The Concave Parabolic Mirror

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Abstract

The diffraction of a plane electromagnetic wave axially incident on the concave side of a perfectly conducting parabolic reflector is solved exactly and in closed form in frequency domain. The method of solution follows the technique originally proposed by Horace Lamb in 1906 who, however, was unable to solve the problem addressed herein. The present solution is obtained by matching the left- and right-propagating fields at the focal line of the mirror.

1. Introduction

The solution to two-dimensional scattering by a perfectly conducting parabolic cylinder is known in terms of infinite series of parabolic-cylinder functions when the primary field is either a plane wave incident on the convex side of the cylinder or a line source parallel to the cylinder generators and located on either the convex or the concave side of the cylinder (see [1] and references therein). In revisiting Sommerfeld's problem of diffraction by a half-plane, Lamb [2] obtained a closed-form solution for a time-harmonic plane wave symmetrically incident on the convex side of a parabolic mirror, in which case the focal line of the mirror acts as a virtual source for the field reflected by the mirror. Problems involving a penetrable parabolic-cylinder interface separating two different uniform isotropic media that are either isorefractive or anti-isorefractive to each other have been solved recently by using Lamb's technique, but again only for those cases in which there is no actual focusing of electromagnetic energy at the focal line of the mirror [3-5]. Lamb was unable to solve the more difficult problem of symmetrical incidence on the concave side of the mirror, and stated explicitly that such a case could not be solved by his method. A solution to Lamb's unsolved problem is obtained herein. The analysis is conducted in frequency domain with either the electric or the magnetic field parallel to the generators of the mirror's surface.

2. Solution of the Concave Problem

It must be understood that a concave parabolic mirror is not a truly open surface; rather, it is the limit of a closed elliptic cylinder when the interfocal distance between the focal lines tends to infinity while the difference between the major axis and the interfocal distance remains finite. Thus, any primary source is really located inside a closed surface and the only possible direction for plane wave incidence is that perpendicular to the generators and joining the focal line to the vertex line of the mirror. This plane wave is everywhere affected by the presence of the reflecting mirror. The application of Lamb's technique yields the superposition of two fields. The first field consists of a plane wave and the product of that same plane wave times a Fresnel integral, and propagates symmetrically toward the mirror; it can be interpreted as a plane wave superimposed to its reflection at the mirror's surface that produces a field imploding on the focal line. The second field consists of a plane wave propagating symmetrically away from the mirror plus the product of that same plane wave times a Fresnel integral; it can be interpreted as a field radiating from the focal line and producing a plane wave after reflection at the mirror. Obviously, this second field is what the first field becomes after crossing the focal line, so the two fields must be connected. Lamb did not succeed in connecting these two fields, and in fact stated that "it is not possible to combine {the two fields} so as to represent the case of an originally parallel beam reflected *through* the real focus and returning finally, after a second reflection, back on its original course." In this work, the solution is obtained by imposing the boundary condition for each field at the mirror's surface and the additional condition that the two fields be identical at the focal line. All integration constants are determined exactly. As verification, it is shown that at large distances from the mirror, the first and second fields tend asymptotically to two plane waves propagating in opposite directions and having the same amplitudes, thus ensuring energy conservation.

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3. References

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