Equivalent Circuit Synthesis for Microstrip Structures Design and Optimisation

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Abstract

The paper suggests network synthesis algorithm used limited band scattering parameters of microwave devices. The proposed approach is based on the simple description of the microwave structures using lumped element network. The submitted example of synthesis illustrates the microstrip filter design.

1. Introduction

Devising a model of a microwave device consists in synthesis of an equivalent circuit implemented through radio electronic components and delay lines. Such model can describe characteristics of losses microwave devices with required accuracy in broad operational frequency band. One approach suggests combining lumped-elements circuits and transmission lines. One of the advantages of this models may be their compactness and easiness of analysis. In parallel, presentation of the model in the form of an equivalent electric circuit maintains its feasibility as a microwave device under variation of parameters in the process of optimization. In some applications transmission lines can be neglected and the microwave structure can be described only by lumped elements model. The network synthesis algorithm will be proposed in the paper as well as demonstration its appliance to the microstrip band-pass filter.

2. Basic Approaches to Synthesis of a microwave device equivalent circuit

There are two approaches to synthesis of a microwave device equivalent circuit as per a given Z – parameters matrix. The first of them consists in presenting the device as certain "black box" with unknown structure terminated on R_r resistance (Fig. 1a). For such a model, the Z_{in} full input impedance function is calculated for a known Z – parameters and given terminating resistor:

$$Z_{in} = Z_{11} - \frac{Z_{12} \cdot Z_{21}}{Z_{22} + R_r}, \tag{1}$$

where Z_{11}, Z_{12}, and Z_{22} are the open circuit driving point and transfer impedances of the network. For this function, its feasibility conditions are verified, and, in case they are feasible, synthesis of equivalent circuit is performed using one of the methods, like Darlington method [1,2] or Brune method [1-3].

Fig. 1. Reactive network terminated in a single resistance (a) and equivalent bridge circuit of a symmetric reciprocal two-port (b).

The second approach for two-port lumped element synthesis is to start from an equivalent circuit containing one-port elements only and subsequently performing a lumped-element one-port synthesis. For example for symmetric two-ports with $Z_{12} = Z_{21}$ a symmetric bridge or lattice represntation as shown in Fig. 2b can be chosen. The impedance and admittance matrices of the symmetric bridge representation are given by

$$\mathbf{Z} = \frac{1}{2}\begin{bmatrix} Z_2 + Z_1 & Z_2 - Z_1 \\ Z_2 - Z_1 & Z_2 + Z_1 \end{bmatrix}. \tag{2}$$

3. Brune's Synthesis

The Brune's synthesis is a method that realizes for a positive real (p.r.) impedance $Z_{in}(s)$ an equivalent input impedance to a reactive network terminated in a single resistance [2,3]. Otto Brune was first to demonstrate that every positive real (p.r.) function can be realized as the driving-point impedance of an *RLC* circuit, though ideal ransformers are needed. It is a canonical realization in that the number of elements equals the number of coefficients of the impedance function.

The Brune's synthesis algorithm may be conditionally divided into 2 steps: preliminary and main. During the preliminary step, $Z_{in}(s)$ poles and zeros located on characteristic points of s-plane must be dentified and transformed into realizable equivalent circuit elements in the following way:

- If $Z_{in}(s)$ has a pole at infinity, remove a series inductance and obtain a positive real remainder $Z(s)$.
- If $Z_{in}(s)$ has a pole at zero, remove a series capacitor and obtain a p.r. remainder $Z(s)$.
- If $Z_{in}(s)$ has a zero at infinity, remove a shunt capacitor from $Y(s) = 1/Z(s)$ and obtain a p.r. remainder $Y(s)$.
- If $Z_{in}(s)$ has a zero at zero, remove a shunt inductor from $Y(s) = 1/Z(s)$ and obtain a p.r. remainder $Y(s)$.
- If $Z_{in}(s)$ has poles at $s = \pm j\omega_0$, remove a parallel *LC* impedance and obtain a p.r. remainder Z(s).
- If $Z_{in}(s)$ has zeros $s = \pm j\omega_0$, remove a series *LC* admittance from $Y(s) = 1/Z(s)$ and obtain a remainder Z(s).

After all these removals, we are left with a remainder which does not have any poles or zeros on the $j\omega$ axis, including the points $s = \infty$ and $s = 0$. Such an impedance function is called a *minimum* reactance and *minimum susceptance* function. The main step of Brune's synthesis begins at this point.

During the second step, the remaining minimum reactance function is decomposed into a ladder structure looking as follows:

$$Z(s) = R + sL_1 + \cfrac{1}{\cfrac{s/L_2}{s^2 + 1/(L_2C_2)} + \cfrac{1}{sL_3 + Z_1(s)}}. \tag{3}$$

Since in such decomposition some inductances have negative sign, they are transformed into either mutual inductance, or ideal transformer. Fig. 2 gives an example of Brune's realization of the driving-point impedance of a one-port.

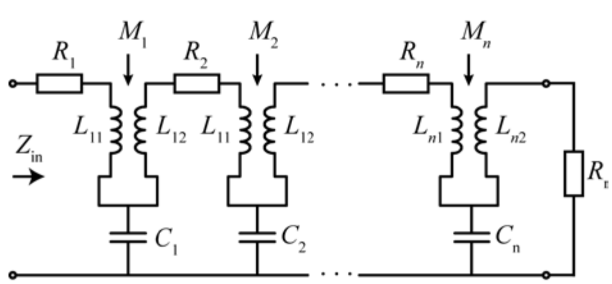


Fig. 2. The Brune's equivalent circuit.

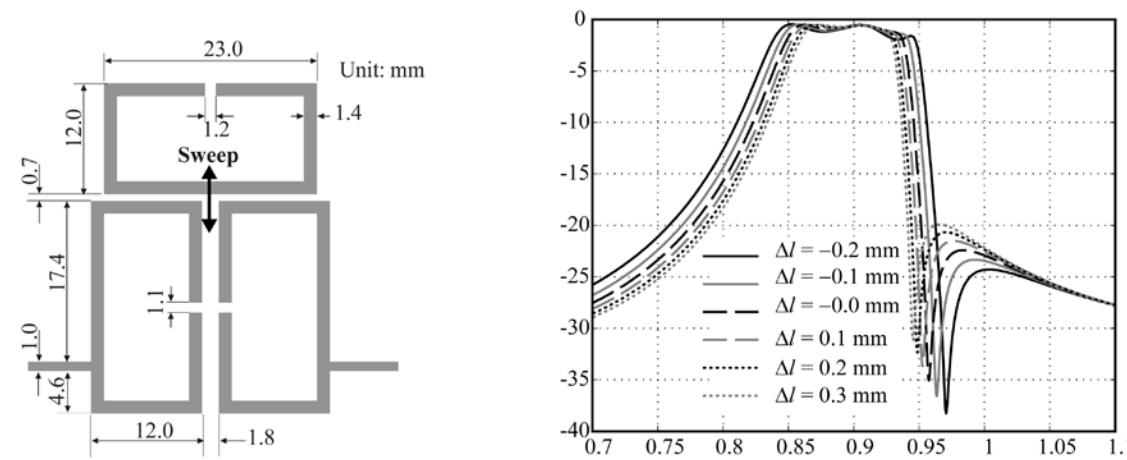
4. Proposed Technique of Equivalent Circuit Synthesis

The main steps of the proposed algorithm are:

- If input data are given in the form of a scattering matrix, conversion of the scattering matrix [4] into a *impedance* matrix.
- Selection of equivalent circuit type.
- Processing all pole model parameters of the circuit input impedance function(s) in the limited frequency band using Vector Fitting Method [5]. During this step the order of equivalent circuit matching physical poles, but not approximation poles should be determined. That can be done using stability criterion, since physical poles are stable against small fluctuations of the band, while approximation poles are not.
- Verification of the feasibility conditions for resulting input functions (in case of discrepancies, return to step 2 of the algorithm).
- Selection of method of input impedance function realization.
- Determination of the equivalent circuit parameters, verification for positive values of resistances, inductances and capacitances.
- Checking the results against input data. Determination of relation between the model parameters and microwave device parameters.

5. Numerical Example

The paper considers the filter having only one transmission zero (or attenuation pole). According to the description represented in [6] this filter is the intermediate between the Chebyshev and elliptic-function filters. Fig. 3a shows the layout of designed microstrip filter. The distance between lower loops and upper loop is a parameter which should be optimized for bandwidth control. For the research purpose we designed own CAD model of this filter and compute the scattering parameters using fullwave EM simulation. The set of $|S_{12}|$ parameter is shown in Fig. 3b for different sweep Δl from the initial position.



(a) (b)

Fig. 3. Trisection open-loop filter (a) and it's measured S-parameters (b).

The simulated data was processed using proposed algorithm above. Bridge circuit shown in Fig. 1b was chosen due to geometrical symmetry of filter configuration. The network model was performed both for Z_1 and Z_2 elements using Brune's synthesis. The 2nd order equivalent circuit for Z_1 is shown in Fig. 4a. Fig. 4b shows equivalent circuit for Z_2 having 1st order. These circuits were obtained for $\Delta l = 0$. Also the equivalent circuits of Z_1 and Z_2 were obtained for others values of parameter by optimization. The corresponding circuit element values are shown in Tab. 1.

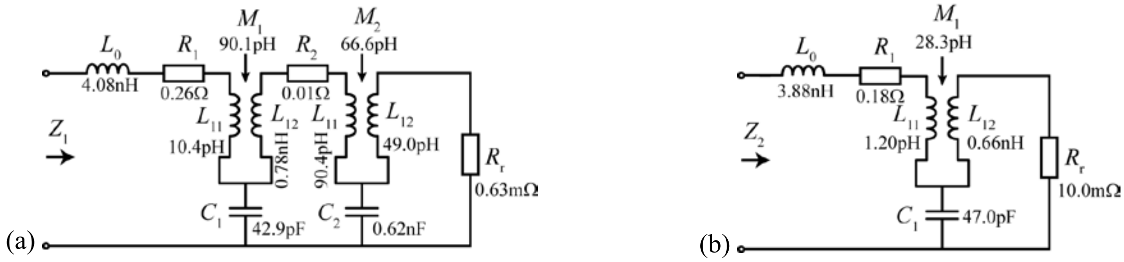


Fig. 4. Equivalent circuits of Z_1 (a) and Z_2 (b).

Table 1. The parameters of lumped-element equivalent circuits for Z_1 and Z_2.

Δl, mm	L_0, nH	R,Ω	L_1, pH	L_2, pH	M, pH	C, pF	R_r, mΩ	L_0, nH	R,Ω	L_1, pH	L_2, nH	M, pH	C, pF	R_r, mΩ
-0.2	4.1	0.26	1.34	692.5	30.51	49.20	1	3.9	0.2	2.31	0.69	40.18	44.94	11
		0.01	152.4	92.07	118.4	326.0								
-0.1	4.1	0.26	6.45	751.1	69.58	44.94	0.8	3.9	0.2	1.82	0.67	32.24	46.57	10
		0.01	115.8	65.75	87.25	460.2								
0	4.1	0.26	10.37	781.9	90.05	42.93	0.6	3.9	0.2	1.20	0.66	28.25	47.03	10
		0.01	90.42	49.05	66.6	619.5								
0.1	4.1	0.26	15.33	815.1	111.8	41.0	0.5	3.9	0.2	1.14	0.66	27.47	47.20	10
		0.01	74.69	39.16	54.08	777.8								
0.2	4.1	0.26	24.63	873.4	146.7	38.17	0.5	3.9	0.2	1.15	0.66	27.56	47.72	10
		0.01	68.13	35.36	49.08	862.5								
0.3	4.1	0.26	20.00	845.6	130.0	39.39	0.4	3.9	0.2	1.27	0.66	29.14	46.88	10
		0.01	55.33	28.32	39.58	1076								

The analysis of equivalent circuit element values confirms monotonic dependence of element values related to the bandwidth on varied microstrip structure parameter and independence of other element values, for example R and L_0.

6. Conclusion

The full wave electromagnethic simulation of microwave devices can be compared with network model on the assumption of narrow band used. The physical interpretation based on the circuit helps efficiently design microstip units of microwave systems. The proposed synthesis algorithm was demonstrated with known filter. And equivalent circuit was obtained for future optimization.

7. References

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