

# Six-Port Wave Correlator Theory and Practical Application to RF Network Analysis

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## Abstract

This tutorial lecture is intended to introduce the basic concept and principle of the six-port wave correlator, and to present the latest advancements in its theory and techniques. The lecture starts with some fundamentals on the six-port reflectometer dating back to its first proposal by G. F. Engen. The key feature of the scheme is to measure the reflection coefficient of RF circuits without resorting to heterodyne conversion. This implies a possibility to measurement of S11 in both magnitude and phase, free from frequency limitation. Next, the concept is extended to a six-port wave correlator on which we focus from the viewpoint of establishing a new scheme for characterizing the full S matrix, like what can be carried out by vector network analyzers. Finally, we explore some specific microwave applications of the six-port network, such as beam direction finding, Doppler frequency measurement, etc.

## 1. Introduction

The six-port technology and its many useful applications are now being recognized once again in modern society [1], and are published in book form [2]. These introduced applications are very interesting for us, however, it is important to understand a basic theory firmly first of all. This lecture starts with the basic concepts and fundamentals on both the six-port reflectometer [3], [4], [5], and the six-port wave correlator [6]. Next, as one of useful applications, we focus on a new scheme for vector network analyzers (VNAs), i.e., six-port reflectometer & wave correlator based VNA [7] and Prototype MMIC six-port wave correlator based VNA [8], [9]. Finally, we explore some specific microwave applications of the six-port network, such as beam direction finding [6], Doppler frequency measurement [10], etc. The lecture is full of interesting ideas and techniques for a variety of applications in microwave and millimeter-wave scientific fields.

## 2. Six-Port Fundamentals

### 2.1 Six-Port Reflectometer

As an alternative measurement technique for the complex reflection coefficient  $\Gamma$  of a device under test (DUT), the six-port reflectometer was proposed by Engen and Hoer in the 1970s [3], [4].

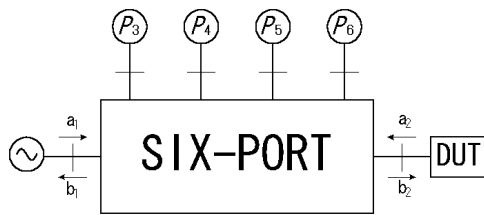


Fig1. Six-port reflectometer

The six-port reflectometer comprises a signal source port, a measurement port, and four sidearm ports to which power detectors are connected as shown in Fig. 1. The incident and emergent scattering variables at the measurement port are denoted by  $a_2$  and  $b_2$  respectively. When the DUT of reflection coefficient  $\Gamma = a_2/b_2$  is connected to the port, the four sidearm port power readings  $P_h$ , ( $h=3, 4, 5, 6$ ) may be written as

$$P_h = \alpha_h |A_h a_2 + B_h b_2|^2 = \alpha_h |b_2|^2 |A_h \Gamma + B_h|^2, \quad (1)$$

where  $A_h$  and  $B_h$  are complex constants depending on the six-port reflectometer and  $\alpha_i$  are the power conversion parameters of the power meter connected to the sidearm ports. If we denote an arbitrary sidearm port by subscript  $h$ , and one of the other four ports by  $i, i=3, 4, 5, 6, i \neq h$ , then the power ratio  ${}_h P_i \equiv P_i / P_h$  with port  $h$  as reference port can be expressed as

$${}_h P_i \equiv \frac{P_i}{P_h} = \left| \frac{A_i \Gamma + B_i}{A_h \Gamma + B_h} \right|^2 = {}_h K_i \left| \frac{1 + k_i \Gamma}{1 + k_h \Gamma} \right|^2, \quad (2)$$

where  ${}_h K_i, k_i$ , and  $k_h$  are calibration parameters of the six-port reflectometer which should be determined in advance [5].

## 2.2 Six-port Wave Correlator

A six-port wave-correlator is shown in Fig. 2. Here the six-port wave correlator is operated as a two-channel wave receiver. Two ports incident waves are denoted by  $a_2$  and  $a_1$ , and defined their complex wave ratio  $W = a_2/a_1$ , then the four sidearm port power readings  $P_h$ , ( $h=3, 4, 5, 6$ ) may be written as same formulas as (1) and (2),

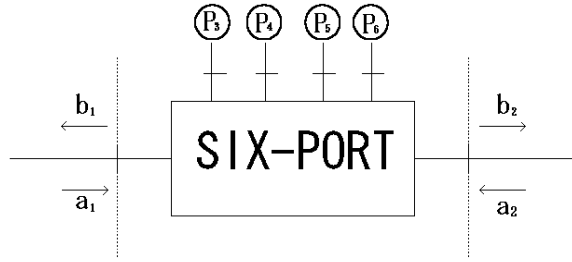


Fig. 2. Six-port wave correlator  
In (4), although the choice of  $h$  is wholly arbitrary, we set  $h=4$  ( $i=3,5,6$ ) in harmony with Engen's notation [4]. Then, (4) can be rewrite,

$$P_h = \beta_h |C_h a_2 + D_h a_1|^2 = \beta_h |a_1|^2 |C_h W + D_h|^2, \quad (3)$$

$${}_h P_i \equiv \frac{P_i}{P_h} = \frac{|C_i \Gamma + D_i|^2}{|C_h \Gamma + D_h|^2} = {}_h T_i \left| \frac{1 + t_i W}{1 + t_h W} \right|^2, \quad (4)$$

where  $\beta_h$  are the power conversion parameters.  $C_h$  and  $D_h$  are complex constants, and  ${}_h T_i$ ,  $t_i$ , and  $t_h$  are calibration parameters of the six-port wave-correlator which should be determined in advance [6].

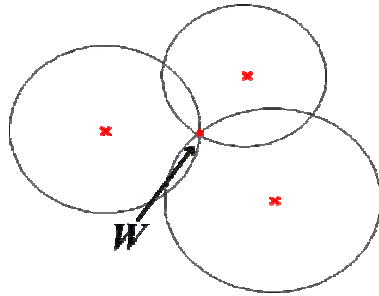


Fig. 3. Complex  $W$  plane  
Since the locus of each circle represents the possible values for  $W$ , the complex  $W$  is the intersection of three circles as shown in Fig. 3.

$${}_4 P_i = {}_4 T_i \left| \frac{1 + t_i W}{1 + t_4 W} \right|^2, \quad (5)$$

where real  ${}_4 T_3$ ,  ${}_4 T_5$ ,  ${}_4 T_6$  and complex  $t_3$ ,  $t_4$ ,  $t_5$ ,  $t_6$  are the system parameters of the wave correlator, and  $W$  is the correlation ratio of the two incident waves  $a_2$  and  $a_1$ . By expanding (5) into quadratic form, we have three circles in the complex  $W$  plane with the centers  $z_i$  and radii  $L_i$  described as

$$z_i = -\frac{t_4 {}_4 P_i - t_i {}_4 T_i^2}{4 {}_4 P_i - 4 {}_4 T_i}, \quad L_i = \frac{|t_4 - t_i|}{4 {}_4 P_i - 4 {}_4 T_i} \sqrt{{}_4 T_i {}_4 P_i}. \quad (6)$$

## 3. Six-Port Based Vector Network Analyzers

### 3.1 Six-Port Refractometer & Wave Correlator based VNA

The system block diagram of the six-port refractometer & wave correlator based VNA is shown in Fig. 4 [7]. This configuration is composed of power divider, directional couplers, matching loads, switches, phase shifter, and the six-port junction. The two-port DUT with scattering parameters  $S_{ij}$ ,  $i, j = 1, 2$ , is connected at reference planes P1 and P2. The source signal is split into two parts using a two-way power divider. One of them directly feeds the six-port junction, while the other is routed to the DUT's ports through a sliding phase shifter and switch, SW1. Two directional couplers are connected to the DUT so that the reflected or transmitted waves of the DUT can be sampled, and routed via switch SW2 to the other port of the six-port junction. This circuit configuration is not an optimized one with respect to circuit performance and circuit simplicity. The calibration steps for  $S_{11}$  (or  $S_{22}$ ) measurements are as follows, and the connections are shown in Fig. 4. First connect a short circuit to port 1, P1 in Fig. 2, and set the switches SW1 and SW2 as shown in Fig. 2. Then the powers at the side arms of the six-port junction are measured with moving the phase shifter in equal increments over at least one wavelength. As one can see the six-port junction is in fact functioning as a reflectometer in this case, and the calibration is similar to [5].

Next the calibration steps for  $S_{21}$  (or  $S_{12}$ ) measurements are as follows, and the connections are shown in Fig. 4, except setting the switches SW1 and SW2 to the corresponding opposite positions. The detailed calibration steps for  $S_{12}$  measurements are as follows. First make a "thru" connection between the points

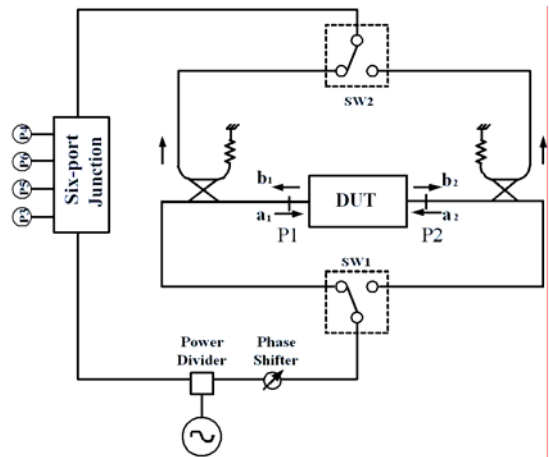


Fig. 4. System block diagram for  $S_{11}$  measurements

where the DUT will be connected, i.e., P1 and P2 in Fig. 4. Then the powers at the side arms of the six-port junction are measured with moving the phase shifter in equal increments over at least one wavelength. As one can see, the six-port junction is in fact functioning as a wave-correlator in this case, and the above calibration is same as the calibration method for calibrating a six-port based wave-correlator by using only one phase shifter, which we discussed in [6].

### 3.2 Prototype MMIC Six-Port Wave Correlator based VNA

A Prototype MMIC six-port correlator based VNA to measure the scattering parameters of a two-port DUT is shown in Fig. 5 [8], [9]. This configuration is composed of directional couplers, switches, isolators, circulators, and matching loads.

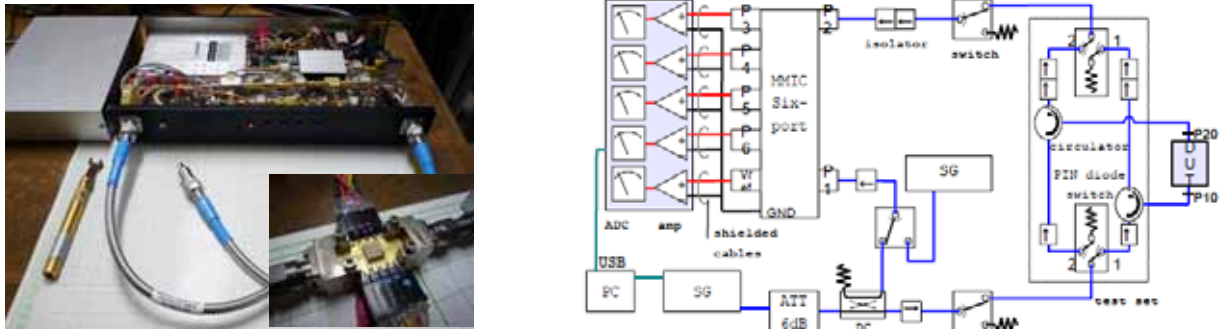


Fig. 5. Scheme of the MMIC six-port correlator based vector network analyzer

For determining the system parameters of the six-port correlator, three lines of different length are used as standards for measuring the transmission S-parameters, and three shorts with different length are used as standards for measuring the reflection S-parameters. The system performance was evaluated by measuring various two-port passive components. Figure 6 illustrates the measured S-parameters of a UWB filter. The measurement is conducted from 9 to 12 GHz with 151 points. For comparison, the measured results using a commercial VNA (Agilent N5230C) are also shown in the same figure. One can see the agreement between the two measured data is very good up to the dynamic range of 60 dB for S21. The measurement results of S11 are also in very good agreement.

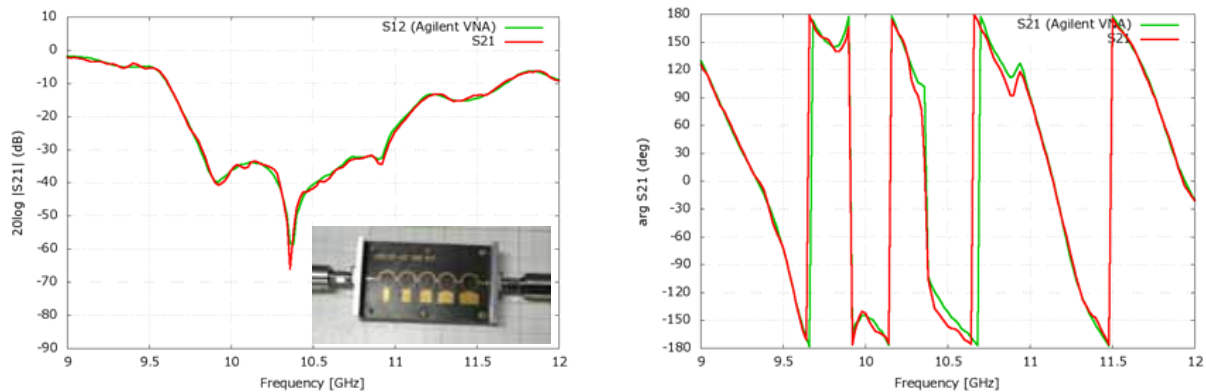


Fig. 6. S-parameters of a UWB filter measured by the developed six-port VNA and a commercial VNA

## 4. Six-Port Microwave Applications

For circumstances of the space, we will show you only two references as specific microwave applications of the six-port network, such as beam direction finding [6], Doppler frequency measurement [10].

An application of the proposed six-port based wave correlator to beam direction finding was discussed in [6]. Experimental proof at X-band frequency was carried out and satisfactory results were obtained. Another specific microwave application of the six-port wave correlator for a very low velocity measurement using the Doppler effect

was discussed in [10]. The experiment and simulation results at 10 GHz show that the target velocity around 0.2 mm/s has been successfully inferred, which proved the validity of the proposed scheme.

## 5. Conclusion

The six-port technology is a powerful measurement tool in microwave and millimeter-wave scientific fields. This lecture started with the basic concepts and fundamentals on both the six-port reflectometer, and the six-port wave correlator. Next, as one of useful applications, we focused on a new scheme for vector network analyzers, i.e., six-port reflectometer & wave correlator based VNA and Prototype MMIC six-port wave correlator based VNA. Finally, we explored some specific microwave applications of the six-port network, such as beam direction finding, Doppler frequency measurement.

## 6. Acknowledgments

The author wishes to thank Profs. Hatsuo Yabe, Fadhel M. Ghannouchi and Fengchao Xiao for their advice and support to his work, and Dr. Kohei Fujii for MMIC six-port wave correlator designs and trials.

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