# Transformation Electromagnetics in Antenna Engineering: Theory and Implementation

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## Abstract

Current designs of electromagnetic cloaks are largely based on the use of metamaterials and a technique called "transformation optics/electromagnetics". Free space cloaks require materials with extreme properties and, hence, they are difficult to implement in practice. However, the theory of "transformation optics/electromagnetics" offers a useful design tool for antenna engineers, and enables them to develop novel antennas. In this paper, we will review some research activities at Queen Mary, University of London, regarding applications of transformation electromagnetics in the antenna and microwave engineering. Design examples such as flat reflectors, lenses and sub-wavelength antennas will be introduced. Novel FDTD techniques to deal with the design of gradient index metamaterials will be also demonstrated and used to evaluate the performance of transformation-based antennas. In particular, the tradeoff in antenna performance whether or not metamaterials are required in the design will be discussed.

## 1. Introduction

Electromagnetic Transformation utilizes a symmetry property of EM fields embodied in the fact that Maxwell's equations are form-invariant under coordinate transformations. The power of Electromagnetic Transformation is that it may be used to specify the required electromagnetic properties of a material in order to control in a predefined manner the path of an electromagnetic beam. These specifications typically require anisotropic and exotic material parameters that can only be achieved by the fabrication of novel materials, but lead to novel ways to control "electromagnetic space". In addition to the well-publicised cloaking phenomenon [1], [2], other applications of transformation electromagnetics include all-angle absorbers [3] and retro-reflectors [4], reflection-less beam shifters and beam-splitters [5], and "flat" parabolic mirrors [6]. Many of these applications have been also confirmed by numerical simulations [7, 8]. Some recent work at Queen Mary, University of London demonstrates how electromagnetic transformation concepts can be employed by distorting EM space using an all-dielectric approach [9]. In this paper, some design examples such as flat reflectors, lenses and sub-wavelength antennas will be presented. They are characterized by using gradient index metamaterials, based on both numerical modeling and experiments. In particular, the tradeoff in antenna performance whether or not metamaterials are required in the design will be discussed.

## 2. Theoretical Background

Let's assume a 3D volume of space described by a Cartesian coordinate system (x, y, z), and a second volume described by a distorted coordinate system (x′, y′, z′). 2D cut-planes in those spaces are shown in Fig. 1. The two coordinate systems are related through a general transformation function x′ = x′(x, y, z), y′ = y′(x, y, z), z′ = z′(x, y, z). Assume that, in the original space (x, y, z), it contains materials having $\overline{\overline{\varepsilon}}$ and $\overline{\overline{\mu}}$. According to the theory of transformation electromagnetics [1], the resulting permittivity and permeability tensors in a distorted coordinate system (x′, y′, z′) are given by

$$\overline{\overline{\varepsilon}}' = \frac{J\overline{\overline{\varepsilon}}J^T}{\det(J)} \quad , \quad \overline{\overline{\mu}}' = \frac{J\overline{\overline{\mu}}J^T}{\det(J)} \tag{1}$$

where $J$ is the Jacobian transformation matrix between the two coordinate systems, defined as

$$J = \begin{pmatrix} \frac{\partial x'}{\partial x} & \frac{\partial x'}{\partial y} & \frac{\partial x'}{\partial z} \\ \frac{\partial y'}{\partial x} & \frac{\partial y'}{\partial y} & \frac{\partial y'}{\partial z} \\ \frac{\partial z'}{\partial x} & \frac{\partial z'}{\partial y} & \frac{\partial z'}{\partial z} \end{pmatrix} \tag{2}$$

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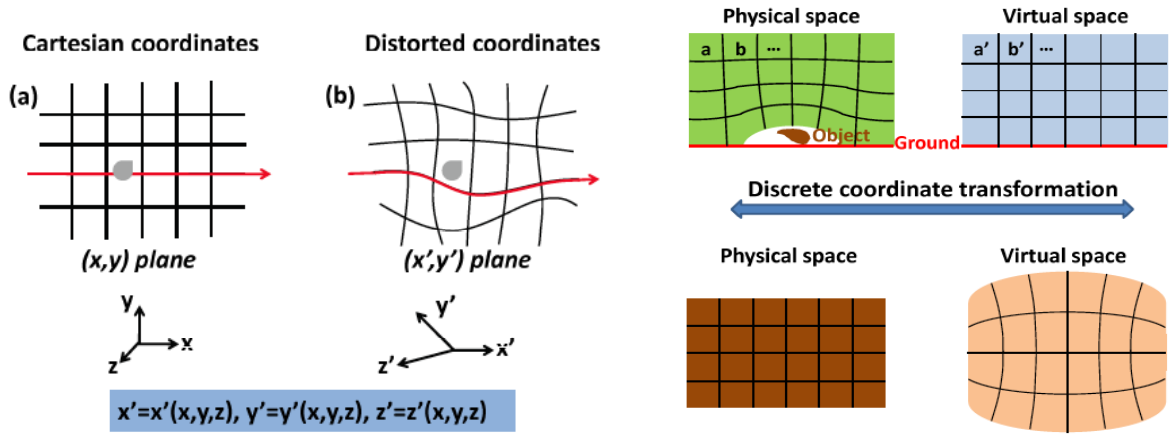


Fig. 1. Two coordinate systems. (a) The Cartesian coordinates and (b) the distorted coordinates. The incident electronic waves travel from point to point in both the two coordinates, but their routes are different. In the distorted coordinate, the wave will not reach the grey object.

In fact, the underlying theory of transformation electromagnetics is quite familiar to the computational electromagnetics community. In 1983, Holland [10] modified the classic Yee's Finite-Difference Time-Domain (FDTD) algorithm in the generalized non-orthogonal coordinates and it was identified as Nonorthogonal FDTD (NFDTD). In NFDTD, the electric and magnetic fields are analysed using their covariant and contravariant components respectively. The scheme has been refined by several groups including Lee [11] and Hao [12] *et al*. The relationship between contravariant and covariant components of the electric and magnetic fields can be described using metric tensors $\left(\overline{\overline{g_{ij}}}\right)$ and its determinant ($g$) of each distorted cell in the NFDTD grid. In [6], a detailed discussion on how to relate the metric tensor to equations (1) and (2) was presented. It should be noted that one can devise *all dielectric* materials to control the E-polarized waves, as long as the grid in the distorted space is quasi-orthogonal. In the following section, several design examples will be introduced.

## 3. Design Examples

### 3.1 Flat "Parabolic" Reflectors

The parabolic reflector antenna is one of the most widely applied antennas. In this section, an all-dielectric flat reflector is designed from the discrete coordinate transformation as an example. A sample conventional reflector is

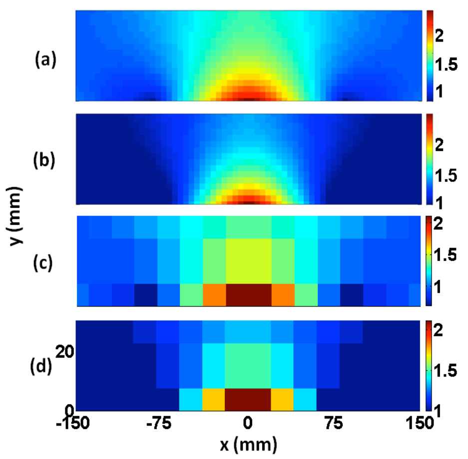


Fig. 2. (a) Permittivity map consisting of 64×16 blocks, (b) 64×16 blocks, without less-than-unity values, (c) 16×3 blocks and (d) 16×3 blocks, without less-than-unity values.

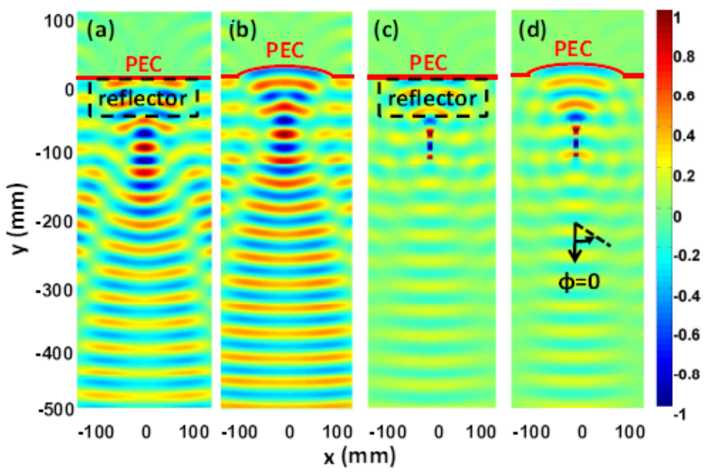


Fig. 3 The real part of the Ez field at 8 GHz. (a) A plane wave illuminates to a flat reflector. The focal length is 102.6mm. (b) A plane wave illuminates to a conventional reflector. The focal length is 102.7mm. A horn antenna is also applied at the focal point to feed the (c) flat reflector and (d) conventional reflector.

designed to working around 10GHz with an aperture of 180 mm (illustrated as the PEC in Fig. 2(b)). Coordinate transformation is applied in a region of 300mm×75 mm, and the relative permittivity distribution is given in Fig. 2. Reasonable simplifications and approximations are employed to remove the less-than-unity permittivity values and reduce the flat reflector into a cluster of dielectrics [9], as shown in Fig. 2(d), which is much more realizable. FDTD method based simulations are used to compare the performances of the flat reflector with the conventional parabolic one and the electric field distributions are plotted in Fig. 3. The two reflectors have very similar focal lengths with the incidence of a plane wave, as shown in Fig. 3 (a) and (b). When a horn source is located at their focal points, quasi-plane waves are observed in (c) and (d). This performance indicates that the flat reflector can excellently transform an incident spherical wave into a plane wave as the conventional one, and so as to achieve highly directive radiations. The bandwidth of the flat reflector is also tested. It has good directivities from 4 GHz to 12 GHz, meanwhile the reflection coefficient is kept below -20dB over this frequency band. These numerical simulation results have verified that the all-dielectric flat reflector have similar broadband performances compared to the conventional ones, while possessing the advantages of flat profiles and easy construction.

## 3.2 Lens Antennas

In this section, we extend the above principle to the compressed flat lens design [15]. As shown in Figs. 4(a)-(c), 14×2 conventional dielectric blocks instead of metamaterials with simultaneously dispersive permittivity and permeability are employed to the flat lens aperture. Building on the merits of such an efficient transformation, a potential realization using a microstrip array as depicted is carried out. We show that the microstrip array (Fig. 4(d)) presents considerably the same performance as the conventional convex lens and can function well over a broad frequency bandwidth (Fig. 4(e)).

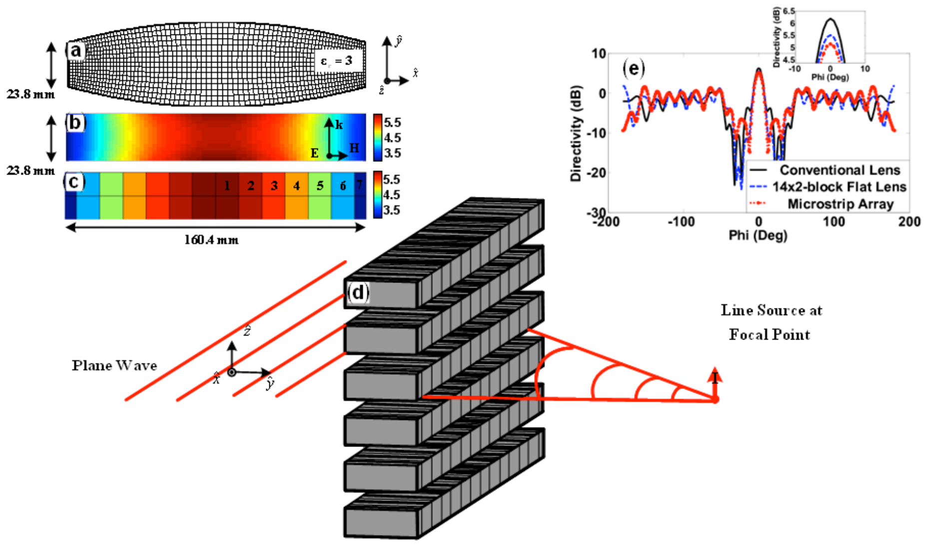


Fig. 4. Coordinate transformation for the convex and flat lens design and microstrip array mimic. (a) Convex lens with nearly orthogonal mapping. (b) Flat lens with the permittivity map consisting of 80×15 blocks. (c) Flat lens with the permittivity map consisting of 14×2 blocks. (d) Schematic showing a line source located at focal point of the microstrip array. (e) Radiation patterns of the conventional convex lens, the simplified 14×2-block flat lens and the flat lens based on a mictrostrip array at 8 GHz.

## 3.3 Enhanced Transmission from a Single Subwavelength Slit

There have been intensive studies on different technologies for guiding electromagnetic waves through sub-wavelength apertures, channels or waveguides in recent times. The theory of coordinate transformation provides a novel way to obtain extraordinary transmission (ET) over a broad frequency band. The main scheme to enhance the transmission is to build a physical space containing a sub-wavelength aperture (see Fig. 5 (a)), and a virtual space containing a much larger aperture comparable to the wavelength. By engineering the background medium in the physical space, the two spaces have the same electromagnetic properties to the environment, which means, the transmitted energy to the outer space through the sub-wavelength aperture is similar to that through the large aperture. In this way, the transmission is increased dramatically [13]. Discrete coordinate transformation is utilized to decide the permittivities of the background medium in the physical space, and the space is further reduced to be a small device composed of a few isotropic dielectrics as shown in Fig. 5 (b). The ET device is experimentally demonstrated in an X band waveguide [14] and the measured results in Fig. 5 (d) have proved that the device can provide transmission with the -3 dB bandwidth over more than 1 GHz, in a region which would otherwise be a stop band due to the introduction of a copper plate with a sub-wavelength slit in the cross section of the waveguide.

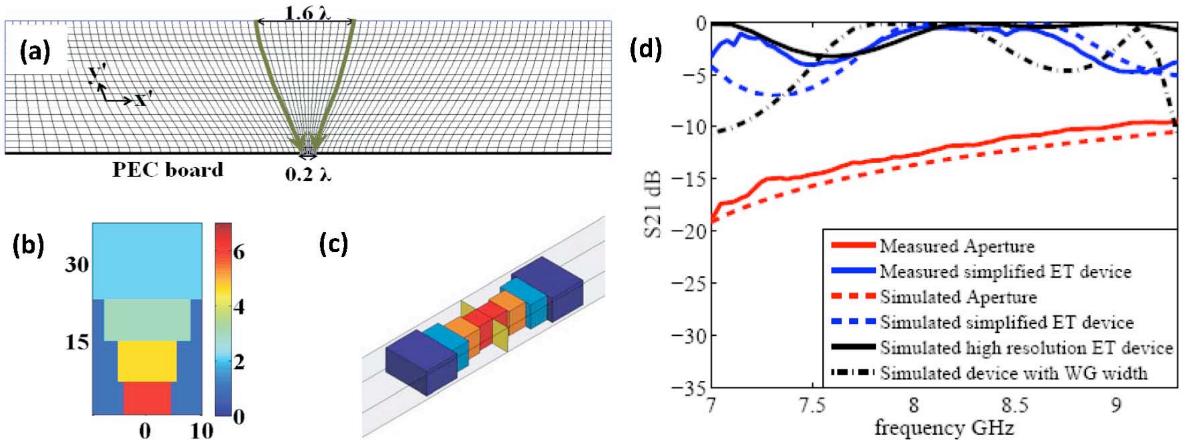


Fig. 5 (a) The physical space with the distorted coordinate system. (b) The relative permittivity map of the simplified ET device. (c) The configuration of the ET device and its arrangement in the X band waveguide. (d) The measured and simulated results with and without the ET device.

## 4. Conclusion

Some practical design examples based on electromagnetic transformation have been presented. It is shown that the electromagnetic transformation technique can be used to manipulate wave propagation using both novel engineering materials and conventional dielectrics. Slight degradations in performance from all dielectric devices can be mitigated due to their broadband performance, which are usually failed to deliver by the metamaterials.


## 6. Acknowledgments

The work is partially supported by EPSRC and TSB from the UK. The contributions from Wenxuan Tang, Di Bao, Christos Argyropoulos, Themos Kallos, Rui Yang are gratefully acknowledged.


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