Transformation Media for Finite Element Solution of Multi-scale Electromagnetic Boundary Value Problems

Mustafa Kuzuoglu [1], ***Ozlem Ozgun*** [2]

[1] Dept. of Electrical Engineering, Middle East Technical University, 06531, Ankara, Turkey

[2] Dept. of Electrical Engineering, Middle East Technical University, Northern Cyprus Campus
Guzelyurt, Mersin 10, Turkey

{kuzuoglu, ozozgun}@metu.edu.tr

Abstract

This paper presents coordinate transformation techniques for solving multi-scale electromagnetic boundary value problems involving fine geometrical features. The major purpose of this study is to get rid of fine mesh and to allow uniform and easy-to-generate meshes in the finite element solution of the multi-scale problems by introducing metamaterial regions into the computational domain. The permittivity and permeability parameters of the metamaterial are obtained by utilizing the coordinate transformation technique, which is based on the form-invariance property of Maxwell's equations. The medium where the coordinate transformation is applied is equivalent to an anisotropic medium whose parameters are determined by the Jacobian of the transformation. Several numerical simulations are illustrated in the context of finite element solution of electromagnetic scattering problems.

1. Introduction

One of the major goals of the computational electromagnetics (CEM) community is to devise efficient simulation tools that can handle various cases, especially involving geometrically-complex objects. Conventional CEM methods are usually in trouble while dealing with multi-scale structures, especially when small-scale features are to be modeled with adequate numerical precision without increasing the computational resources. For example, if a small antenna mounted on a large platform is to be simulated, a fine mesh should be generated around the antenna to properly capture its small-scale geometry with high resolution. This increases the number of unknowns, computation time and memory, as well as, introduces ill-conditioned matrices as a result of poor mesh quality. Some alternative approaches have been proposed to overcome such difficulties in the literature [1-3].

The main motivation behind the development of the techniques presented in this study is to achieve efficient numerical simulation of the multi-scale problems having uniform and easy-to-generate meshes and less number of unknowns, by modifying the computational domain so as to include metamaterial regions without changing the desired field behavior. Constitutive parameters of the metamaterial regions are designed by employing the form-invariance property of Maxwell's equations under coordinate transformations. The main idea in this approach is that Maxwell's equations retain their form in the modified coordinate system, but the medium becomes an anisotropic medium accordingly. Transformation-based techniques have received considerable attention in recent years, especially after the development of the invisibility cloak [4]. There exist numerous approaches in the literature, some of which have been proposed by us and listed in [5-11].

Below, we first briefly formulate the constitutive parameters of a transformation medium under a general coordinate transformation. Afterwards, we present the techniques in the context of two-dimensional electromagnetic scattering problems exhibiting multi-scale behavior, together with some representative finite element simulations.

2. Determination of Constitutive Parameters

Under a general coordinate transformation $\vec{r} \rightarrow \tilde{\vec{r}} = \mathrm{T}(\vec{r})$, where $\vec{r}$ and $\tilde{\vec{r}}$ are the coordinates in the original and transformed domain, respectively, original medium can be replaced with an anisotropic medium so that transformed fields satisfy original forms of Maxwell's equations. This property is known as the form-invariance property of Maxwell's equations under space transformations. The permittivity and permeability tensors of the anisotropic medium can be determined by [12]

$$\overline{\overline{\varepsilon}} = \varepsilon \overline{\overline{\Lambda}} \quad \text{and} \quad \overline{\overline{\mu}} = \mu \overline{\overline{\Lambda}}, \quad \text{where} \quad \overline{\overline{\Lambda}} = (\det \overline{\overline{\mathrm{J}}}) \left(\overline{\overline{\mathrm{J}}}^{\mathrm{T}} \cdot \overline{\overline{\mathrm{J}}} \right)^{-1} \tag{1}$$

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Here, ε and μ are the constitutive parameters of the original medium, and $\overline{\overline{\mathrm{J}}}$ is the Jacobian tensor which is $\overline{\overline{\mathrm{J}}} = \partial(\tilde{x}, \tilde{y}, \tilde{z}) / \partial(x, y, z)$ in Cartesian coordinates.

Original fields in transformed coordinates , $\vec{E}(\tilde{\vec{r}})$, and the transformed fields in original coordinates, $\tilde{\vec{E}}(\vec{r})$, are interdependent, in the sense that the original fields inside the transformed space can be recovered by using the fields inside the anisotropic material by using the following relation [12]:

$$\vec{E}(\vec{r}) \to \tilde{\vec{E}}(\vec{r}) = \overline{\overline{\mathrm{J}}}^{\mathrm{T}} \cdot \vec{E}(\tilde{\vec{r}}) \qquad \text{and} \qquad \vec{H}(\vec{r}) \to \tilde{\vec{H}}(\vec{r}) = \overline{\overline{\mathrm{J}}}^{\mathrm{T}} \cdot \vec{H}(\tilde{\vec{r}}) \tag{2}$$

3. Transformation Media for Multi-Scale Problems

This section presents coordinate transformation techniques in designing metamaterial regions for the numerical solution of scattering problems involving (i) electrically-small objects; (ii) an object coated by electrically-thin dielectric layers; and (iii) an object with electrically-thin feature. Although these techniques are special to the problems of our interest, a wide range of problems can be handled by suitable modifications. Hence, our purpose is to give a reader an intuition on how to make use of transformation media in handling multi-scale problems, by means of some representative applications.

In the first application, we consider a scattering problem where an electrically-small object (whose size is "small" compared to wavelength λ) is illuminated by a plane-wave. Accurate solution of this problem requires a large number of unknowns inside the computational domain that is not occupied by the object, because of the fact that the mesh must be refined over the fine sections of the electrically-small objects, and the truncation boundary must be located sufficiently far away from the object to eliminate spurious reflections. In the proposed technique, an equivalent problem with uniform mesh is designed by locating an anisotropic layer at a distance from the object (see Fig. 1) [7]. Both original and equivalent problems yield identical field values in their common free-space regions, and the fields inside the metamaterial region are related to the original near-fields at the close vicinity of the object through the field equivalence in (2). It is worthwhile to note that the same mesh can be used for any arbitrarily-shaped object by simply changing the constitutive parameters of the anisotropic layer according to the geometry of the object. In designing the metamaterial region, each point P inside Ω_{M} is mapped to $\tilde{\mathrm{P}}$ inside the transformed region $\tilde{\Omega} = \Omega \cup \Omega_{\mathrm{M}}$, by using the following coordinate transformation $\mathrm{T}: \Omega_{\mathrm{M}} \to \tilde{\Omega}$

$$\tilde{\vec{r}} = (d_{ac}/d_{ab})(\vec{r} - \vec{r}_{\mathrm{b}}) + \vec{r}_{\mathrm{c}} \qquad \text{where} \qquad d_{ac} = \|\vec{r}_{\mathrm{a}} - \vec{r}_{\mathrm{c}}\| \qquad \text{and} \qquad d_{ab} = \|\vec{r}_{\mathrm{a}} - \vec{r}_{\mathrm{b}}\| \tag{3}$$

The unit vector $\hat{a}$ is emanated from a point inside the innermost domain (such as the center-of-mass point) in the direction of the point P.

The finite element simulations are illustrated in Fig. 1, where a plane wave ($\varphi^{inc} = 180^{\mathrm{o}}$) is incident to a circular conducting object, whose radius is λ/20 (λ=1m). The element size in the equivalent problem is approximately λ/40, but the element size in the original problem is gradually decreased from the boundary of the outermost boundary to the boundary of the object. To measure the performance of the proposed method, a mean-square error $\mathrm{Err} = \sum_{\Omega_{\mathrm{FS}}} \left| E^{\text{equivalent}} - E^{\text{original}} \right|^2 \Big/ \sum_{\Omega_{\mathrm{FS}}} \left| E^{\text{original}} \right|^2$ is defined, where $E^{\text{equivalent}}$ and E^{original} are the electric fields calculated in the equivalent and original problems, respectively, inside the common free-space region. The error is computed as 0.1256%. The reduction in unknowns in the equivalent problem is obtained as 40%, compared to the original problem.

In the second application, we consider a problem involving an object coated by electrically-thin dielectric layers. Since the wavelength decreases inside thin dielectrics, smaller-sized elements must be used for good numerical precision. In the proposed technique, an equivalent problem is designed by locating a metamaterial layer at a distance from the object (see Fig. 2). Each point P inside Ω_{M} is mapped to $\tilde{\mathrm{P}}$ inside the transformed region $\tilde{\Omega} = \Omega \cup \Omega_{\mathrm{M}} \cup \Omega_{\mathrm{d}}$, by using the same coordinate transformation in (3). Here, Ω_{d} is the region of the dielectric layers. When the transformed point $\tilde{\mathrm{P}}$ falls into the dielectric region, the parameters of the metamaterial layer are computed by using the dielectric constant of the dielectric layer. It is again useful to emphasize that different problems (such as multiple layers with different thicknesses and dielectric constants) can be simulated by employing a single mesh, and by changing only the material parameters accordingly. The finite element simulations are presented in Fig. 2, where a plane wave whose angle of incidence is 90° illuminates a circular conducting object coated by two dielectric layers. The dielectric constant and thickness of the inner layer is 8 and λ/20, respectively. The dielectric constant and thickness of the outer layer is 4 and λ/20, respectively. The mean-square error is computed as 0.205%.

In the last application, we deal with the scattering from an object having an electrically-thin feature, which must be modeled by fine mesh in conventional methods. In the proposed technique, an equivalent problem is constructed as shown in Fig. 3. Each point P inside Ω_M is mapped to $\tilde{P}$ inside the transformed region $\tilde{\Omega} = \Omega \cup \Omega_M$ [Ω includes the thin region], by using the same coordinate transformation in (3). When the transformed point $\tilde{P}$ lies inside the thin region, the material properties of this region are used. For example, if the thin structure is conducting, the boundary condition that the tangential component of the total electric field is zero is imposed on the boundary. The finite element simulations are presented in Fig. 3, where a plane wave whose angle of incidence is 45° illuminates an object with thin feature whose thickness is λ/15. The mean-square error is computed as 0.192%.

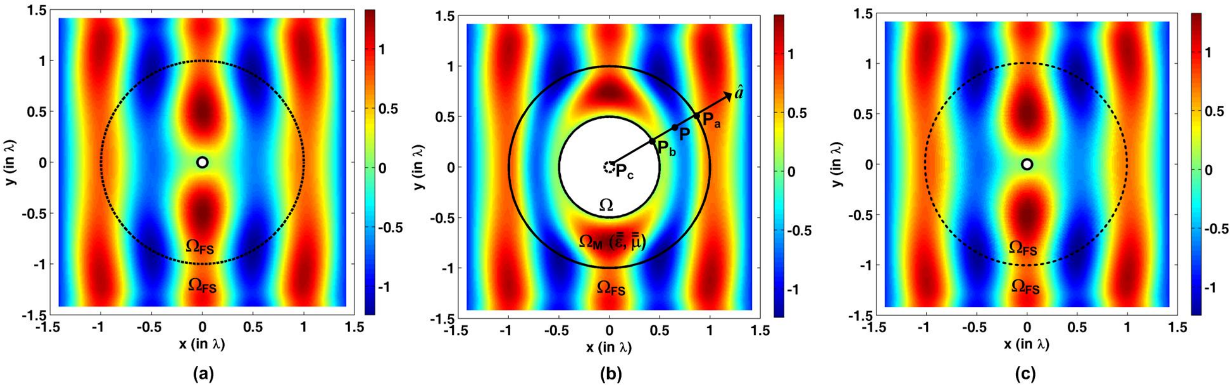

Fig. 1. Real part of total electric field for scattering from an electrically-small circular object: (a) Original problem, (b) Equivalent problem in original coordinates, (c) Equivalent problem after transforming the field values (in transformed coordinates) by using (2).

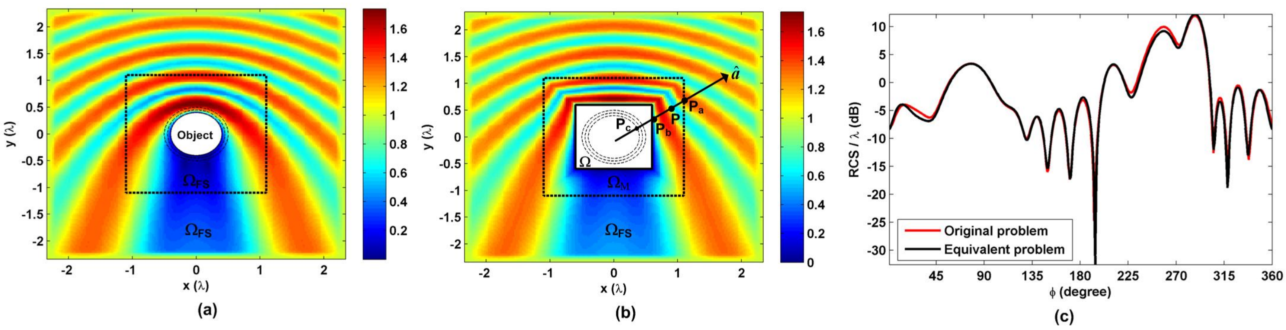

Fig. 2. Finite element simulation of scattering from a circular object coated by two dielectric layers: (a) Magnitude of total electric field in original problem, (b) Magnitude of total electric field in equivalent problem, (c) Radar cross section profiles.

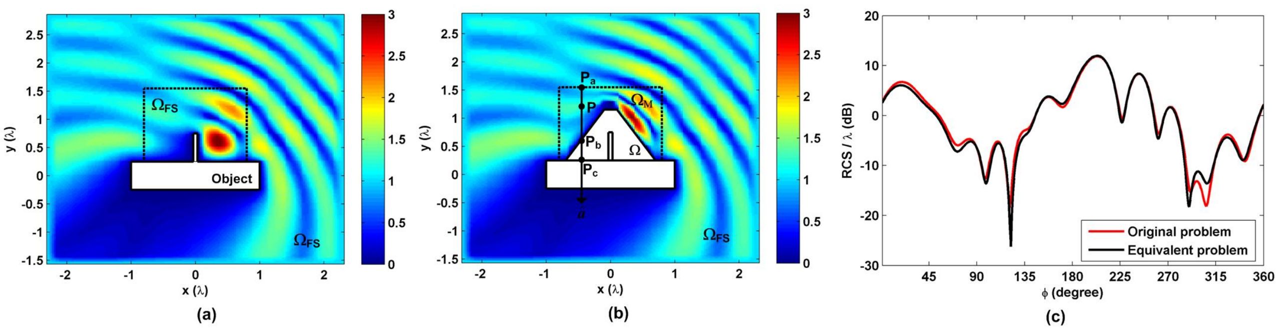

Fig. 3. Finite element simulation of scattering from an object with thin feature: (a) Magnitude of total electric field in original problem, (b) Magnitude of total electric field in equivalent problem, (c) Radar cross section profiles.

4. Conclusions

We have presented simulation tools in numerical modeling of electromagnetic boundary value problems via finite methods by placing special transformation media inside their computational domains for the purpose of uniform and easy-to-generate meshes and less number of unknowns. We have numerically explored the functionality of all techniques with the help of finite element simulations. We have observed good agreements between the analytical formulations and the numerical simulations.

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6. References

1. H. Wu, and A.C. Cangellaris, “Efficient Finite Element Electromagnetic Modeling of Thin Wires,” *Microwave Opt Technol Lett*, 50, 2008, pp. 350-354.

2. Z.-G. Qian, and W.C. Chew, “Fast Full-Wave Surface Integral Equation Solver for Multiscale Structure Modeling,” *IEEE Trans. Antennas Propagat*, 57, 2009, pp. 3594-3601.

3. O. Ozgun, R. Mittra, and M. Kuzuoglu, “Finite Element / Dipole Moment Method for Efficient Solution of Multiscale Electromagnetic Problems,” *IEEE International Symposium on Antennas and Propagation and USNC/URSI National Radio Science Meeting*, July 11 – 17, 2010, ON, Canada.

4. J. B. Pendry, D. Schurig and D. R. Smith, “Controlling electromagnetic fields,” *Science*, 312, 2006, pp. 1780-1782.

5. O. Ozgun, and M. Kuzuoglu, “Electromagnetic metamorphosis: Reshaping scatterers via conformal anisotropic metamaterial coatings,” *Microwave Opt. Technol. Lett.*, 49, 2007, pp. 2386-2392.

6. O. Ozgun, and M. Kuzuoglu, “Utilization of anisotropic metamaterial layers in waveguide miniaturization and transitions,” *IEEE Microw. Wireless Comp. Lett*., 17, 2007, pp. 754-756.

7. O. Ozgun, and M. Kuzuoglu, “Efficient Finite Element Solution of Low-Frequency Scattering Problems via Anisotropic Metamaterial Layers,” *Microwave Opt. Technol. Lett.*, 50, 2008, pp. 639-646.

8. O. Ozgun, and M. Kuzuoglu, “Form-invariance of Maxwell’s Equations in Waveguide Cross-section Transformations,” *Electromagnetics*, 29, 2009, pp. 353-376.

9. O. Ozgun, and M. Kuzuoglu, “Domain Compression via Anisotropic Metamaterials designed by Coordinate Transformations,” *Journal of Computational Physics*, 229, 2010, pp. 921-932.

10. O. Ozgun, and M. Kuzuoglu, “Form Invariance of Maxwell’s Equations: The Pathway to Novel Metamaterial Specifications,” *IEEE Antennas and Propagation Magazine*, 52, June 2010, pp. 51-65.

11. O. Ozgun, and M. Kuzuoglu, “Form-invariance of Maxwell's Equations in Coordinate Transformations: Metamaterials and Numerical Models” (Chapter 3), *Metamaterials: Classes, Properties and Applications*, Editor: E. J. Tremblay, Nova Science Publishers Inc., 2010.

12. I.V. Lindell, *Methods for Electromagnetic Field Analysis*, Oxford Univ. Press, 1992.