Circular Antenna Array for Microwave Bessel Beam Generation

***Pierre Lemaître-Auger*[1-2], *Samer Abielmona*[2] *and Christophe Caloz*[2]**

[1]Laboratoire de Conception et d'Intégration des Systèmes, Grenoble INP-Esisar, 50 Barthélémy de Laffemas, BP54, 26902 Valence, France, visiting professor at Poly-Grames, pierre.lemaitre-auger@esisar.grenoble-inp.fr

[2]Poly-Grames Research Center, Department of Electrical engineering, Ecole Polytechnique de Montréal, Montréal, Québec, H2T 1J3, Canada

Abstract

A circular antenna array (CAA) is proposed and demonstrated for the generation of optimal pseudo-Bessel beams at millimeter-wave frequencies. Numerical simulations show that a 91-element array produces a Bessel beam of a 7λ main lobe width over a distance of 180λ. Based on this report, it is suggested that Bessel beams may provide a unique solution to millimeter-wave quasi-optical systems by providing highly focused beams with small-sized antennas.

1. Introduction

Millimeter-wave (or THz) technology is most promising for next-generation high-speed short range communication system, high-resolution radar, high-precision instrumentation and spectroscopy following the recent development of efficient sources and detectors. In this context, it may be anticipated that quasi-optical THz systems will generate great interest in the forthcoming decades. However, high beam directivity is incompatible with reasonable device sizes at these frequencies. For example, a Gaussian beam (one of the less divergent beams) at 100 GHz would require a beamwidth diameter (at $1/e^2$) of ~2,5m in order to provide the same directivity as a standard laboratory laser (divergence less than 1.5 mrad)!

In order to address this challenging issue, we speculate here that a solution could be found in the utilization of Bessel beams, which are non-diffracting beams first predicted in the early 1980ies [1]. The Bessel beam is a solution to the scalar wave equation in a homogeneous medium and was first reported in 1987 by Durnin [2]. Although requiring a source of infinite extent, the Bessel beam can fortunately be laterally truncated and still exhibit a non-diffractive behavior of its main lobe over a distance long enough for practical interest [3].

Further pioneering contributions on non-diffracting waves or pulses were made by Brittingham in 1983 [4] and by Ziolkowski in 1985 [45]. In 1992, Lu and Greenleaf reported the X-wave which is a time-pulsed version of a Bessel beam [6]. From that point, quasi non-diffracting waves, also nowadays called localized waves, were then studied in theoretical physics, optics and acoustics. However, very few works concern microwaves.

We propose here the concept of a circular antenna array (CAA) as a pseudo-Bessel beam launcher. The CAA has several advantages over classical production techniques. First, an antenna array can emulate arbitrary field distributions. Second, its polarization is easily controlled by the design of the individual antennas: both linear and circular polarizations are possible. Third, it occupies a small volume and is not subject to misalignment problems.

2. Bessel Beam Generation Techniques

The Bessel beam is described by the following equation [2]:

$$E(\rho, z, t) = AJ_0(k_\rho \rho)e^{i(\omega t - \beta z)}, \tag{1}$$

where $J_0(u)$ is the Bessel function of the first kind of order 0, A is the amplitude, ρ is the radial coordinate, ω the angular frequency, and k_ρ and β are the transverse and the longitudinal components, respectively, of the wave number, k, which are related by

$$k_\rho^2 + \beta^2 = k^2 = \left(\frac{\omega}{c}\right)^2, \tag{2}$$

978-1-4244-6051-9/11/$26.00 ©2011 IEEE

where c is the velocity of light in vacuum. A pseudo-Bessel beam is a Bessel beam which is transversally truncated to a disk area of radius R. Equation (2) indicates that all the vectors $\boldsymbol{k}$ of the plane-wave spectrum of the Bessel beam lie on the surface of a cone, which is a fundamental requirement for the generation of a Bessel beam.

The first Bessel beam launcher ever reported consists of a diffracting ring placed at the focal distance of a convergent lens and illuminated by a plane wave, as shown in Fig. 1(a) [3]. Light diffracts when passing through the ring. If the ring width is small enough, it can be considered as an infinitesimal annular source. After passing through the lens, all the diffracted rays satisfy (2) and therefore form a pseudo-Bessel beam. This launcher suffers from very low efficiency because most of the incoming beam power is blocked by the diaphragm. To overcome this problem, the axicon lens, illustrated in Fig. 1(b), is more commonly used today. It is a lens with a flat entrance surface and an output volume shaped into a cone. When a plane wave hits the conic surface, the light refracts across it, satisfying again condition (2), so as to form a pseudo-Bessel beam.

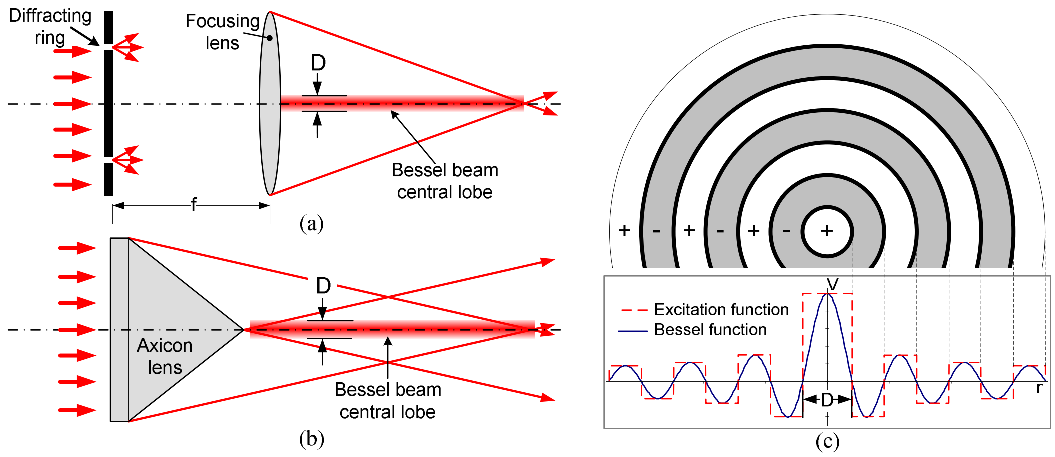

Figure 1 Schematic representation of different mechanisms for the generation of a Bessel beam. (a) Diffractive ring with focusing lens [3]. (b) Axicon lens [1]. (c) Piezo-electric rings for the generation of an acoustic pseudo-Bessel beam from a truncated Bessel excitation [7].

Bessel beams were also produced with acoustic waves, using a very different launching technique: concentric circular piezo-electric rings are electrically excited with a voltage distribution that emulates the Bessel function described by (1), as depicted in Fig. 1(c) [7, 8].

Very few experimental works on the generation of Bessel beams at microwaves or THz have been done. Mugnai *et al.* reported an experiment with the objective to prove that the group velocity of the Bessel beam is greater than c [9]. They used a modified version of the Durnin diffractive ring setup, employing a parabolic mirror instead of the lens. The diffractive ring was placed in the focal plane of the mirror and was illuminated by a horn antenna. As previously mentioned, such an apparatus suffers from poor efficiency; moreover, its performance is further reduced from the blockage of the horn antenna. Trappe *et al.* used a microwave axicon lens together with a horn antenna and an elliptical mirror at 0.1 THz [10]. Similar techniques were also reported at THz with axicon lenses and laser sources and quasi-optical components [11].

3. Circular Antenna Array Launcher

With all the techniques discussed in the previous section, the amplitude of the pseudo-Bessel beam main lobe exhibits strong oscillations along the propagation direction, especially close to the source. Numerical simulations reveal that this oscillatory behavior also comes with a lateral deformation of the main lobe. In a sense, the main lobe “breathes” in synchronization with the oscillation: it gets larger/smaller when the oscillation is at a minimum/maximum. This undesired effect may be mitigated by optimizing the excitation by fully taking into account the truncated nature of the source. As will be shown next, this optimal function significantly deviates from a simple truncated Bessel function for small aperture radius.

An antenna array offers an alternative paradigm to the aforementioned techniques. It may provide an arbitrary phase and magnitude distribution and thereby, by sampling the optimal continuous source function, lead to an optimal pseudo-Bessel beam for a given truncated source area. The proposed pseudo-Bessel beam circular antenna array (CAA) launcher is shown in Fig. 2(a). The circular shape is naturally selected for its azimuthal symmetry. Patch antennas are uniformly distributed on the circular disk area according to a hexagonal lattice. All the patch antennas are designed to produce the same linear polarization. As a consequence, the problem can be considered scalar in a first approach. The corresponding optimal source amplitude and phase functions are plotted in Fig. 2(b).

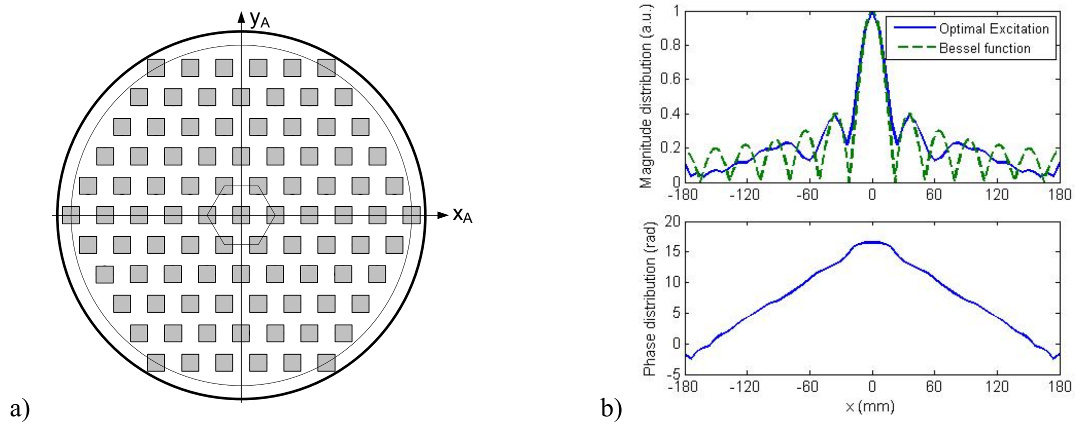

Figure 2 Proposed CAA. (a) Schematic front-view of the CAA with 91 patch antennas arranged according to a hexagonal grid (array diameter: $60\lambda_0$). (b) Optimal excitation function along the *x*-axis of the CAA.

Numerical simulation results are presented in Fig. 3, where the array has been optimized to induce minimal amplitude and phase deformations of the main lobe for the best possible agreement with (1). For the proof-of-concept, the patch antennas were first replaced by infinitesimal dipoles and next by λ/2 dipole antennas. No noticeable difference was observed between the two cases, as may have been anticipated from the electrically large size of the array. E_x and E_z were ~1000 times smaller than E_y while H_y was strictly equal to zero and H_z was ~100 times smaller than H_z, corresponding to a quasi-TEM wave.

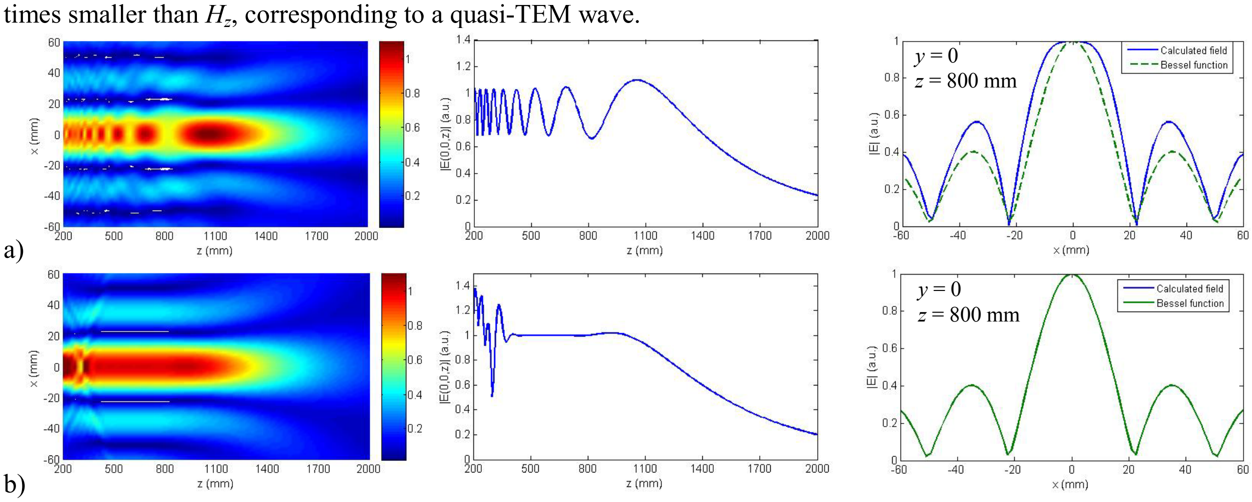

Figure 3 Numerical simulation results for pseudo-Bessel beams generated with two different excitation functions. (a) Truncated Bessel amplitude function. (b) Optimal amplitude and phase excitation, of the type shown in Fig. 2(b). Frequency: 50GHz, array diameter: $60\lambda_0$, 3259 point sources, $k_\rho = k \sin(6°)$.

Three different excitation functions were compared: 1) a constant amplitude and a radially linearly increasing phase (obtained at the apex of the axicon); 2) a Bessel amplitude distribution with a constant phase, corresponding to (1); 3) an optimized truncated-surface excitation of the type shown in Fig. 2(b).

This optimal distribution was found using the following procedure [12]: The total field at a given observation point in a transverse (*xy*) plane located at z_0 is expressed as the product of an unknown complex amplitude function and of the isotropic radiation term (e^{-ikr}) for all the point sources of the array. Then, the mean-square error between the observed field in the observation plane and the Bessel distribution is minimized with respect to the complex amplitude unknowns. This leads to a linear system whose solution provides the desired complex amplitude function of the source.

The results obtained for cases 2) and 3) are shown in the first and second rows, respectively, of Fig. 3. They show the scalar field space distribution in the longitudinal (*x0z*, $y = 0$) plane, the evolution of the maximum field value of the main lobe along the *z*-axis and the absolute value of the amplitude along the *x*-axis for $y = 0$ and $z = 800$ mm. k_ρ is equal to $k \cdot \sin(6°)$ for a frequency of 50 GHz. To emulate an ideal continuous source function immune from spatial discretization effects, 3259 ideal sources were used. Comparing Fig. 3(a) and Fig. 3(b), the disappearance of the oscillation and improved agreement with the Bessel function is clearly observed. The width of the main lobe is 44 mm (7.3λ) while the propagation range is approximately 1700 mm (~280λ), a much longer distance than the one that could be obtained with a Gaussian beam. More practical results with less point sources (e.g. 91) exhibit a relatively similar behavior, but with a shorter propagation distance of 1500 mm.

4. Conclusion

We have shown that the generation of a pseudo-Bessel beam with a CAA is advantageous over previously reported techniques and while also being realistic, with less than 100 radiating elements. Experiments are currently under way.

5. References

[1] H. E. Hernandez-Figueroa, M. Zamboni-Rached, and E. Recami, Edts., *Localized Waves.* Hoboken, N.J.: Wiley-Interscience, IEEE Press, 2008.

[2] J. Durnin, "Exact solutions for nondiffracting beams. I. The scalar theory," *Journal of the Optical Society of America A (Optics and Image Science),* vol. 4, pp. 651-4, 1987.

[3] J. Durnin, J. J. Miceli, Jr., and J. H. Eberly, "Diffraction-free beams," *Physical Review Letters,* vol. 58, pp. 1499-501, 1987.

[4] J. N. Brittingham, "Focus waves modes in homogeneous Maxwell's equations: transverse electric mode," *Journal of Applied Physics,* vol. 54, pp. 1179-89, 1983.

[5] R. W. Ziolkowski, "Exact solutions of the wave equation with complex source locations," *Journal of Mathematical Physics,* vol. 26, pp. 861-3, 1985.

[6] J. Y. Lu and J. F. Greenleaf, "Nondiffracting X waves-exact solutions to free-space scalar wave equation and their finite aperture realizations," *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control,* vol. 39, pp. 19-31, 1992.

[7] J. Y. Lu and J. F. Greenleaf, "Ultrasonic nondiffracting transducer for medical imaging," *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control,* vol. 37, pp. 438-47, 1990.

[8] D. K. Hsu, F. J. Margetan, and D. O. Thompson, "Bessel beam ultrasonic transducer: fabrication method and experimental results," *Applied Physics Letters,* vol. 55, pp. 2066-8, 1989.

[9] D. Mugnai, A. Ranfagni, and R. Ruggeri, "Observation of superluminal behaviors in wave propagation," *Physical Review Letters,* vol. 84, pp. 4830-3, 2000.

[10] N. Trappe, R. Mahon, W. Lanigan, J. A. Murphy, and S. Withington, "The quasi-optical analysis of Bessel beams in the far infrared," *Infrared Physics & Technology,* vol. 46, pp. 233-47, 2005.

[11] M. U. Shaukat, P. Dean, S. P. Khanna, M. Lachab, S. Chakraborty, E. H. Linfield, and A. G. Davies, "Generation of Bessel beams using a terahertz quantum cascade laser," *Optics Letters,* vol. 34, pp. 1030-2, 2009.

[12] J. E. Hernandez, R. W. Ziolkowski, and S. R. Parker, "Synthesis of the driving functions of an array for propagating localized wave energy," *Journal of the Acoustical Society of America,* vol. 92, pp. 550-62, 1992.