A cost function expression for SDR multi-standard systems design using directed hypergraphs

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Abstract

The design of future multi-standard systems is very challenging. Flexible architectures exploiting processing commonalities of the different set of standards cohabiting in the device offer promising solutions. In this paper, graph theory aspects are introduced with the stress on the notion of directed hypergraphs which will help in showing how the graph structure of the multi-standard system can be viewed as a theoretical directed hypergraph. This graph description provides all the alternatives capable of implementing the design. However, a cost function needs to be defined to calculate the cost of each possible option chosen. Thus, we explain the sound concept of the computation process of a cost function introduced in previous papers, before transforming it into a formal expression with the aid of various definitions and notations of directed hypergraphs.

1 Introduction

Graph theory is rapidly moving into the mainstream of mathematics mainly because of its diverse applications in different fields. Graph theory [1] is the study of graphs used to model pairwise relations between objects from a certain collection. A "graph" in this context refers to a collection of vertices and a collection of edges that connect pairs of vertices. A graph may be undirected, meaning that there is no distinction between the two vertices associated with each edge, or its edge may be directed from one vertex to another in which case it is called a digraph. Hypergraphs [1] and directed hypergraphs [2] are generalizations of graphs and digraphs respectively.

Radio system designers nowadays focus their attention on developing flexible equipments that support as much air interface standards as possible in order to cope with the daily accelerating rate of technology innovation. The Software-defined Radio (SDR) concept [3] was emerging as a potential and efficient solution for making wireless networks highly adaptable and flexible. One of the possibilities to design software radio architectures is the *"parametrization"* approach [3], which is an optimal way of realizing a multi-standard terminal by identifying the appropriate common functions and operators between and inside the standards in which their behavior can be simply adjusted by a simple parameter modification. For this purpose, it is required to illustrate a graph structure representing the decomposition, into less and less complex operators, of each standard which has to be realized in the design [4]. Such a graph structure of a multi-standard system provides all the alternatives capable of implementing the design. However, after choosing a certain option of implementation, its cost has to be calculated via a certain suggested cost function.

The rest of this paper is organized as follows. First, formal definitions of hypergraphs and directed hypergraphs are presented with the focus on some basic definitions concerning directed hypergraphs, which are essential for our work. The next section describes the graph structure of the SDR multi-standard system theoretically as a directed hypergraph. This step includes the drawing of a graph that represents various design alternatives. In the subsequent section, first we address the cost function, reported in [5], which is accompanied with the specification of appropriate cost parameters to the different entities of the multi-standard graph structure. Afterwards, necessary derivations are performed on the calculation process of this cost function in order to eventually give it a theoretical form with the help of several definitions and notations of directed hypergraphs. Finally, a conclusion's section ends this paper.

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2 Definitions and Notations concerning Directed Hypergraphs

A hypergraph H [1] is defined by a pair $H = (V, E)$, where $V = \{x_1, x_2, \cdots, x_n\}$ is the set of vertices of H and $E = \{E_1, E_2, \cdots, E_m\}$, with $E_i \subseteq V$, $E_i \neq \phi$ for $i = 1, 2, \cdots, m$, denotes the set of hyperedges of H. A directed hypergraph is a hypergraph but with directed hyperedges, where a directed hyperedge e (or hyperarc) is an ordered pair $e = (X, Y)$ of (possibly empty) disjoint subsets of vertices; X is the tail of e denoted by $T(e)$ and Y is its head denoted by $H(e)$.
Let $v \in V$. The Forward Star and the Backward Star of node v are defined by $FS(v) = \{E_i \in E, v \in T(E_i)\}$ and $BS(v) = \{E_i \in E, v \in H(E_i)\}$ respectively.
Given a hyperarc $E_i = (T(E_i), H(E_i))$ in a directed hypergraph H, a BF-reduction of E_i is a hyperarc of the form $(\{x\}, \{y\})$ where $x \in T(E_i)$ and $y \in H(E_i)$.
In a directed hypergraph $H = (V, E)$ a path, of length q, from r to n $(r, n \in V)$ denoted by P_{rn} is defined by a sequence of nodes and hyperarcs $P_{rn} = (v_1 = r, E_{i1}, v_2, E_{i2}, v_3, \cdots, E_{iq}, v_{q+1} = n)$ such that: $r \in T(E_{i1})$, $n \in H(E_{iq})$ and $v_j \in H(E_{i(j-1)}) \cap T(E_{ij})$ $j = 2, \cdots, q$. This path is called an rn-path, where r is its origin and n is its destination. We'll denote $V(P)$ and $E(P)$ by the set of vertices and hyperarcs respectively traversed via the path P. Accordingly, we have $V(P_{rn}) = \{v_1, v_2, \cdots, v_{q+1}\}$ and $E(P_{rn}) = \{E_{i1}, E_{i2}, \cdots, E_{iq}\}$.
For more details and examples regarding the previous definitions, refer to [2].
Let P be a path in a directed hypergraph H & let $E_{ij} \in E(P)$. We'll define the BF-reduction via the path P of E_{ij} by its particular BF-reduction obtained by selecting the predecessor vertex to E_{ij} in the path P as its specific tail node and the successor vertex to E_{ij} in P as its head node. Let's denote $BF_P(E_{ij})$ by this BF-reduction of E_{ij} via P. Accordingly we get: $BF_{P_{rn}}(E_{ij}) = (\{v_j\}, \{v_{j+1}\})$ $j = 1, 2, \cdots, q$.
Suppose that we have a directed hypergraph $H = (V, E)$ in which a positive integer weight is assigned to every BF-reduction of all hyperarcs in E. Let P be a path from r to n $(r, n \in V)$, we'll denote the weight of P by the product of the weights on the BF-reductions via P of all the hyperarcs in $E(P)$. So we can write: $w(P) = \prod_{E_{ij} \in E(P)} w(BF_P(E_{ij}))$, where $w(P)$ denotes the weight of the path P and $w(BF_P(E_{ij}))$ stands for the weight of $BF_P(E_{ij})$ in H.

All the above definitions and notations are required in the following two sections. In section 3, they are helpful to present the graph structure of the SDR multi-standard system as a theoretical directed hypergraph. Whereas in section 4, they are used to provide a formal representation of the cost function proposed in [5], which computes the cost of any one of the alternatives illustrated in the graph structure of section 3.

3 Graph Modeling of a Multi-Standard system

The multi-standard reconfigurable system was represented as a graph with many different layers (or levels) in [4]. Each Processing Element (PE) occupies a certain layer depending upon its granularity level, where more complex PEs have higher granularity levels than less complex ones that can form their functionalities. The goal of this approach is to provide the options to the designer, to select a set of operators, each of them at the most appropriate level of granularity, as dictated by his needs.
A graph structure of two standards S & T is presented in Fig. 1. Each node represents an elementary PE. In order to perform the functionalities of this PE, it can be installed by itself in the design, as a unified nondivisible block, or it can be realized by some lower-level building blocks. A node of a higher level, called a parent node, may have dependencies with nodes of underlying levels, called descendant nodes. Two node dependencies occur between nodes of different levels. An "OR" dependency (direct arrow), as the hyperarc between $A2$ & $B1$ in Fig. 1, means that one descendant node ($B1$) called several specific times is necessary to implement the parent node ($A2$). However, an "AND" dependency ("inverted Y" connection), as the hyperarc between $A2$ and $B2$ & $B3$ in Fig. 1, signifies that all descendant nodes via the "AND" dependency ($B2$ & $B3$) are needed to implement the parent node ($A2$) accompanied with certain number of calls.
Formally speaking, the graph structure of a multi-standard system can be viewed as a directed hypergraph defined by the couple (V, E), where the set of vertices V includes the blocks (functions and operators) present in the graph structure (example $V = \{S, T, A1, A2, A3, B1, \cdots, D4, D5\}$ in the graph structure of Fig. 1)

and a directed hyperedge $e \in E$ will include the parent node as a tail node while all the necessary descendent node(s) capable of performing its task will form the head node(s) of e. So this means that whenever we have an "AND" dependency, the hyperarc is formed such that the parent node is the tail node and all the descendent nodes via this "AND" dependency are its head nodes. Whereas when faced with an "OR" dependency, the hyperarc will have the parent node as the tail node and only one of its descendent nodes (if more than one exists) via the corresponding "OR" dependency will be the head node. In this way, we form the set of hyperarcs E including all the "OR" and "AND" dependencies present in the corresponding graph structure. For instance, we have $(\{A2\}, \{B1\})$ and $(\{A2\}, \{B2, B3\})$ belong to the set of hyperarcs E of the directed hypergraph of Fig. 1.

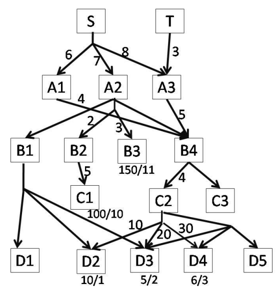

Figure 1: A multi-standard directed hypergraph. The arbitrary numerical values under the PEs represent BC/CC and on the BF-reductions are the NoCs. These costs are the only necessary ones to calculate the cost function in section 4.

4 The Cost Function

Some necessary cost parameters were considered in the cost function which is introduced in [5]. There was associated on each PE of the system a Building Cost (BC), which is the cost of the building PE capable of realizing a certain function and is just paid once independently of the number of times in which this PE is going to be called, and a Computational Cost (CC), which is considered to be the time taken by a PE to compute a function and it's calculated every time a PE is called by higher level PEs. Moreover, there was associated on every BF-reduction of each hyperarc a parameter called the Number of Calls (NoC), which is the necessary number of times which children nodes at lower levels will be called by their parent node to perform the corresponding function which is equivalent to the parent's node functionality. Note that in the following, the term weight will be used to represent the NoCs. For instance, the NoCs (3) on the BF-reduction $(\{A2\}, \{B3\})$ of the hyperarc $(\{A2\}, \{B2, B3\})$ will be expressed as $w(A2, B3) = 3$.

Suppose that the operators $D2, D3, D4, C1, \& B3$ were chosen to be installed in the design to implement S & T in Fig. 1. Then the cost of implementation via this choice (according to [5]) was calculated as follows:
Cost $= (((CC(D2)\times w(C2,D2)+CC(D3)\times w(C2,D3)+CC(D4)\times w(C2,D4))\times w(B4,C2))\times w(A3,B4))\times w(S,A3)+(((CC(D2)\times w(C2,D2)+CC(D3)\times w(C2,D3)+CC(D4)\times w(C2,D4))\times w(B4,C2))\times w(A1,B4))\times w(S,A1)+((CC(C1)\times w(B2,C1))\times w(A2,B2)+CC(B3)\times w(A2,B3))\times w(S,A2)+(((CC(D2)\times w(C2,D2)+CC(D3)\times w(C2,D3)+CC(D4)\times w(C2,D4))\times w(B4,C2))\times w(A3,B4))\times w(T,A3)+BC(D2)+BC(D3)+BC(D4)+BC(C1)+BC(B3)$
$= (((1\times 10+2\times 20+3\times 30)\times 4)\times 5)\times 8+(((1\times 10+2\times 20+3\times 30)\times 4)\times 4)\times 6+((10\times 5)\times 2+11\times 3)\times 7+(((1\times 10+2\times 20+3\times 30)\times 4)\times 5)\times 3+10+5+6+100+150$
If we expand the above factors we'll get:
$= CC(D2)\ [w(C2,D2)\times w(B4,C2)\times w(A3,B4)\times w(S,A3)\ +\ w(C2,D2)\times w(B4,C2)\times w(A1,B4)\times w(S,A1)]\ +\cdots+CC(D2)\ [w(C2,D2)\times w(B4,C2)\times w(A3,B4)\times w(T,A3)]+\cdots$
$= \{CC(D2)\ [\sum_{P\ SD2-path} w(P)]\ +\cdots+CC(B3)\ [\sum_{P\ SB3-path} w(P)]\}\ +\{CC(D2)\ [\sum_{P\ TD2-path} w(P)]\ +\cdots+ CC(D4)\ [\sum_{P\ TD4-path} w(P)]\}\ +BC(D2)+BC(D3)+BC(D4)+BC(C1)+BC(B3)$

$$= \sum_{x\ installed\ block} CC(x) \sum_{P\ Sx-path} w(P)\ + \sum_{x\ installed\ block} CC(x) \sum_{P\ Tx-path} w(P)\ + \sum_{x\ installed\ block} BC(x)$$

. So generally, we can write the cost function (CF) as follows:

$$CF = \sum_{y/BS(y)=\phi} \Big(\sum_{x\ installed\ block} CC(x) \sum_{P\ yx-path} w(P)\Big) + \sum_{x\ installed\ block} BC(x). \quad (1)$$

- $\sum_{x\ installed\ block} BC(x)$ represents the total sum of Building Costs of the installed blocks in the design.
- $CC(x) \sum_{P\ yx-path} w(P)$ is the necessary computational cost imposed by the installed block x responsible for realizing the standard y (y is a highest level block because $BS(y) = \phi$ so it's a standard).
- $\sum_{x\ installed\ block} CC(x) \sum_{P\ yx-path} w(P)$ stands for the total computational cost imposed by all the installed PEs x in the design to perform the functionality of the standard y.
- $\sum_{y/BS(y)=\phi} (\sum_{x\ installed\ block} CC(x) \sum_{P\ yx-path} w(P))$ represents the total computational cost paid for all the standards together.

5 Conclusion

We have theoretically presented the graph structure of the SDR multi-standard system as a directed hypergraph $H = (V, E)$ with V its set of vertices and E its set of hyperarcs. This approach aims at providing the options to the designer, to select the most appropriate levels of granularity as dictated by his needs. Furthermore, a formal equation for the cost function introduced in [5] has been derived which can calculate the cost once a designer has chosen a certain alternative of implementation.
In the future, we plan to search for an appropriate algorithm that can optimize the presented cost function to its minimum. This will enable to choose the most appropriate common operators from the most convenient granularity levels which will yield an improved flexible SDR multi-standard system. Moreover, the complexity of such an algorithm has to be examined in order to check its applicability difficulties.

6 Reference

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