An approach to finding the correct branch from the forest of possible solutions for extracted effective material parameters

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Abstract

In the classical Nicolson-Ross-Weir electromagnetic material parameter extraction technique the effective material parameters are obtained through reflection and transmission measurements of a planar material sample. One of the advantages of this technique is that it provides the result over a broad frequency band with just one measurement. This technique, however, does not provide us unambiguous results of the effective permittivity and permeability but the correct solution needs to be found through an additional deduction process. Here we present a derivative of the Nicolson-Ross-Weir extraction technique that can overcome this problem related to the infinitely many branches of the solution.

1 Introduction

The electromagnetic properties of a simple conventional material (excluding magnetoelectric coupling here) can be described in terms of permittivity and permeability. The correct evaluation of these parameters is of foremost importance for many applications and many methods to extract these parameters from a material sample has been developed. Among the different methods, one possibility is to extract the material parameters from the reflection and transmission measurements of the sample [1–5]. Named after the authors of the original papers introducing this technique [1, 2], the Nicolson-Ross-Weir (NRW) technique has the advantage over some other techniques for extracting the results over a broad frequency band. It has, however, one intrinsic problem relating to the the periodicity of the phase factor for the wave propagating through the measured material sample. Namely, the resulting solution has infinitely many roots and to find the correct one, one needs to obtain some additional information about the sample or measurement [6–8].

We present here an alternative approach to overcome the ambiguity in the extraction results based on our recent work [9]. Rather than calculating the phase factor for the wave propagating through the measured material sample at every frequency point, we concentrate on the difference of the phase factor between two preceding frequency points. This way we do not need to worry about the periodicity of the phase factor given that some preconditions are fulfilled. The proposed extraction technique is therefore capable of extracting the material parameters from a sample without additional branch-seeking algorithms. In this presentation we will discuss the proposed technique in detail and describe its performance and drawbacks with various examples.

2 The modified Nicolson-Ross-Weir extraction technique

Being a derivative of the NRW technique, the proposed technique resolves the electromagnetic material parameters from the inverse of the phase factor $e^{j\omega\sqrt{\mu\varepsilon}d} = e^{\alpha d}e^{j(\beta d+2\pi m)}$ for the electromagnetic wave propagating through the measured material sample and the wave impedance, both of which have been extracted from the reflection and transmission measurements/simulations of the material sample (see Fig. 1(a) for a schematic of a typical measurement/simulation setup). Unlike in the classical NRW technique, where the phase factor is solved for each individual frequency point separately, in the proposed technique the phase factor is rather solved from the difference between the arguments at two preceding frequency points. When

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calculating the refractive index n for the sample point N at the angular frequency ω_N, this leads to, instead of the plain logarithmic function of the phase factor, a slightly different form [9]:

$$n_N = \sqrt{\varepsilon_{\mathrm{r}}\mu_{\mathrm{r}}} = \frac{1}{k_0 d}\left[-j\ln\left(\left|e^{j\omega_N\sqrt{\mu\varepsilon}d}\right|\right) + \phi_0 + \sum_{i=1}^{N}\arg\left(\frac{e^{j\omega_i\sqrt{\mu\varepsilon}d}}{e^{j\omega_{i-1}\sqrt{\mu\varepsilon}d}}\right)\right], \tag{1}$$

where d is the thickness of the material sample, k_0 is the wave number in free space, and ϕ_0 is the argument of the phase factor at the first sample frequency ω_0 (for each sample frequency $\omega_0, \omega_1, \omega_2, \ldots \omega_N$ we have a corresponding argument $\phi_0, \phi_1, \phi_2, \ldots \phi_N$ and amplitude of the phase factor). Further, the phase factor can be calculated from the retrieved $S-$parameters as

$$e^{j\omega\sqrt{\mu\varepsilon}d} = \frac{1 - S_{11}^2 + S_{21}^2}{2S_{21}} + \frac{2S_{11}}{\left(z - \frac{1}{z}\right)S_{21}} \tag{2}$$

and the relative wave impedance z still needed for the retrieval of the relative material parameters can be calculated as [4, 5]

$$z = \sqrt{\frac{\mu_{\mathrm{r}}}{\varepsilon_{\mathrm{r}}}} = \sqrt{\frac{\left(1 + S_{11}\right)^2 - S_{21}^2}{\left(1 - S_{11}\right)^2 - S_{21}^2}}. \tag{3}$$

In this case the passivity condition needed to remove the ambiguity in (3) becomes a trivial case in many software packages since the standard procedure $w = \mathrm{sqrt}(z)$ maps the complex plane z onto the positive real half of the complex plane w ($\Re(w) \geq 0$). There are, however, some pre-conditions that need to be satisfied in order for the proposed extraction technique to work. Namely, in order to justify the choice of the branch $m = 0$ for the argument ϕ_0 of the phase factor of the first sample point, at this frequency ω_0 the electrical thickness of the sample should be less than $\lambda/2$. Further, the difference of the arguments of the two preceding sample points i and $i-1$ should not exceed π. These restrictions will be discussed in more detail in the presentation.

3 Examples of the extraction procedure

The performance of the proposed extraction technique was studied with a couple of material examples. With the conventional material there were no difficulties in finding the correct branches for samples that were even large in electrical thickness. The proposed extraction technique was tested also for a slab of wire medium. The wire medium slab comprised parallel copper wires embedded in silicon-dioxide having $\varepsilon_{\mathrm{r}} = 4$ (see Fig. 1(b)). The radius of the wires was chosen to be $r_0 = 0.1\,\mathrm{mm}$ and the wires were positioned in a two-dimensional square lattice with a lattice constant $p = 3.5\,\mathrm{mm}$ in both directions. In the simulations the wire medium sample consisted of five unit cells and the total thickness of the sample was $d = 17.5\,\mathrm{mm}$. We extracted the $S-$parameters from the simulations for a plane wave with normal incidence having the electric

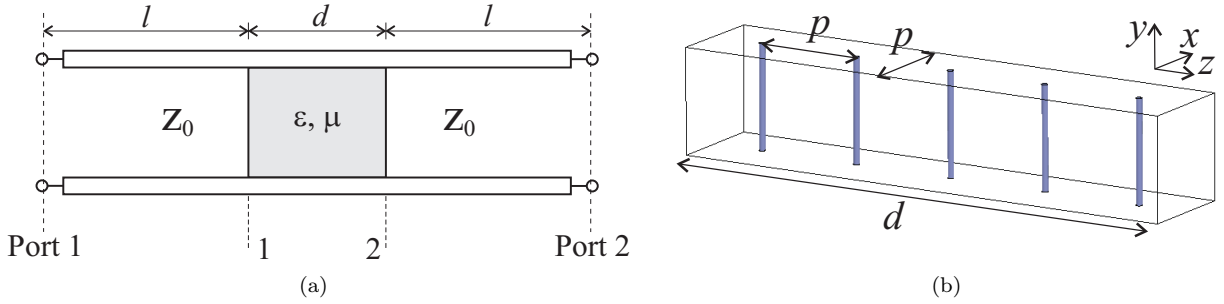

Figure 1: (a) A schematic figure of the setup used for extraction of the material parameters. (b) The simulation model of the wire medium slab. The slab is infinite in both x- and y-directions.

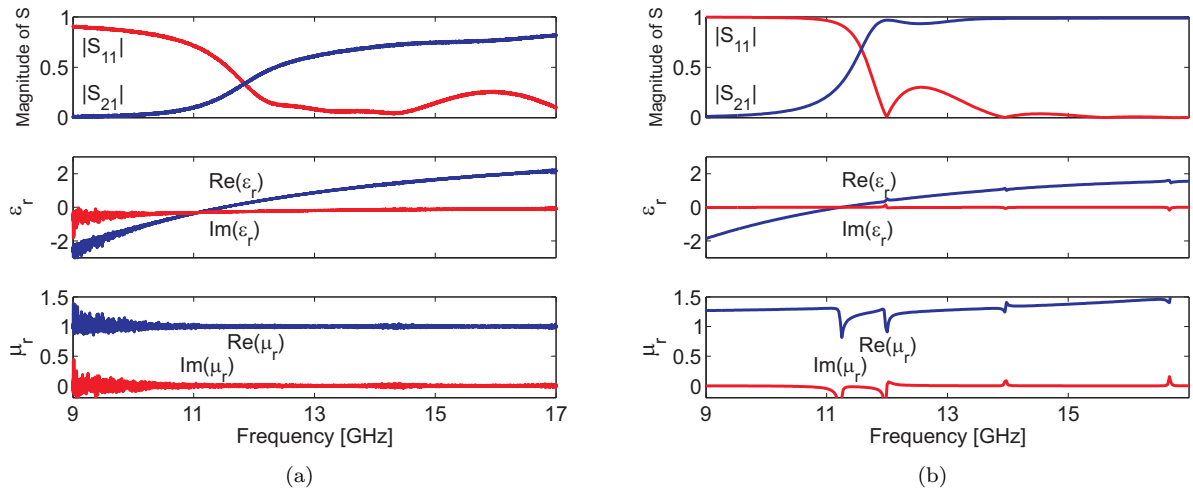

Figure 2: The reflectance and transmittance, complex relative permeability, and complex relative permittivity extracted from the (b) analytically calculated and (b) simulated $S-$parameters for a slab of wire medium

field component parallel to the wires. For comparison we calculated the $S-$parameters also analytically for the same structure modeling the wire medium as a slab of dielectric following the Drude dispersion. The dielectric permittivity and the angular plasma frequency [10, 11] are given, respectively, as

$$\varepsilon_{\mathrm{r}}(\omega) = 1 - \frac{\omega_{\mathrm{p}}^2}{\omega\left(\omega - j\gamma_{\mathrm{p}}\right)}, \quad \omega_{\mathrm{p}} = \frac{c}{p}\frac{1}{\sqrt{2\pi \ln\left(\frac{p^2}{4r_0(p-r_0)}\right)}}, \tag{4}$$

where $\gamma_{\mathrm{p}} = 5\,\mathrm{GHz}$ and $c = 1/\sqrt{\varepsilon\mu}$. For our parameters the angular plasma frequency for the wire medium becomes $\omega_{\mathrm{p}} \approx 2\pi \times 11.5\,\mathrm{GHz}$. Further, we added uncorrelated random normally distributed noise ($\mu_{\mathrm{d}} = 0$, $\sigma = 0.005$) into the analytically calculated $S-$parameters in order to simulate typical measurement setup.

In Fig. 2(a) the analytically calculated reflectance and transmittance, and the complex relative permittivity and permeability extracted from them, are shown. In Fig. 2(b) the similar data extracted from the simulations is shown. We see that extracted material parameters follow the expectations even though the thickness of the material sample exceeds multiples of $\lambda/2$, especially in the analytical case of homogeneous material. However, for the permeability extracted from the simulations of discrete vias, we notice that the value differs from the expected one. This happens due to the discreteness of the wire medium sample: Although the period of the sample is much less than the wavelength, the Bloch impedance of the unit cell differs considerably from the averaged wave impedance and leads to the situation where the extracted material parameters differ from the ones obtained through effective-medium theory (see also [10, 11]). This phenomenon will be discussed in more detail in the presentation. At the thickness resonances and in the stop band, however, we see that in the analytical case the accuracy of the extraction results deteriorates due to the noise added to the system. In the simulated case we see also inaccuracy in the extracted material parameters due to the numerical noise. Despite the noise in the system, the proposed extraction technique manages to remain on the correct branch.

4 Conclusions

An approach to extraction of electromagnetic material parameters is studied. This approach is a derivative of the classical NRW extraction technique, but does not suffer from the same intrinsic problem of its predecessor relating to the multiple branches of the solution. This is because instead of calculating the

phase factor of the electromagnetic wave propagating through the measured material sample for each sample point separately, the proposed technique rather calculates the phase factor for each sample point using the information from the preceding points. Namely, we make use of the difference between the arguments of adjacent sample points. This way, given that this difference is not too large, we are able to remain on the correct branch of the solution throughout the extraction procedure.

References

[1] A. M. Nicolson and G. F. Ross, "Measurement of the intrinsic properties of materials by time-domain techniques," *IEEE Trans. Instrum. Meas.*, vol. IM-19, no. 4, pp. 377–382, 1970.

[2] W. B. Weir, "Automatic measurement of complex dielectric constant and permeability at microwave frequencies," *Proc. IEEE*, vol. 62, no. 1, pp. 33–36, Jan. 1974

[3] W. Barry, "A broad-band, automated, stripline technique for the simultaneous measurement of complex permittivity and permeability," *IEEE Trans. Microw. Theory Tech.* vol. MTT-34, no. 1, pp. 80–84, Jan.1986.

[4] D. R. Smith, S. Shultz, P. Markoš, and C. M. Soukoulis, "Determination of effective permittivity and permeabiity of metamaterials from the reflection and transmission coefficients," *Phys. Rev. B*, vol. 65, 195104, 2002.

[5] D. R. Smith, D. C. Vier, Th. Koschny, and C. M. Soukoulis, "Electromagnetic parameter retrieval from inhomoheneous metamaterials," *Phys. Rev. E*, vol. 71, 036617, 2005.

[6] T. L. Blakney and W. B. Weir, Comments on "Automatic measurement of complex dielectric constant and permeability at microwave frequencies," *Proc. IEEE*, vol. 63, pp. 203–205, 1975.

[7] J. Baker-Jarvis, E. J. Vanzura, and W. A. Kissick, "Improved technique for determining complex permittivity with the transmission/reflection method," *IEEE Trans. Microw. Theory Tech.,* vol. 38, no. 8, pp. 1096–1103, 1990.

[8] X. Chen, T. M. Grzegorczyk, B.-I. Wu, J. Pacheco, Jr. and J. Kong, "Robust method to retrieve the constitutive effective parameters of metamaterials," *Phys. Rev. E 70*, 016608, 2004.

[9] O. Luukkonen, S. I. Maslovski, and S. A. Tretyakov, "An improved material parameter extraction technique based on the transmission and reflection measurement of a material sample," *under review*

[10] S. Maslovski and S. Tretyakov, "Wire media with negative effective permittivity: A quasi-static model," *Microwave and Optical Technology Letters,* vol. 35, pp. 47–51, 2002.

[11] S. Maslovski and M. Silveirinha, "Nonlocal permittivity from a quasistatic model for a class of wire media," *Physical Review B*, vol. 80, 245101, 2009.