}^3 A_i z^{-i}} \quad (2)$$

where,

$$B_0 = b \frac{a_{z0} a_{z1} a_{z2}}{a_{p0} a_{p1} a_{p2}}, A_1 = -(\beta_{p0} + \beta_{p1} + \beta_{p2}), A_2 = \beta_{p0} \beta_{p1} + \beta_{p0} \beta_{p2} + \beta_{p1} \beta_{p2}$$

$$A_3 = -\beta_{p0} \beta_{p2} \beta_{p3}, B_1 = -B_0 (\beta_{z0} + \beta_{z1} + \beta_{z2}), B_2 = B_0 (\beta_{z0} \beta_{z1} + \beta_{z0} \beta_{z2} + \beta_{z1} \beta_{z2})$$

$$B_3 = -B_0 \beta_{z0} \beta_{z2} \beta_{z3}, a_{zi} = \frac{2 - s_{zi} \Delta t}{\Delta t}, a_{pi} = \frac{2 - s_{pi} \Delta t}{\Delta t}, \beta_{zi} = \frac{2 + s_{zi} \Delta t}{2 - s_{zi} \Delta t}, \beta_{pi} = \frac{2 + s_{pi} \Delta t}{2 - s_{pi} \Delta t}$$

The form in (2) establishes the scattering coefficient i.e. the relationship between  $V^r$  (signal reflected from a cell) and  $V^i$  (signal incident on next cell) depicted in Fig 1,

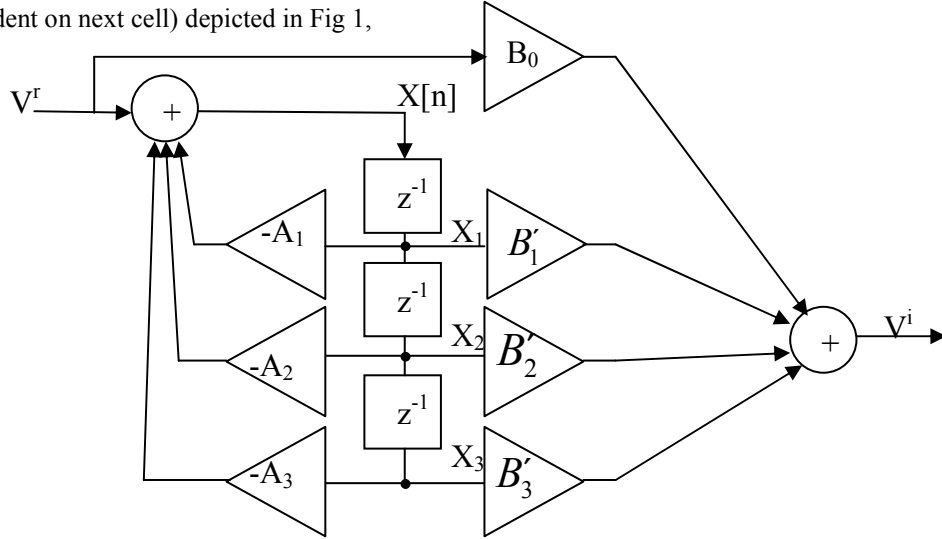


Fig 1 Schematic of the DF implementing the TF in (1) in the time-domain

Embedding the procedure in Fig 1(macro-model) into the time-domain code is straight forward. At time step n we calculate  $X[n]$  from values of the state variables at previous time steps,

$$X[n] = -A_1 X_1[n-1] - A_2 X_2[n-2] - A_3 X_3[n-3] + V^r[n]$$

The state variable at time step n are similarly obtained from,

$$X_1[n] = X[n-1], X_2[n] = X[n-2], X_3[n] = X[n-3]$$

The  $V^i$  is obtained from,

$$V^i[n] = B_0 V^r[n] + B'_1 X_1[n] + B'_2 X_2[n] + B'_3 X_3[n]$$

Naturally, higher order filters may be incorporated to accommodate more complex TFs where more poles/zeros are required. This simple procedure allows for the descriptions of very complex materials properties [5], conducting panels with perforations for the calculation of shielding effectiveness [6], composite panels e.g. CFC layers [7], meta-materials where the fine geometrical structure is approximated efficiently by DFs [8] etc. In all these cases a substantial reduction in model complexity is achieved by the extraction of poles and zeros of the responses and the conversion to time-domain procedures via the bilinear transformation as typified by the procedure outlined above.

### 3. Stochastic EMC

The inherent uncertainty in many configurations encountered in engineering practice has already been mentioned. In electromagnetic compatibility, the problem may be posed as follows. Given an engineering system with certain parameters being random variables with a known probability density function (pdf) obtain the pdf of the output (response) of the system. A simple example is that of a cabinet that has an aperture with dimensions being random variables with known pdf, and the pdf of the shielding effectiveness is required. The straightforward approach to such a problem would be to sample the space of the input random variable(s), obtaining in each case the response, and thus systematically through a number of trials obtain the pdf of the response. This is the well known Monte-Carlo (MC) approach and requires a large number of trials. In the kind of models required in EMC system studies, where doing a single deterministic simulation is a major computational task, the idea of doing thousands of simulations to establish the pdf of the response is completely unrealistic. In problems of such complexity what is needed is a different approach which allows us to establish the pdf, or at least some of its moments, from a small number of simulations. It turns out that such an approach is possible and one of several methods for doing this, the so called Unscented Transform (UT) is described here [9-11]. Consider a random variable  $x$  (the input) and a random variable representing the output  $y$ ,

$$y = g(x) = g(\bar{X} + \hat{x}) \quad (3)$$

where  $\bar{X}$  is the expected value (mean) and  $\hat{x}$  is a symmetrically distributed zero mean Gaussian random variable with variance  $\sigma^2$ . This non-linear mapping can be expressed in a Taylor series as,

$$g(\bar{X} + \hat{x}) = g(\bar{X}) + \frac{dg}{dx} \hat{x} + \frac{1}{2!} \frac{d^2g}{dx^2} \hat{x}^2 + \dots = g(\bar{X}) + a_1 \hat{x} + a_2 \hat{x}^2 + \dots$$

The expected value and variance of the output are then,

$$\bar{g} = g(\bar{X}) + a_2 \sigma^2 + \dots$$

$$\sigma_g^2 = a_1^2 \sigma^2 + 2a_2 \sigma^4$$

We estimate the two moments of the output by evaluating the function at three points as shown below,

$$\bar{g} = E\{g(\bar{X} + \hat{x})\} = w_0 g(\bar{X}) + w_1 g(\bar{X} + S) + w_1 g(\bar{X} - S)$$

$$\sigma_g^2 = w_0 [g(\bar{X}) - \bar{g}]^2 + w_1 [g(\bar{X} + S) - \bar{g}]^2 + w_1 [g(\bar{X} - S) - \bar{g}]^2$$

where  $S$  are the sigma points and  $w$  are the weights. What this means in terms of computation is that the response is evaluated three times only. Demanding that the moments evaluated by the last two sets of equations are the same after some manipulation the sigma points and weights are calculated,

$$w_1 = \frac{1}{6}, w_0 = \frac{2}{3}, S = \sigma\sqrt{3} \quad (4)$$

Thus through only three full scale simulations the first two moments of the output response pdf can be estimated thus saving considerably in computation. Moreover, the entire pdf can be reconstructed. The inverse of (3) is

$$x = h(y) \quad (5)$$

The pdf of  $y$  is related to the pdf of  $x$  through

$$f_y(y) = \sum f_x(x) \left| \frac{dh(y)}{dy} \right|$$

where the summation on the RHS is done over all the roots of  $x$  [12]. We approximate the non-linear mapping through a polynomial,

$$y = g(x) = a_0 + a_1 x + a_2 x^2 \quad (6)$$

To invert, for any value of  $y$  we need to find  $x$  that makes,

$$a_0 - y + a_1 x + a_2 x^2 = 0$$

Finding the roots,

$$h(y) = x = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_2(a_0 - y)}}{2a_2}$$

assuming that the pdf of the input is a Gaussian,

$$f_x(x) = \frac{1}{\sqrt{2\pi}} e^{x^2/2}$$

and taking account of the two roots of x we thus have the pdf of the output.

$$f_y(y) = f_x(x) \frac{dh(y)}{dy} \Big|_{x=x_1} + f_x(x) \frac{dh(y)}{dy} \Big|_{x=x_2}$$

$$= \frac{1}{\sqrt{2\pi}} \left\{ \frac{e^{\frac{1}{2} \left( \frac{a_1 + \sqrt{a_1^2 - 4a_2a_0 + 4a_2y}}{2a_2} \right)^2}}{\sqrt{a_1^2 - 4a_2a_0 + 4a_2y}} + \frac{e^{\frac{1}{2} \left( \frac{-a_1 + \sqrt{a_1^2 - 4a_2a_0 + 4a_2y}}{2a_2} \right)^2}}{\sqrt{a_1^2 - 4a_2a_0 + 4a_2y}} \right\} \quad (7)$$

## 4. Conclusion

Recent advances in the treatment of complex problems have been surveyed in this paper. One approach based on the use of digital filters to deal with complex material features was presented. The development of stochastic models suitable for dealing with uncertainties was also emphasized and an approach based on the Unscented Transform was briefly described.

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