Application of the Analytical Expressions of Fields to MOT Solution of Time Domain EFIE and CFIE

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Abstract

The effects of use of the exact closed-form expressions of the electric and magnetic fields (potentials) due to impulsively excited RWG basis functions on the Marching on-in-Time (MOT) solutions of the EFIE and CFIE are presented. The solutions via analytical expressions of fields are verified and compared with conventional (numerical) MOT solutions. It is shown that the accuracy and stability of the solutions obtained with the analytical-based method are better than those obtained with numerical-based conventional method. Also the dependency of the MOT solution of the time domain EFIE to time step size is investigated and it is shown that the analytical based solutions are less sensitive to selection of the time step size.

1. Introduction

In the marching-on-in-time (MOT) method, the stability is a major problem [1]. Recent studies show that the accurate calculation of the MOT matrix elements contributes to stability of the MOT solutions [2-5]. Accurate calculation of the matrix elements can be reached by the analytical expressions of the fields which are developed in [2-3]. In this study two main topics are investigated: (i) investigation of the benefits of using the closed-form expressions of the fields in the MOT solution of the CFIE and (ii) exploring the whether improved accuracy in the MOT matrix elements of the EFIE is sufficient for stability.

The analytical expressions of the electric and magnetic fields (and/or potentials) are first developed in [2-3] for impulsively excited Rao-Wilton-Clisson (RWG) bases [6]. The basic idea is that the spatial basis integral alone is a form of Radon Transform. This spatial integral, over the spatial basis function, can be regarded as electric and magnetic fields due to impulsively excited RWG basis function. As long as the temporal basis function is chosen as piece-wise polynomial functions, as in [7], it is possible to determine the temporal convolution of the analytical expressions of the fields and temporal basis function analytically. There are several advantages of the developed analytical formulae: (i) the analytical formulae do not depend on the time step size, there must not be any partitioning operation performed and analytical expressions of the impulsively excited fields can be used with any kind of temporal basis function. The convolution can be determined either numerically or analytically. (ii) If the temporal basis function consists of monomials, temporal convolution can be determined analytically and the fields due to related basis functions can be determined analytically. (iii) The analytical expressions of the potentials due to impulsively excited RWG currents do not have any apparent singularity. Also once the temporal convolution is determined analytically; there is no need to perform any singularity treatment for magnetic field.

With the analytical expressions of the magnetic field, it is shown in [4] that the stability of the MOT solution of magnetic field integral equation (MFIE) is improved. However effects of the inner resonance problem are still seen in the solution. In this study, the effects of the analytical expressions are investigated for time domain EFIE and CFIE. It will be shown that with the use of the analytical formulae, accuracy and stability of the MOT solution of the CFIE will be improved, as in [4]. In the application of the analytical expressions to the MOT solution of EFIE, the dependency of the solution to selection of time step size is investigated. For this case, it will be shown that analytical-based solutions of EFIE are more immune to the selection of time step size. Also an adverse example will be shown that the analytical-based solution of EFIE can be unstable. For the same case numerical-based solution of EFIE is corrupted (although stable). It can be concluded that the accuracy of the MOT matrix elements for EFIE does not guarantee stability.

2. Determination of MOT Matrix Elements

MOT matrix elements for time domain surface integral equations contains five dimensional integrals, i.e., two surface integrals over spatial basis function and testing function, one temporal integral related to convolution. In the

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conventional MOT algorithm, first the temporal convolution is evaluated analytically, and then two surface integrals are determined numerically. However, in the proposed method, first the surface integral over spatial basis function is evaluated analytically via Radon Transform approach. Then the convolution integral is determined analytically, if the temporal basis functions are consist of monomials. At last the integral over testing function is calculated numerically.

In the MOT method, the unknown current density is discretized with spatial and temporal basis functions as

$$\mathbf{J}(\mathbf{r},t) = \sum_i \sum_n I_{n,i} T_i(t) \mathbf{f}_n(\mathbf{r}) \tag{1}$$

where $\mathbf{f}_n(\mathbf{r})$ is the n[th] spatial basis function, $T_i(t)$ is the i[th] temporal basis function and $I_{n,i}$ are the unknown coefficients. In this study, spatial basis functions are chosen as RWG basis functions [6] in space and temporal basis functions are chosen as piece-wise polynomial interpolation functions [7]. The MOT matrix elements for time domain EFIE and MFIE can be given as

$$\begin{aligned} Z_{mn,ji}^{EFIE} &= -\frac{\mu}{4\pi} \int_{S_m} \mathbf{f}_m(\mathbf{r}) \cdot \left[\partial_t^2 T_i(t) * \mathbf{A}_n(\mathbf{r},t) \right]_{t=t_j} d\mathbf{r} \\ &\quad -\frac{1}{4\pi\varepsilon} \int_{S_m} \nabla \cdot \mathbf{f}_m(\mathbf{r}) \left[T_i(t) * \phi_n(\mathbf{r},t) \right]_{t=t_j} d\mathbf{r}, \end{aligned} \tag{2}$$

$$\begin{aligned} Z_{mn,ji}^{MFIE} &= \frac{1}{2} \int_{S_m} \mathbf{f}_m(\mathbf{r}) \cdot \mathbf{f}_n(\mathbf{r}) \partial_t T_i(t_j) d\mathbf{r} \\ &\quad - \int_{S_m} \mathbf{f}_m(\mathbf{r}) \cdot \hat{\mathbf{n}} \times \left[\partial_t T_i(t) * \mathbf{H}_n(\mathbf{r},t) \right]_{t=t_j} d\mathbf{r} \end{aligned} \tag{3}$$

where $\mathbf{E}^i(\mathbf{r},t)$ and $\mathbf{H}^i(\mathbf{r},t)$ is the incident electric and magnetic field, respectively, ∂_t denotes temporal derivative, $*$ denotes convolution with respect to time, $\hat{\mathbf{n}}$ is the outward unit normal vector of the scatterer surface S at the observation point $\mathbf{r}$, ε is the permittivity, μ is the permeability of the surrounding medium, and c is the speed of light. $\mathbf{f}_m(\mathbf{r})$ is m[th] testing (weighting) function. In (2) and (3), $\mathbf{A}_n(\mathbf{r},t)$, $\phi_n(\mathbf{r},t)$ and $\mathbf{H}_n(\mathbf{r},t)$ are the magnetic vector potential, electric scalar potential and magnetic field due to impulsively excited n[th] basis function, respectively. Since the RWG basis function is defined on a pair triangle ($S_n = S_n^+ + S_n^-$), $\mathbf{A}_n(\mathbf{r},t) = \mathbf{A}_n^+(\mathbf{r},t) + \mathbf{A}_n^-(\mathbf{r},t)$ and $\phi_n(\mathbf{r},t) = \phi_n^+(\mathbf{r},t) + \phi_n^-(\mathbf{r},t)$. In accordance with Figure 1 and Figure 2, analytical expressions of $\mathbf{A}_n^\pm(\mathbf{r},t)$ and $\phi_n^\pm(\mathbf{r},t)$ can be determined as

$$\mathbf{A}_n^\pm(\mathbf{r},t) = \frac{\pm c l_n}{2A_n^\pm} \left[\mathbf{e}^\pm(\mathbf{r},t) + \left(\rho - \rho_n^\pm \right) \alpha^\pm(\mathbf{r},t) \right], \tag{4}$$

$$\phi_n^\pm(\mathbf{r},t) = \pm \frac{c l_n}{A_n^\pm} \alpha^\pm(\mathbf{r},t), \tag{5}$$

where $\mathbf{e}^\pm(\mathbf{r},t)$ is the bisecting vector and $\alpha^\pm(\mathbf{r},t)$ is the arc length for patch $S_n^\pm$. Closed-form expressions of $\mathbf{e}^\pm(\mathbf{r},t)$ and $\alpha^\pm(\mathbf{r},t)$, as well as the analytical expression of $\mathbf{H}_n(\mathbf{r},t)$, are derived in [3]. Once the analytical expressions for (4) and (5) are determined, the determination of the convolution integrals is straight forward, and described in [2-3].

The MOT matrix elements for CFIE can be determined by a linear combination of the matrix elements of EFIE and MFIE, given in (2) and (3), respectively:

$$Z_{mn,ji}^{CFIE} = \nu Z_{mn,ji}^{EFIE} + (1-\nu)\eta Z_{mn,ji}^{MFIE} \tag{6}$$

where $\nu \in [0,1]$ is the combination coefficient, $\eta = \sqrt{\mu/\varepsilon}$ is the intrinsic impedance of the surrounding medium.

3. Numerical Results

In this section, the aforementioned issues, improvement of the stability in the CFIE, the increased accuracy in the MOT matrix elements of EFIE is not sufficient for stability, and the dependency of the EFIE solution to time step size will be shown with numerical examples. In numerical examples, the incident wave is chosen as a modulated Gaussian plane wave:

$$\mathbf{E}^i(\mathbf{r},t) = \hat{\mathbf{p}} \cos\left[2\pi f_0 (t - \mathbf{r}\cdot\hat{\mathbf{k}}/c)\right] e^{-\frac{(t-t_p-\mathbf{r}\cdot\hat{\mathbf{k}}/c)^2}{2\sigma^2}} \tag{7}$$

where f_0 is the center frequency, $\hat{\mathbf{k}}$ is direction of travel and $\hat{\mathbf{p}}$ the is polarization of the incident wave. $\sigma = 7/(2\pi f_{bw})$, $t_p = 3.5\sigma$, and f_{bw} will be referred to as the effective bandwidth. In the examples, while evaluating the numerical integrals, the triangles are divided to sub-triangles that have dimensions much smaller than the minimum wavelength, and implementing the 7-point Gauss quadrature rule over these sub-triangles. In the examples, "A" denotes the results based on analytical expressions of the fields, proposed in this study, "N" denotes results obtained by the conventional numerical based MOT solution.

In the first example, scattering from a sphere with radius 1 m, discretized with 290 triangular patches, is investigated. The frequency properties of the incident wave are 75 MHz and 50 MHz. The other properties of the incident wave are chosen as in the previous example. In Figure 3, one of the self term elements of the impedance matrix of EFIE is given for different time step sizes. It can be seen that MOT matrix elements obtained with analytical and numerical based methods for EFIE is diverging with the decrease in the time step size. In Figure 4 solutions for current density is shown for different time step sizes. It can be seen that the A-EFIE results are coherent even if the time step size decreases, whereas N-EFIE results become corrupted.

In the second example, a NASA almond with maximum height of 0.0575 m and maximum width of 1.15 m, discretized with 704 triangular patches is analyzed. The properties of the incident field are chosen as $\hat{\mathbf{k}} = (-1,0,0)$, $\hat{\mathbf{p}} = (0,1,0)$, 200 MHz, and 150 MHz. As seen in Figure 5, the CFIE results do not coincide and N-CFIE result is growing exponentially with respect to time.

4. Conclusions

Effects of the application of analytical expressions of the electric and magnetic field are investigated. It can be concluded that using analytical expressions of the fields improves the stability of the EFIE and CFIE, also analytical based EFIE solutions are less sensitive to time step size selection. More numerical examples will be presented to show the claims in the paper.

5. References

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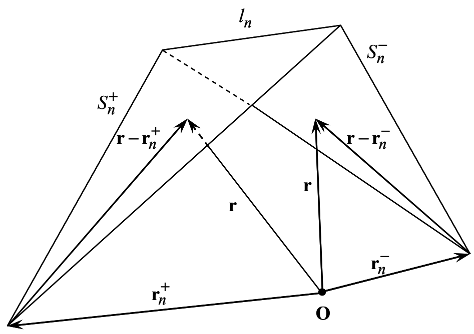


Figure 1. Definition of the RWG basis.

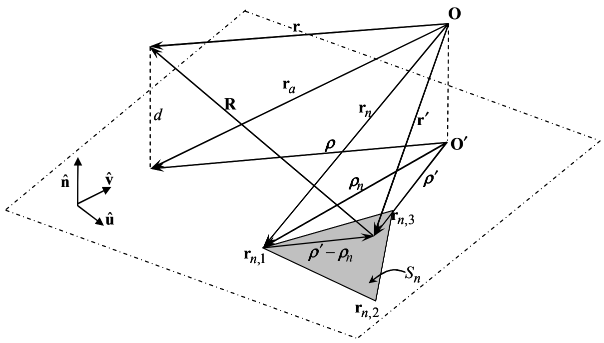
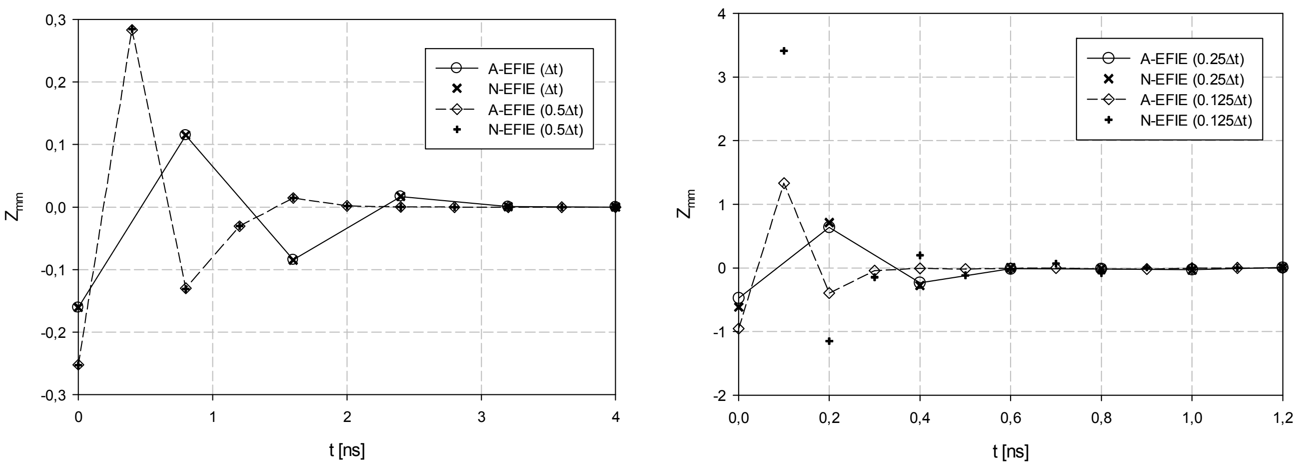


Figure 2. Vector definitions for a triangular patch.



(a) (b)

Figure 3. MOT matrix elements of EFIE for different time step sizes.

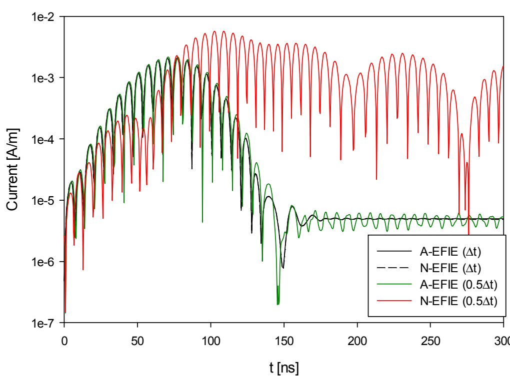


Figure 4. EFIE results for different time step sizes.

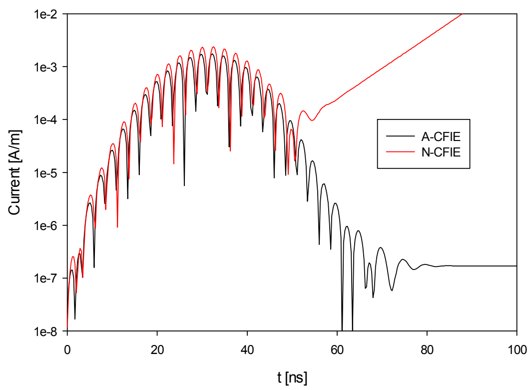


Figure 5. CFIE solution of NASA almond.