Analysis of Metamaterials using Analytic Properties

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Abstract

In this contribution it is investigated, how a metamaterial structure with a finite number of repeated cells can be analyzed efficiently using analytic properties. The method of lines (MoL) is utilized, which is a semi-analytical method. As the propagation in propagation direction is described analytically, Floquet`s Theorem can be used to transform the field at input and output of one cell into a set of Floquet modes. The propagation of the Floquet modes throughout the structure consisting of N periods can easily be calculated by the resulting phase difference. Thus the computation time is independent from the number of periods. With increasing number of periods the MoL becomes more and more competitive compared to commercial tools, where the computation time increases with higher number of periods. Results for a metamaterial CRLH transmission line are presented and compared with results obtained by commercial tools and measured results from literature.

1. Introduction

Metamaterials are of increasing interest in order to realize novel components. They are artificial structures with electromagnetic properties not commonly found in nature. Typical realizations are three-dimensional structured materials containing cells with split-ring resonators (SRRs) or composite right/left-handed transmission lines. CRLH transmission lines, which are circuit oriented and non-resonant, can efficiently be used to build leaky wave antennas and other practical applications of metamaterials. CRLH transmission lines exhibit left-handed (LH) and right-handed (RH) behavior in the lower and upper frequency range. In the LH range, the transmission line is formed by a series capacitance and a shunt inductance in contrast to the ordinary RH range [1,2].

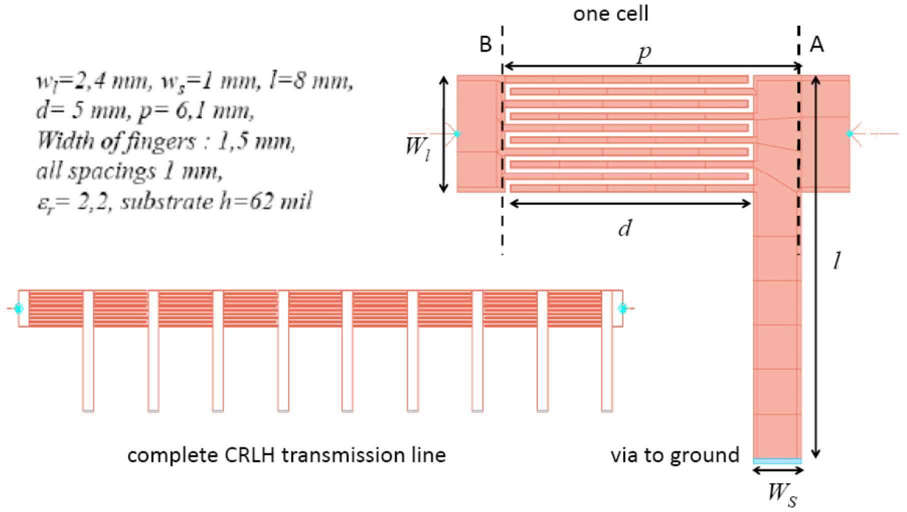

Fig. 1: Complete CRLH microstrip line consisting of 9 cells, connected to 75 Ω input and output lines in microstrip technique and one single cell with series interdigital capacitor with 10 fingers and inductance to ground [1].

A typical example is shown in Fig. 1, where we have a concatenated structure consisting of 9 cells. Each cell is built of a series interdigital capacitor in microstrip technology and a shunt inductance to ground. Input and output are connected to 75 Ω transmission lines. The simulation of this kind of structure can be done using commercial tools in reasonable time, however, the numerical effort using a 2½ or 3D commercial solver goes up extremely with the number of periods. The use of the typical periodic boundary conditions is not possible, as the periods are not repeated outside the computational domain, but are connected to input and output lines.

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In this contribution the Method of Lines will be introduced for an efficient simulation of metamaterial components. The Method of Lines is a fullwave numerical method, based on a finite difference discretization scheme. The difference to commercially used finite difference solvers is that the discretization is only performed in 2 directions of the relevant coordinate systems, whereas in the remaining direction the calculation is analytical. If this direction is chosen to be the propagation direction of the wave in the considered structure, the 2D analysis of the cross-section performed with finite differences gives a set of propagating modes for each homogeneous part of the device. In the second step the related field is propagated to the interfaces of this homogeneous section and matched to the fields of the adjacent section. Thus the scattering parameters are derived by the repeated matching and propagation procedure throughout the structure. As it will be shown, Floquet`s theorem can be efficiently used to account for the limited periodicity of the structure under consideration and thus can reduce the computational effort and calculations times , even compared to commercial solvers.

2. Analysis of one cell

In the MoL the fields are computed by a 2D discretization of the cross-section. For each homogeneous section of a concatenated structure a set of eigenvalues is computed, describing the different propagating and attenuated modes in this section. The amplitudes of the different modes are determined by the matching of the tangential field components at the interfaces between the sections. The matching takes place in the spatial domain, not in the modal domain, thus we can establish a relationship between the fields at input and output of one section. In the concatenation process all the sections are combined by a successive matching and propagation. In this way one period as shown in Fig. 1 can easily be analyzed using the conventional MoL algorithm.

The result is a relationship between the tangential fields at input of output of the complete cell (eq. 1). Where z_i consist of multimode impedance matrices [3].

$$\begin{bmatrix} \mathbf{E}_B \\ \mathbf{H}_B \end{bmatrix} = \begin{bmatrix} z_1 & z_2 \\ z_3 & z_4 \end{bmatrix} \begin{bmatrix} \mathbf{E}_A \\ \mathbf{H}_A \end{bmatrix} \tag{1}$$

When the cell is connected to input and output lines and the structure is fed by the fundamental mode of the connected line only, we can derive the scattering parameters of the whole cell. This is presented in Fig. 2a. Here the results are compared to an ADS Momentum simulation. A good agreement can be observed between the two simulations.

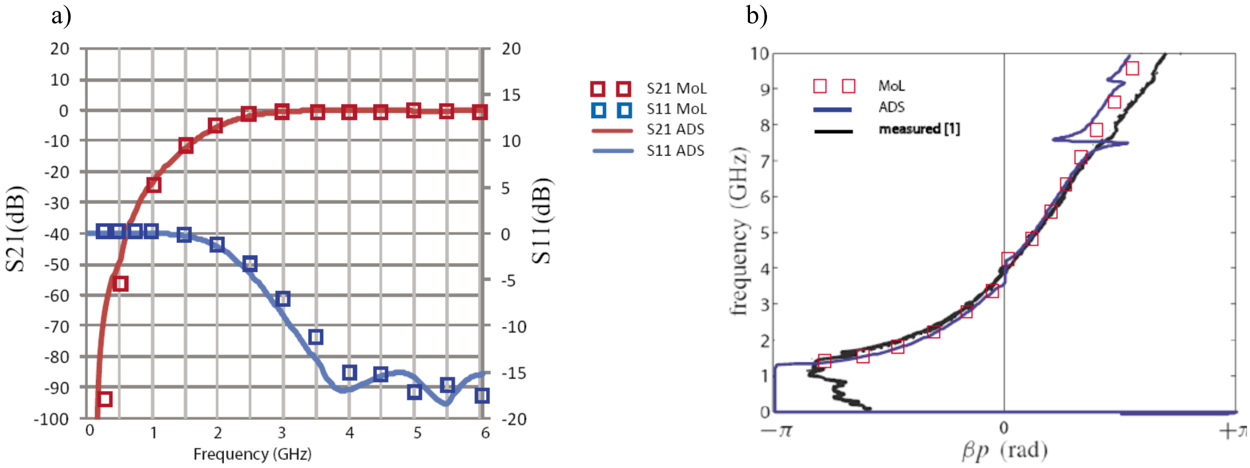

Fig. 2: a) Scattering parameters b) Dispersion characteristic for one cell. Comparison between MoL, ADS Momentum and measured results [1].

Relevant in order to evaluate the behavior of the metamaterial cell is to investigate the dispersion characteristic. This is shown in the following plot Fig. 2b). The Mol result is compared to an ADS Momentum simulation and to measured results from [1]. A good agreement can be observed.

A comparison between the computation time between MoL and ADS shows that ADS is faster. Both methods are using only a 2D siscretization. Where in the Mol the complete cross-section is discretized, in ADS only the metallization needs a 2D discretization. Due to the fact that the MoL requires some matrix inversions and is programmed in MATLAB, the resulting computation time is slower and gives no benefit, however, the results are comparable.

4. Analysis of periodic structure

For the analysis of the periodic structure consisting of a finite number of cells, we can exploit the analytic properties of the MoL. In the resulting algorithm we now utilize Floquet`s theorem.

$$\tilde{E}_i^{\pm}(z_B = z_A + L) = \exp(\pm\Delta\tilde{\varphi}_i)\tilde{E}_i^{\pm}(z = z_A) \tag{2}$$

where $+$ and □ stand for the forward and backward propagating modes, respectively. Floquet's theorem means that the modal fields in two points that lie one period apart (here shown for the electric field) only differ by a phase factor. The quantities in the Floquet domain are here denoted with a ~. Therefore we have to transform our fields in space domain to the modal domain by a suitable transformation. The number of Floquet modes equals the number of discretization points in space domain. The phase relation yields for each mode separately. The correspondence in modal domain is shown in eq. 3.

$$\begin{bmatrix} \tilde{\mathbf{E}}_B \\ \tilde{\mathbf{H}}_B \end{bmatrix} = \begin{bmatrix} \cosh(\Delta\tilde{\varphi}) & -\tilde{Z}_0 \sinh(\Delta\tilde{\varphi}) \\ -\tilde{Y}_0 \sinh(\Delta\tilde{\varphi}) & \cosh(\Delta\tilde{\varphi}) \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{E}}_A \\ \tilde{\mathbf{H}}_A \end{bmatrix} \tag{3}$$

with the modal characteristic impedance matrix and the modal phase shift matrix

$$\tilde{Z}_0 = diag(\tilde{Z}_{0i}) \quad (4) \qquad\qquad \Delta\tilde{\varphi} = diag(\Delta\tilde{\varphi}_i) \quad (5)$$

This can be achieved by a transformation to principal axis, which results in a diagonalization of the transfer matrix, describing the relation in the original domain. From the eigenvalues of this transfer matrix we can deduct the propagation constants in the "Floquet" domain.

$$\mathbf{E}_{A,B} = S_E \tilde{\mathbf{E}}_{A,B} \qquad \mathbf{H}_{A,B} = S_H \tilde{\mathbf{H}}_{A,B} \tag{6}$$

The corresponding eigenvalue problems results from the equivalence of the two transfer relations (1) and (3) and gives us the necessary phase factors:

$$z_1 S_E = S_E \cosh(\Delta\tilde{\varphi}) \qquad z_4 S_H = S_H \cosh(\Delta\tilde{\varphi}) \tag{7}$$

Now we have an appropriate description of the fields at input and output of one cell in the transformed "Floquet" domain and can apply Floquet´s Theorem. As this applies to any period of the structure, we can repeat the application of eq. (2) from the first to the last period (namely the N-th period) in order to obtain a relation from the input port C of the finite periodic circuit to its output port D

$$\tilde{E}_i^{\pm}(z_D = z_C + NL) = \exp(\pm N\Delta\tilde{\varphi}_i)\tilde{E}_i^{\pm}(z = z_C) \tag{8}$$

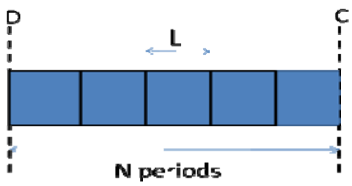

Fig. 3: Finite periodic structure with N periods of length L

Resulting in the following relation between the modal fields at C and D:

$$\begin{bmatrix} \tilde{\mathbf{E}}_D \\ \tilde{\mathbf{H}}_D \end{bmatrix} = \begin{bmatrix} \cosh(N\Delta\tilde{\varphi}) & -\tilde{Z}_0 \sinh(N\Delta\tilde{\varphi}) \\ -\tilde{Y}_0 \sinh(N\Delta\tilde{\varphi}) & \cosh(N\Delta\tilde{\varphi}) \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{E}}_C \\ \tilde{\mathbf{H}}_C \end{bmatrix} \tag{9}$$

The propagation concept of the Floquet modes is very simple. When we have the Floquet modes, any length of N cells can be described by just a simple multiplication of the phase by a scalar multiplication with the factor N. At the outer boundary of the periodic section, here at the interfaces C and D, we can switch back to the original description in space domain by multiplying the Floquet fields by the corresponding eigenvectors. Back in the space domain, we can continue with the usual Mol algorithm an concatenate the periodic section with non-periodic sections like the 75 Ω transmission line at input and output. This procedure can be repeated any time a periodic section occurs. Even for just 2 or 3 cells the periodic MoL algorithm gives advantage in computation time compared to the conventional algorithm.

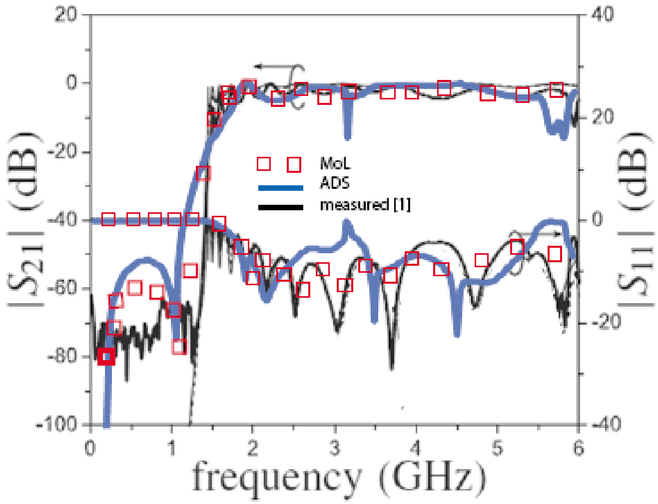

Fig. 4. Comparison of the scattering parameters of a 9 cell structure of MoL with ADS and measured values.

With the new periodic algorithm the computation time is now independently from the number of periods. Due to the additional eigenvalue problem to be solved is nearly double the time of the analysis for one cell with the MoL. However, up from structure with more than 5 cells it becomes competitive with commercial solvers, although the MATLAB based MoL program is not optimized in all respects and the MATLAB routines are also not very fast.

5. Conclusion

An algorithm is shown which is very suitable and efficient for the analysis of metamaterial components. Metamaterials usually consist of the concatenation of suitable cells, which are repeated a number of times. With the MoL based periodic solver we can tackle analytically this finite periodicity, which is not possible with usual commercial solvers or traditional periodic boundaries. By using Floquet`s Theorem to describe the propagation properties of the cell, we can make the computation time independent from the number of required periods. With increasing number of periods the proposed methods becomes competitive and even advantageous compared to commercial solvers. The obtained accuracy in the method of lines is usually high due to the semi-analytically nature of the method and the only 2D discretiztion needed.

6. Acknowledgement

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7. References

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