A comparison of implementations of a combined charge and current formulation of the method of moments

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1 Introduction

The *Method of Moments* (MoM, also referred to as the *Boundary Element Method* outside of electromagnetics [1]) is a well-known and oft-used technique for complex electromagnetics simulations based on a numerical approach to Maxwell's equations. Results can be very accurate, but as it requires solving a dense matrix equation it is very computationally intensive. MoM has traditionally been used for narrow-band antenna simulations, but thanks to advances in processing power and in the technique itself, it is slowly becoming feasible to perform simulations of electrically large objects and of diverse scenarios over a broad frequency band.

In [2], a variant of the MoM is proposed that uses both the *Electric* and *Magnetic Field Integral Equation* (E- and MFIE) in order to improve the low frequency stability. In implementing this method, however, we repeatedly ran into difficulties with the discretization step (basis and testing functions). Even using point matching, in theory a very simple technique, can lead to inaccuracies if care is not taken. These inaccuracies and their solutions and/or alternatives will be discussed further.

2 Testing Functions

The form of MoM used is shown in eqn. (1). Here, the MFIE is used to calculate the induced current. The modified EFIE is then used to calculate the induced charge. The Lorenz gauge $\nabla \cdot \mathbf{A} = -j\omega\mu\epsilon\phi$ is not used, and thus instability due to any $1/\omega$ terms is avoided. The current and charge can be found for any closed surface and at any frequency, and from this the radiated electric and magnetic field can be calculated in a stable manner.

$$\mathbf{E} = -\left(j\omega\mathbf{A} + \nabla\phi\right) \tag{1}$$

$$\mathbf{A}(\mathbf{r}) = \mu \int_S \mathbf{J}_s(\mathbf{r}')G(\mathbf{r},\mathbf{r}')\, dS' \tag{2}$$

$$\phi(\mathbf{r}) = \frac{1}{\epsilon} \int_S \rho_s(\mathbf{r}')G(\mathbf{r},\mathbf{r}')\, dS' \tag{3}$$

When implementing this *charge and current* based EFIE, choosing appropriate basis and testing functions is non-trivial. The standard testing technique—Galerkin—is no longer immediately applicable, as the charge is discretized with a different (scalar) basis function compared to the current. A scalar basis function could be found with gradients similar to a RWG [1] function, thus fitting both the $\mathbf{A}$ and the $\nabla\phi$ term, but attempts have not been positive so far. After searching in various directions, point matching was selected for efficiency of implementation to allow us to prove the validity of the method.

Point matching, although not always as accurate as other methods, usually results in a simple and well-conditioned matrix. Here, the scalar product of (1) with an appropriate vector is made to hold in the triangle center. Either the surface normal or surface tangent may be used, the second giving a rectangular matrix if both orthogonal tangential vectors are used. A square matrix is more desirable, but as will become apparent normal matching is not always the best, or even a possible choice.

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2.1 Normal Point Matching

It should be immediately apparent that normal point matching cannot be used to solve the magnetic equation when the current is discretized using an edge basis function such as the well-known RWG function. The number of surface current unknowns would outnumber the number of tested equations, giving an infinite number of solutions. The electric field, on the other hand, can be tested with normal matching, given the appropriate boundary condition $\mathbf{n} \cdot \mathbf{E}_{total} = \rho/\epsilon$. The total field is discontinuous at the surface, so care must be taken to evaluate the EFIE in the limit approaching from the outside of the surface, similar to what is done with the MFIE. The self term is constant, almost eliminating the need for special handling of the $1/r^2$ term. Results are close to the analytical solution.

Applying normal matching results in eqn. (4). The vector potential $\mathbf{A}$ is tangential to the surface, and will therefore be zero for the self term of the matrix. The gradient of the scalar potential ϕ is also tangential for all points outside of the match point itself, and therefore the only term that needs special handling is the limit of the singular potential gradient as the match point approaches the surface. This limit can be found analytically for the self term and is equal to $-\rho/(2\epsilon)$, a result analogous to that found for the current when deriving the MFIE. The right hand side of (4) then reduces to (5), with $\mathbf{r}_n$ the center of the circle inscribed in triangle n and using a block basis function for the charge. Eqn. (5), being non-singular, can be evaluated numerically.

$$\mathbf{n} \cdot \mathbf{E}_i(\mathbf{r}) - \lim_{\mathbf{r} \to S} j\omega \mathbf{n} \cdot \mathbf{A}(\mathbf{r}) = \frac{1}{\epsilon}\rho(\mathbf{r}) + \lim_{\mathbf{r} \to S} \mathbf{n} \cdot \nabla \phi(\mathbf{r}) \tag{4}$$

$$\hat{E}_i\left(\mathbf{r}_n\right) = \frac{1}{2\epsilon}\rho_n + \frac{1}{\epsilon}\sum_{m \neq n} \rho_m \int_{S_m} \mathbf{n}_n \cdot \nabla G(\mathbf{r}_n, \mathbf{r}')\, dS' \tag{5}$$

2.2 Tangential Point Matching

Tangential matching may be applied to both the E- and MFIE. For the magnetic field, the resulting integral is similar to but simpler than that for Galerkin matching. The electric field will be continuous in this case, so no limit is needed when approaching the surface. The self term is nonzero, however, as the gradient of (3) lies in the surface, so the $1/r^2$ term must be extracted and dealt with. The analytical equations are stable in terms of frequency, but the numeric implementation becomes inaccurate at low frequencies.

A first step in handling the singularity is recognizing that the gradient of the Green's function is directed towards the match point. ρ being constant over the triangle, this means that the integral over the inscribed circle of the triangle (the *incircle*) with a match point placed in its center will be zero. Only the corner areas will contribute to the integral.

Equation (3) can be evaluated analytically for these corner areas with the help of [3] and an approximation for the exponent. Switching to polar coordinates centered in the inscribed circle reduces the integral to (6), with α the angle of the corner, $\mathbf{i}_b$ the unit vector along the angle bisector and d_{min} and d_{max} the distance to the edge and to the corner respectively. Simplifying further, evaluating the integrals analytically where possible, and using a series expansion of e^x, the final expression is show in (7), with the infinite sum evaluated until the additional term is below machine precision. Bringing the factorial into the recursive evaluation of V_n can also help to reduce precision errors.

$$I = \frac{\rho}{2\pi\epsilon}\mathbf{i}_b \int_{d_{min}}^{d_{max}} e^{-\gamma R}\left(\frac{1}{R} + \gamma\right)\sin\left(\theta\left(d\right)\right)\, dR \tag{6}$$

$$= \frac{\rho}{2\pi\epsilon}\mathbf{i}_b \left[d_{min}\cos\frac{\alpha}{2}\left(\frac{e^{-\gamma d_{min}}}{d_{min}} - \frac{e^{-\gamma d_{max}}}{d_{max}}\right) - \sin\frac{\alpha}{2}\left(V_0 + \sum_{n=2}^{\infty}\frac{\left(-\gamma\right)^n\left(1-n\right)}{n!}V_n\right)\right] \tag{7}$$

$$V_n = \frac{\left(d_{max}\right)^n \cos^3\frac{\alpha}{2}}{n} + \frac{\left(d_{min}\right)^2\left(n-3\right)}{n}V_{n-2} \qquad \text{with} \qquad V_0 = \ln\left(\frac{1+\cos\frac{\alpha}{2}}{\sin\frac{\alpha}{2}}\right) - \cos\frac{\alpha}{2} \tag{8}$$

Unfortunately, a direct implementation of equation (7) has so far suffered from convergence problems at low frequency. At low frequency the exponent terms will grow excessively large and overflow, implying that an alternate method of computation, such as inverse recursion, is required.

2.3 Comparison

Figure 1 shows the condition (the ratio of the extreme singular values) of the two matrices for various types of matching. Normal matching cannot be applied to the MFIE, so Galerkin is shown instead. The "Mixed" plot shows the result for using both normal and tangential matching together. Also shown as a solid line is the stability of using mixed matching and solving the two equations together in one step, as this is similar to what will happen with dielectrics.

The results are fairly clear: Galerkin is (as known) the best for the MFIE, with tangential matching a close second. This is not unexpected, as the vector potential induced in a point by the self term and nearby basis functions is predominantly tangential. In contrast, surface normal and mixed testing work better for the EFIE. Using the mixed formulation is a good compromise when it is required that both equations are tested in the same way.

Figure 2 gives an indication of the accuracy of the EFIE for normal and tangential matching. Note the chaotic result for tangential matching, specifically the alternation between positive and negative (green and blue) charge density. This rapid oscillation where the result should be more smooth indicates a problem with this solution. That tangential matching does not give a good solution can be seen far more clearly in the bistatic radar cross section, shown in figure 3. Only the EFIE is shown here, as the MFIE always closely matched the shown analytic solution taken from [4]. Note the good match for the normal and mixed methods.

3 Conclusion

A new implementation of an alternate EFIE has been presented. Some of the major implementation difficulties have been discussed, and solutions have been presented. Results have then been shown for simulations of a perfectly conducting sphere. The implementation appears to be stable and the charge and current distribution appear for the most part to be as expected. The scattered field also agrees with known analytic solutions. The accuracy has therefore been verified at the very least for smooth perfectly conducting surfaces.

Acknowledgment

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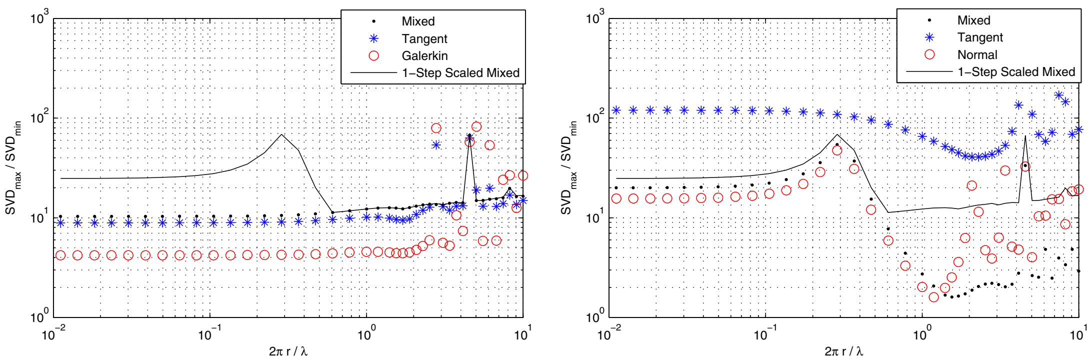
References

[1] R. F. Harrington, D. R. Wilton, C. M. Butler, R. Mittra, and C. L. Bennett, *Lectures on Computational Methods in Electromagnetics*. The Sceee Press, 1981.

[2] J.-w. De Bleser, E. Van Lil, and A. Van de Capelle, “An alternative technique for stabilizing the boundary elements method in electromagnetics,” in *European Conference on Antennas and Propagation*, Barcelona, Apr. 2010.

[3] I. Gradštejn and I. Ryžik, *Table of integrals, series, and products*, A. Jeffrey and D. Zwillinger, Eds. Academic Press, 2000.

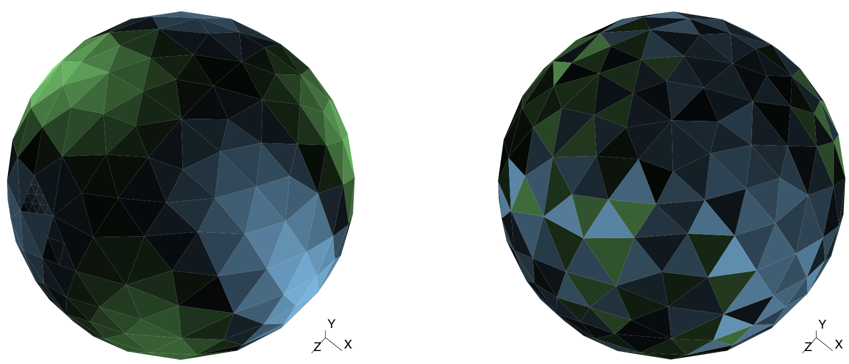
[4] J. A. Kong, *Electromagnetic Wave Theory*. EMW Publishing, 2000.



(a) Current

(b) Charge

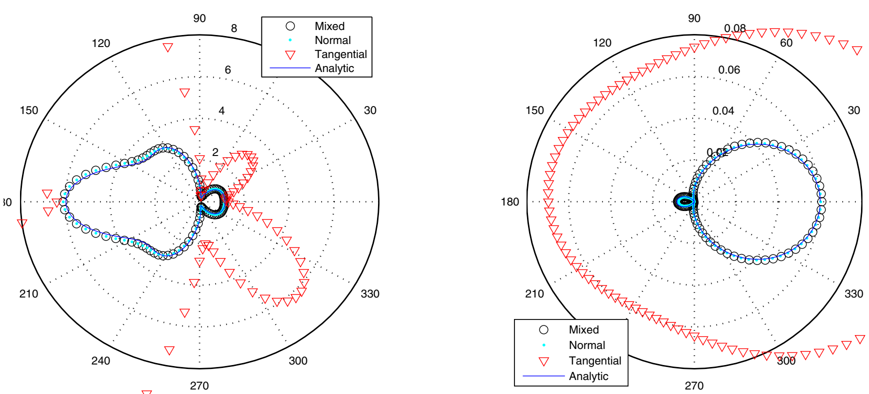
Figure 1: Condition of the MoM matrix of a sphere with different types of testing function



(a) Normal

(b) Tangential

Figure 2: Surface charge for $\lambda = 2.4r$ with different types of testing function



(a) $\lambda = 2.4r$

(b) $\lambda = 20r$

Figure 3: Bistatic RCS for two different wavelengths with different types of testing function