Transient Plane-wave decomposition of reflected TE Gaussian-Beam from Moving Dielectric-Magnetic Planar Interface

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Abstract

This paper is concerned with applying plane-wave decomposition for scattering of a pulsed-beam from a fast moving planar dielectric discontinuity under the frame work of Special Relativity.

1 Introduction

Pulsed-beams (PBs) are the basis for the Phase-pace Pulsed-Beam Summation Method which is a framework for analyzing radiation from extended sources as well as scattering from complex media. In this formulation, the field is expanded into a discrete spectrum of pulsed-beams propagators. These propagators emanate from the aperture plane in a given set of points, in a given set of directions and in a given set of delays [1-3]. Such PB solutions have been obtained in generic media profiles such as inhomogeneous [4-6], anisotropic [7-11] and more.

This paper investigates the reflection of an electromagnetic time-dependent PB by a fast moving planar half-space dielectric-magnetic medium (isotropic at the co-moving frame). The PB field is decomposed into its transient plane-wave (PW) constituents which are transformed into the S'-frame (co-moving) of reference in which the medium is at rest. The reflected PWs are obtained by applying Maxwell's boundary conditions in order to obtained a spectral representation of the reflected PB. The resulting spectral integral is transformed back to the S-frame of reference (the "laboratory-frame"), yielding a PW spectral representation of the reflected PB. In the present paper we present only the TE PB, but the TM PB can be easily obtained in a similar manner

2 Lorentz Transformation of EM Field

The moving reference frame's origin is located at the moving dielectric, $z' = 0$, which is moving with velocity $\boldsymbol{v} = v\hat{\mathbf{z}}$ with respect to the S frame, so that $\hat{\mathbf{x}}' = \hat{\mathbf{x}}$, $\hat{\mathbf{y}}' = \hat{\mathbf{y}}$, $\hat{\mathbf{z}}' = \hat{\mathbf{z}}$. The corresponding Lorentz and inverse Lorentz Transformation is given by

$$\begin{aligned} ct' &= \gamma ct - \gamma\boldsymbol{\beta}\cdot\mathbf{r}, & ct &= \gamma ct' + \gamma\boldsymbol{\beta}\cdot\mathbf{r}', \\ \mathbf{r}' &= \bar{\bar{\boldsymbol{\alpha}}}\cdot\mathbf{r} - \gamma\boldsymbol{\beta}ct, & \mathbf{r} &= \bar{\bar{\boldsymbol{\alpha}}}\cdot\mathbf{r}' + \gamma\boldsymbol{\beta}ct', \end{aligned} \tag{1}$$

where

$$\boldsymbol{\beta} = \frac{\boldsymbol{v}}{c}, \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}, \quad \bar{\bar{\boldsymbol{\alpha}}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \gamma \end{bmatrix}. \tag{2}$$

As outlined in the introduction, the PB is decomposed into there plane wave spectrum constituents. In order to apply Maxwell's boundary conditions at the S' frame's $z' = 0$ plane, the propagating PBs are *a priori* decomposed into TE and TM components with respect to constant z planes at the S frame. Here we present the results for TE PB propagators. Next we transform the fields into the moving S' frame representation using the Lorentz Transformation

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(1) and the Electromagnetic Field Transformation (3)[12].

$$\begin{aligned}
\mathbf{E}' &= \gamma\bar{\bar{\boldsymbol{\alpha}}}^{-1}\cdot\mathbf{E} + c\gamma\boldsymbol{\beta}\times\mathbf{B},\\
\mathbf{B}' &= \gamma\bar{\bar{\boldsymbol{\alpha}}}^{-1}\cdot\mathbf{B} - c^{-1}\gamma\boldsymbol{\beta}\times\mathbf{E},\\
\mathbf{D}' &= \gamma\bar{\bar{\boldsymbol{\alpha}}}^{-1}\cdot\mathbf{D} + c^{-1}\gamma\boldsymbol{\beta}\times\mathbf{H},\\
\mathbf{H}' &= \gamma\bar{\bar{\boldsymbol{\alpha}}}^{-1}\cdot\mathbf{H} - c\gamma\boldsymbol{\beta}\times\mathbf{D},
\end{aligned} \tag{3}$$

where

$$\boldsymbol{\beta} = \frac{\boldsymbol{v}}{c}, \quad \bar{\bar{\boldsymbol{\alpha}}} = \bar{\bar{\boldsymbol{I}}} + (\gamma-1)\beta^{-2}\boldsymbol{\beta}\boldsymbol{\beta}. \tag{4}$$

In (3), the medium in $z' < 0$ is assumed to be vacuum, i.e. $\epsilon = \epsilon_0$ and $\mu = \mu_0$.

3 TE reflected plane wave

The single transient incident PW at the S frame is given by

$$\begin{aligned}
\mathbf{E}^i_{PW}(\mathbf{r},t;\boldsymbol{\kappa}_t) &= \hat{\mathbf{n}}(\boldsymbol{\kappa_t})\overset{+}{f}[t - c^{-1}(\hat{\boldsymbol{\kappa}}^i\cdot\mathbf{r})],\\
\mathbf{H}^i_{PW}(\mathbf{r},t;\boldsymbol{\kappa}_t) &= \eta_0^{-1}\hat{\mathbf{t}}(\boldsymbol{\kappa_t})\overset{+}{f}[t - c^{-1}(\hat{\boldsymbol{\kappa}}^i\cdot\mathbf{r})],
\end{aligned} \tag{5}$$

where $\overset{+}{f}$ denotes some analytic signal[13], $\hat{\boldsymbol{\kappa}}^i = (\xi_1,\xi_1,\zeta)$ is a unit vector with ξ_1 and ξ_2 being the transverse spectral variables and $\zeta = \sqrt{1-\xi_1^2-\xi_2^2}$, and

$$\begin{aligned}
\hat{\mathbf{n}}(\boldsymbol{\kappa_t}) &= \frac{1}{\kappa_t}(\xi_2\hat{\mathbf{x}} - \xi_1\hat{\mathbf{y}}),\\
\hat{\mathbf{t}}(\boldsymbol{\kappa_t}) &= \frac{\zeta}{\kappa_t}(\xi_1\hat{\mathbf{x}} + \xi_2\hat{\mathbf{y}}) - \kappa_t\hat{\mathbf{z}}
\end{aligned} \tag{6}$$

with $\kappa_t = \sqrt{\xi_1^2+\xi_2^2}$.

The $\overset{+}{f}$ argument in (5) is inserted with (1) to produce

$$t - c^{-1}(\hat{\boldsymbol{\kappa}}^i\cdot\mathbf{r}) = \sigma[t' - c^{-1}(\hat{\boldsymbol{\kappa}}^{i'}\cdot\mathbf{r}')] \tag{7}$$

where

$$\hat{\boldsymbol{\kappa}}^{i'} = \frac{\hat{\boldsymbol{\kappa}}^i\cdot\bar{\bar{\alpha}} - \gamma\boldsymbol{\beta}}{\gamma(1-\hat{\boldsymbol{\kappa}}^i\cdot\boldsymbol{\beta})} = \frac{1}{\sigma}[\xi_1,\xi_2,\gamma(\zeta-\beta)], \quad \sigma = \gamma(1-\zeta\beta). \tag{8}$$

Here $\hat{\boldsymbol{\kappa}}^{i'}$ is identified as a unit vector in the direction of propagation in the S'-frame. By inserting (5) into (3) with (7) we obtain the electric incident field at the S' frame

$$\mathbf{E}^{i'}_{PW} = \frac{\gamma}{\kappa_t}(1-\beta\zeta)(\xi_2\hat{\mathbf{x}} - \xi_1\hat{\mathbf{y}})f^+[\sigma(t' - c^{-1}\hat{\boldsymbol{\kappa}}^{i'}\cdot\mathbf{r}')] \tag{9}$$

and similarly

$$\mathbf{H}^{i'} = \frac{1}{\eta_0}\left[\frac{\gamma}{\kappa_t}(\zeta-\beta)(\xi_1\hat{\mathbf{x}} + \xi_2\hat{\mathbf{y}}) - \kappa_t\hat{\mathbf{z}}\right]f^+[\sigma(t' - c^{-1}\hat{\boldsymbol{\kappa}}^{i'}\cdot\mathbf{r}')] \tag{10}$$

By applying the standard procedure for PW scattering from planar dielectrics, we obtain snell's low in S'

$$\sin\theta^{i'} = \sin\theta^{r'}, \quad n_2\sin\theta^{t'} = \sin\theta^{i'}. \tag{11}$$

The resulting reflected PW fields in the S' reference frame are given by

$$\mathbf{E}_{PW}^{r'} = R' \frac{\gamma}{\kappa_t}(1-\beta\zeta)(\xi_2\hat{\mathbf{x}} - \xi_1\hat{\mathbf{y}}) f^+[\sigma(t' - c^{-1}\hat{\boldsymbol{\kappa}}^{r'} \cdot \mathbf{r}')],$$
$$\mathbf{H}_{PW}^{r'} = \frac{R'}{\eta_0\kappa_t}\left[-\frac{\gamma}{\kappa_t}(\zeta-\beta)(\xi_1\hat{\mathbf{x}} + \xi_2\hat{\mathbf{y}}) - \kappa_t^2\hat{\mathbf{z}}\right] f^+[\sigma(t' - c^{-1}\hat{\boldsymbol{\kappa}}^{r'} \cdot \mathbf{r}')], \tag{12}$$

where

$$R' = \frac{\cos^2\theta^{i'} - \sqrt{\cos^2\theta^{i'}\,(n_2^2 - \sin^2\theta^{i'})}}{\cos^2\theta^{i'} + \sqrt{\cos^2\theta^{i'}\,(n_2^2 - \sin^2\theta^{i'})}}, \tag{13}$$

is identified as the reflection coefficient and the propagation angle in S' is given by where

$$\cos\theta^{i'} = \frac{\zeta-\beta}{1-\zeta\beta} = \frac{\cos\theta^i - \beta}{1-\beta\cos\theta^i}. \tag{14}$$

4 Reflected Beam Composition

The TE PB propagator can be defined by its transient PW spectral representation in the form

$$\overset{+}{E}{}^i(\mathbf{r},t) = -\frac{\partial_t^2}{(2\pi c)^2}\int d^2\kappa_t\kappa_t\hat{\mathbf{n}}(\boldsymbol{\kappa_t})\,\overset{+}{\tilde{\psi}}\,(\boldsymbol{\kappa_t} - \bar{\boldsymbol{\kappa}}_t, t - c^{-1}\hat{\boldsymbol{\kappa}}^i \cdot \mathbf{r}), \tag{15}$$

where $\overset{+}{\tilde{\psi}}\,(\boldsymbol{\kappa_t},\tau)$ denotes the transient PW spectral distribution of the synthesis window $\overset{+}{\psi}\,(\mathbf{r},t)$ and $\bar{\boldsymbol{\kappa}}_t$ are the frame directional spectral variables. Next we apply the inverse Lorentz transform in (1) to each reflected transient PW in (9)-(10) and insert the resulting PW in S frame into the spectral representation in (15). The resulting spectral representation of the reflected PB is given by

$$\mathbf{E}^r(\mathbf{r},t) = -\frac{\partial_t^2}{(2\pi c)^2}\int d^2\kappa_t\kappa_t\,\overset{+}{\tilde{\psi}}{}^r(\mathbf{r},t;\boldsymbol{\kappa_t})\hat{\mathbf{n}}(\boldsymbol{\kappa_t})R' \tag{16}$$

where

$$\overset{+}{\tilde{\psi}}{}^r(\mathbf{r},t;\boldsymbol{\kappa_t}) = \overset{+}{\tilde{\psi}}\,(\boldsymbol{\kappa_t} - \hat{\boldsymbol{\xi}}_n, \rho(t - c^{-1}\hat{\boldsymbol{\kappa}}^r \cdot \mathbf{r} + \tau^r)), \tag{17}$$

with

$$\hat{\boldsymbol{\kappa}}^r = \frac{1}{\rho}[\xi_1, \xi_2, \gamma^2[2\beta - \zeta(1+\beta^2)]], \quad \rho = \gamma^2(1 - 2\zeta\beta + \beta^2), \quad \tau^r = \frac{(\beta-\zeta)z_0}{c\gamma(1-2\zeta\beta+\beta^2)}. \tag{18}$$

Equation (16) with (17) and (18) represent the reflected electric PB field in S frame.

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