An Efficient Numerical Approach to the Accurate Analysis of Propagation and Radiation Phenomena in Metamaterial Structures

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Abstract

An overview is presented of a novel implementation for the efficient analysis of metamaterial structures embedded in layered media. Based on a suitable mixed-potential integral-equation formulation, *ad hoc* acceleration procedures for the periodic potentials (expressed through slowly-convergent series when source and observation points lie in the same horizontal plane) have been developed. The approach consists of an asymptotic extraction of homogeneous-medium terms. Numerical results are shown, which prove the computational efficiency of this method and validate different types of propagation and radiation features occurring in metamaterial-like structures. The method is validated with commercial software and data from the literature.

1. Introduction

The characterization and the design of metamaterials require the availability of flexible computational tools which can handle a wide variety of structureconfigurations , shapes, and various kinds of electromagnetic materials (e.g., dispersive or lossy materials having metallic or dielectric periodic inclusions, etc.) [1]. Also, the well-known presence of exotic behaviors (like backward waves), supported by many metamaterial structures, requires rigorous dispersive analyses [2]. The important cases of complex waves, possibly of improper type (i.e., leaky waves), are also of extreme interest and should be taken into account when addressing these kinds of problems. The consideration of complex waves is also required in the implementation of the "array scanning method," which is a method often used for the rigorous analysis of the nonperiodic (single source) excitation of an infinite periodic structure, by means of a continuous superposition of periodic sources [3-5].

The treatment of arbitrary periodic geometries, of possibly complex and improper waves, and of realistic multilayered environments is allowed by suitable integral-equation formulations [1]. Their solution is proposed here through a Galerkin method-of-moments (MoM) approach in the spatial domain, in order to handle objects with arbitrary shape. For an easier evaluation of the reaction integrals filling the impedance matrix, a mixed-potential formulation is chosen [6], granting a milder spatial singularity of the kernels with respect to analogous field formulations. Periodic kernels are unfortunately formulated in terms of series that convergeslowly , especially if the observation and the source points are close. Periodic potentials in free-space have been the object of detailed study for some time [7,8], and one of the most efficient and robust techniques for their computation is the Ewald method, expressing the potential as the sum of two series, both with a Gaussian convergence rate, even for complex waves [8-11].

On the other hand, periodic dyadic potentials in layered media [6] need specific algorithms for acceleration, with the specific treatment different for planar and non-planar problems (i.e., those having vertical currents). Usually, homogeneous-medium asymptotic terms are extracted from the relevant slowly-converging spectral series and are summed back as separate contributions, accelerated with the Ewald method. When vertical currents are present, additional dyadic components exist: for these terms different asymptotic terms should be extracted and the usual Ewald method cannot be applied [12].

In this paper, the general extraction formulation is described with reference to two-dimensional (2D) problems, and a "modified" Ewald method is used. Numerical results show the validity and the efficiency of the approach, with reference both to plane-wave incidence (scattering) problems and to dispersive complex-mode analyses for various periodic multilayer metamaterial-like structures.

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2. Formulation of the Mixed-Potential Integral Equations

In multilayered media, two-dimensional integral equations can be formulated for boundary-value problems through suitable dyadic and scalar mixed-potentials [6]. The continuity of the tangential components of the electric field $\mathbf{E}_{\tan}$ can be imposed from the expressions

$$\mathbf{E}_{\tan} = \Bigg[-j\omega \int_l \underline{\mathbf{G}}_A^{\mathrm{p}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{J}(\mathbf{r}')\,\mathrm{d}l' - \nabla\int_l K_\Phi^{\mathrm{p}}(\mathbf{r},\mathbf{r}')\nabla'\cdot\mathbf{J}(\mathbf{r}')\,\mathrm{d}l' - \nabla\int_l P_z^{\mathrm{p}}(\mathbf{r},\mathbf{r}')\mathbf{z}_0\cdot\mathbf{J}(\mathbf{r}')\,\mathrm{d}l'$$
$$-\frac{1}{\varepsilon}PV\int_l \nabla\times\underline{\mathbf{G}}_F^{\mathrm{p}}(\mathbf{r},\mathbf{r}')\cdot\mathbf{M}(\mathbf{r}')\,\mathrm{d}l' \pm \frac{\mathbf{M}(\mathbf{r}')}{2}\delta_{\mathbf{r},\mathbf{r}'}\Bigg]_{\tan} \quad (1)$$

where $\mathbf{r}$ and $\mathbf{r}'$ are the observation and source points, respectively, and $\mathbf{J}$ and $\mathbf{M}$ are the equivalent electric and magnetic currents flowing on the contour l of a 2D object. The quantities $\underline{\mathbf{G}}_A^{\mathrm{p}}$, K_Φ^{p}, and P_z^{p} are the dyadic, scalar, and corrective potentials associated with electric current sources; the superscript p stands for "periodic." The dual potentials $\underline{\mathbf{G}}_F^{\mathrm{p}}$, K_Ψ^{p}, and Q_z^{p} associated with magnetic currents can be computed with similar schemes and will not be treated here for brevity. A dual expression can also be obtained from (1) for the magnetic field $\mathbf{H}_{\tan}$.

3. Acceleration of Periodic Kernels

The original procedures to achieve significant acceleration in the computation of the Green's function potentials in (1) for periodic structures are outlined in the general case of both transverse (horizontal) and vertical currents.

3.1 Transverse Currents

If the periodicity is assumed along the x direction and the stratification along the (vertical) z direction (see, as a reference structure, Fig. 1(a)), one of the potentials mentioned in the previous section, denoted here as G, can be computed as

$$G(\mathbf{r},\mathbf{r}') = \frac{1}{p}\sum_{n=-\infty}^{+\infty}\left[\tilde{G}(k_{xn};z,z') - \sum_{i=-1}^{+1}\tilde{g}(k_{xn};\Delta z_i)\right]e^{-jk_{xn}\Delta x} + \sum_{i=-1}^{+1} g^{\mathrm{p}}(k_{x0};\Delta x,\Delta z_i) \quad (2)$$

where $\tilde{G}$ is a suitable spectral-domain Green's function, known in closed form for a general stratification. The quantity $k_{xn} = k_{x0}+2\pi/p$ is the wavenumber of the nth harmonic along the x direction, k_{x0} being the phase shift (possibly complex) between adjacent unit cells; $k_{zn} = \sqrt{k_{\mathrm{s}}^2 - k_{xn}^2}$ and k_{s} is the wavenumber of the source medium. The three extracted terms $\tilde{g}$ are homogeneous-medium spectral-domain Green's functions; each one refers to a certain image reflected at the layer interface. $\Delta x = x - x'$ and Δz_i is a vertical distance depending on the particular image term considered. The terms g^{p} are homogeneous-medium periodic Green's functions, computed with the Ewald method [10,11].

3.2 Vertical Currents

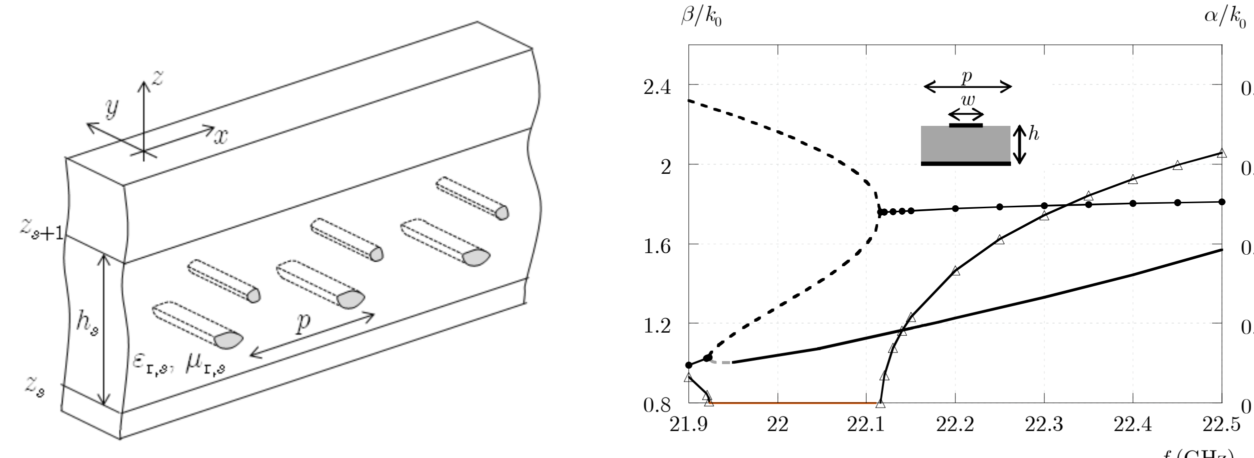
In this case, the corrective potential P_z^{p} requires the extraction of different asymptotic terms:

$$P_z^{\mathrm{p}}(\mathbf{r},\mathbf{r}') = \frac{1}{p}\sum_{n=-\infty}^{+\infty}\left[\tilde{P}_z(k_{xn};z,z') - \sum_{i=-1}^{+1}\frac{1}{jk_{zn}}\tilde{g}(k_{xn};\Delta z_i)\right]e^{-jk_{xn}\Delta x} + \sum_{i=-1}^{+1} g^{z,\mathrm{p}}(k_{x0};\Delta x,\Delta z_i). \quad (3)$$

Every extracted Green's function $\tilde{g}$ is now divided by a factor $1/k_{zn}$; these terms therefore cannot be computed with the Ewald method as done in the transverse case. In fact, the new Green's function $g^{z,\mathrm{p}}$ is the integral along z of the standard Green's function g^{p} [12]:

$$g^{z,\mathrm{p}}(k_{x0};\Delta x,\Delta z) = \int_{|\Delta z|}^{+\infty} g^{z,\mathrm{p}}(k_{x0};\Delta x,\zeta)\,\mathrm{d}\zeta\,. \quad (4)$$

The Ewald method can be suitably modified by integrating both the spectral and spatial series along z: two new series are obtained, again with Gaussian convergence. Their terms are the z-integrals of the respective terms in the standard Ewald method, each one expressed either in closed form (in the spectral series) or through fast-converging integrals (in the spatial series) [12]. The component of the dyadic potential can be accelerated in a similar way.



(a) (b)

Fig. 1: (a) Example of 2D periodic multilayer structure under analysis with relevant coordinate systems and parameters: an array of arbitrarily-shaped cylindrical objects is embedded in a stratified medium. (b) Results of a dispersion analysis for a strip grating (period p = 0.338 cm, strip width w = 0.6 p) on a grounded slab (thickness h = 0.14 cm, relative permittivity ε_r = 20). The normalized phase constant β is shown for the real proper mode (*solid black line*), an improper complex mode (*solid black line with dots*), and improper real modes (*dashed black lines*). The normalized attenuation constant α is shown for the improper complex modes (*solid black lines with triangles*). The analysis is limited here to TE^z modes.

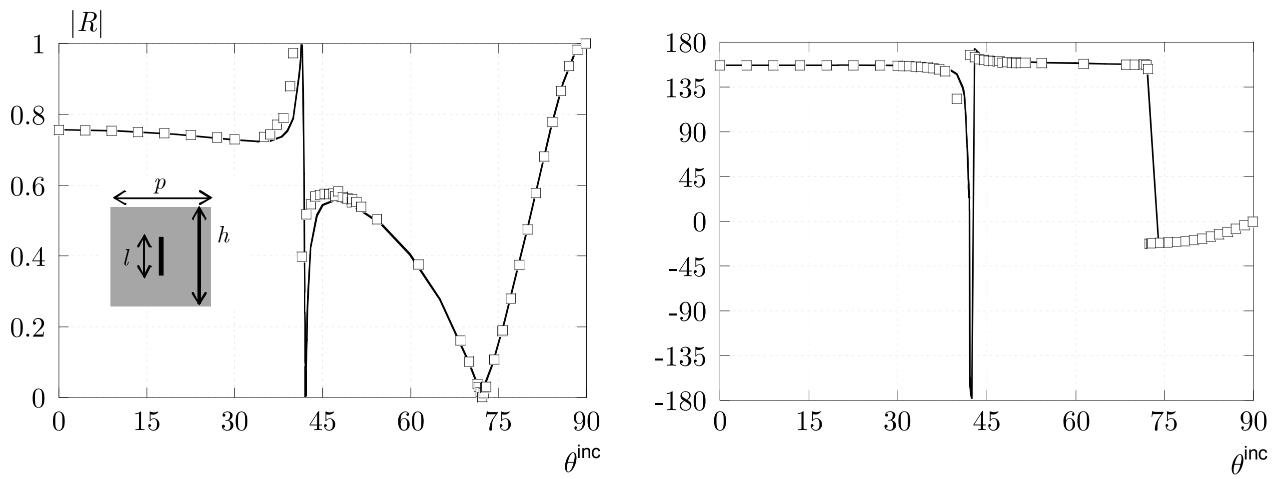
4. Numerical Results

In order to validate the extraction techniques described above, representative numerical results are shown in this section for some canonical multilayer periodic structures.

In Fig. 1(b), a dispersive analysis is plotted for the TE^z modes supported by a planar strip grating printed on a grounded dielectric slab (see the inset of the figure). The computation has been performed with the code EIGER™ [13], implementing the acceleration methods for the Green's functions described in the previous section. The results are in perfect agreement with those discussed in [14], obtained with a different technique. Complex and improper waves are also computed, thus showing the validity of the extraction (2) in all the possible propagation regimes.

Fig. 2 shows the reflection coefficient of the 0th harmonic reflected by a dielectric slab with an array of vertical metallic strips embedded inside (see the inset of the figure). The very good agreement with independent results obtained with the commercial software HFSS™ [15] validates the accuracy of the extraction (3).

Using the proposed formulation, an extremely significant reduction of computation times has been obtained.



(a) (b)

Fig. 2: An array of periodic vertical metal strips (black line in the inset) embedded in a dielectric layer (gray region). The reflection coefficient for the 0th harmonic of an impinging TM^z uniform plane wave vs. the angle of incidence θ is shown. Results from EIGER™ (*solid line*) and from HFSS™ (*squares*) are compared. (a) Magnitude and (b) phase (degrees) at the top interface between air and dielectric. Metal strip length l =10 mm, dielectric-slab thickness h =20 mm and relative permittivity ε_r =10.2. The spatial period is p =20 mm and the operating frequency is f =4 GHz.

5. Conclusion

In this summary an acceleration procedure has been presented for the efficient MoM computation of all the mixed-potential components required to study arbitrarily-shaped cylindrical periodic inclusions in layered media. Useful applications of the approach to the analysis of various metamaterial structures are thus possible. Validations have been presented with reference to guided-wave and scattering problems, validating with data present in the literature and with commercial software. The accuracy and the efficiency of this formulation have thus been verified.

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