Material realizations of extreme electromagnetic boundary conditions and metasurfaces

Ari Sihvola, *Henrik Wallén*, *Pasi Ylä-Oijala*, *Johannes Markkanen*, and *Ismo V. Lindell*
Aalto University School of Electrical Engineering
Department of Radio Science and Engineering
P.O. Box 13000, FI–00076 AALTO (Finland)
ari.sihvola@aalto.fi

Abstract

The paper discusses the correspondence between electromagnetic boundary conditions and interface conditions. In particular, the focus is on the synthetic approach where the interest is in finding material realizations for given boundary conditions. Material realizations are approximative but not unique because, especially if anisotropic and bianisotropic materials are allowed, there are different material classes with which any given boundary condition can be mimicked. As examples, the PEC, PMC, PEMC, and DB boundary conditions are discussed. By comparing the scattering characteristics, it is demonstrated how well certain extreme-parameter material realizations are able to simulate the boundary effect.

1 Introduction

In electromagnetics, the distinction between a boundary and an interface is fundamental. It is essential to emphasize the difference between these two concepts because very often in applied electromagnetics and in metamaterials studies, confusions exist. One often hears the question: "What is the material behind the surface on which a boundary condition is assumed?" The answer is, of course, that such a question is meaningless: nothing in the space on the other side of the boundary affects the fields in the domain of interest. On a boundary of a given spatial domain, electromagnetic fields have to be forced to satisfy a certain boundary condition in order to uniquely determine the field solutions. An interface problem is fundamentally different: the fields in the two domains have an interaction through tangential continuity conditions across the interface.

The boundary–interface issue has a special significance in metamaterials studies where quite often the focus is on media with extreme constitutive parameters. Even if the boundary problem is different from the interface problem, they can be approximations or idealizations of each other. However, here it is important to keep clear what is the starting point and what is the approximation.

If the interface problem is approximated by a boundary problem, which idealizes the situation and hence simplifies the analysis, the approach can be termed *analytic*. The complementary procedure is a *synthetic* approach, where the boundary problem is primary and the question is how to construct and synthesize a real-world material structure that would best approximate the starting-point situation with the ideal boundary [1].

In this paper, we focus on the latter approach and show how well certain boundary conditions and complex surfaces can be simulated by material structures.

2 Boundary Conditions and their Realizations

In classical electromagnetics, several different boundary conditions have been in use. The perfect electric conductor (PEC) condition forces the tangential electric field to be zero, and its dual case

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is the perfect magnetic conductor (PMC) with vanishing tangential component of the magnetic field. These two are special cases of the impedance surface [2] which determines the dyadic relation between the electric and magnetic fields. In connection with metamaterials studies, also more complex metasurfaces have been suggested. Among those are the the perfect electromagnetic conductor (PEMC) surface [3] and the so-called DB boundary condition [4] with its generalizations [5].

2.1 PEC, PMC, and PEMC

Traditionally the PEC boundary condition is associated with the boundary of a good conductor into which the penetration of electromagnetic waves is negligible. Since the complex permittivity $\epsilon = \epsilon' - \mathrm{j}\sigma/\omega$ becomes very large in magnitude for a medium with large conductivity σ, the connection of the PEC boundary with a very large permittivity is natural. Such a medium has been termed EVL medium (Epsilon-Very-Large) [6].

However, as has been shown in [1], in scattering problems the mere increase of permittivity of the scattering object does not uniformly and optimally lead to a PEC-boundary behavior. Rather than an EVL material, a medium with simultaneously large ϵ and small μ simulates the PEC behavior more effectively. Such a medium could be termed ZNZ material (impedance (Z)-Near-Zero). This can be seen from the results in Figure 1 where the scattering efficiency of two types of material spheres are compared as their permittivities increase. The response of a lossy sphere with no magnetic response approaches much more slowly that of the PEC sphere than in the case of a sphere with increasing permittivity and decreasing permeability. The phenomenon is independent of the sphere size, as witnessed by the two figures.

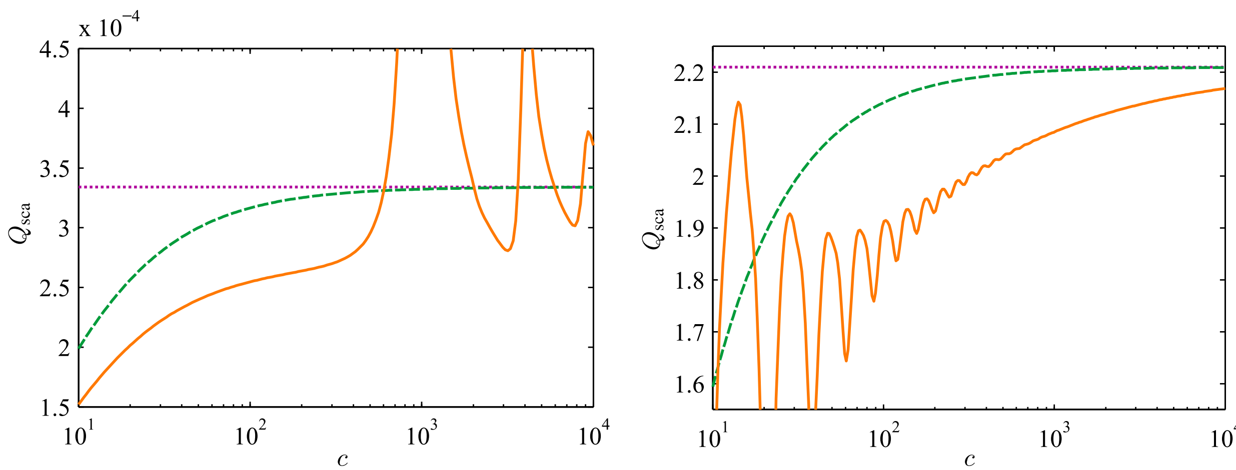

Figure 1: The scattering efficiency of a material sphere compared with a PEC sphere. Dotted: PEC; dashed: $\epsilon_r = c, \mu_r = 1/c$; solid: $\epsilon_r = (1 - 0.1\mathrm{j})c, \mu_r = 1$. (Left: small sphere, size parameter $ka = 0.1$; right: resonant sphere, $ka = 2$; with k being the free-space wave number and a the radius of the sphere.)

Analogously, the optimum PMC surface simulation is to replace the space behind the surface by a homogeneous ZVL medium (impedance (Z)-Very-Large), which means that the permeability is large and the permittivity small in amplitude. The realization of the PMC surface (AMC, "artificial magnetic surface", or HIS "high-impedance surface") is a desired goal in antenna applications. The mushroom structure [7], although not a homogeneous material as such, is one realistic design to generate the PMC surface effect.

The PEMC boundary condition [3] generalizes the PEC and PMC boundaries by an additional parameter M: the condition for the tangential electric ($\mathbf{E}_t$) and magnetic ($\mathbf{H}_t$) fields on the surface is simply $\mathbf{H}_t + M\mathbf{E}_t = 0$. Such a surface has possible applications in, for example, polarization control of antennas. To synthesize a PEMC surface effect, the approach of an extremely uniaxial medium seems promising. This means a conductor-backed layer with "fakir's bed of nails" structure, where both the normal components of the permittivity and permeability grow very large. Extensive calculations of the effect on the performance of the finite parameters of a synthesized anisotropic structure to simulate a PEMC surface are presented in [8]. A PEMC surface fabrication has been published in [9] where the effect is created by Faraday rotation using a grounded ferrite slab structure.

2.2 DB Boundary Condition

The DB boundary condition requires that the normal components of the electric ($\mathbf{D}$) and magnetic ($\mathbf{B}$) flux densities vanish on the surface [4]. Such a surface may have potential applications in backscattering control or spatial filtering of antenna radiation. To mimic the effect of the DB surface using material structures, there are several possibilities. A rather natural approach to satisfy the two DB conditions is to force the normal flux components to be zero by an interface against an anisotropic material whose permittivity and permeability components normal to the surface vanish. However, such a material is not the only possibility: for example, the choice of the so-called bianisotropic IB medium or its generalization, the P-medium [10], can also produce the same DB effect. Figure 2 shows how effectively an INZ (refractive Index-Near-Zero) scatterer shows the same forward scattering as that of a sphere on whose surface the DB boundary condition is forced.

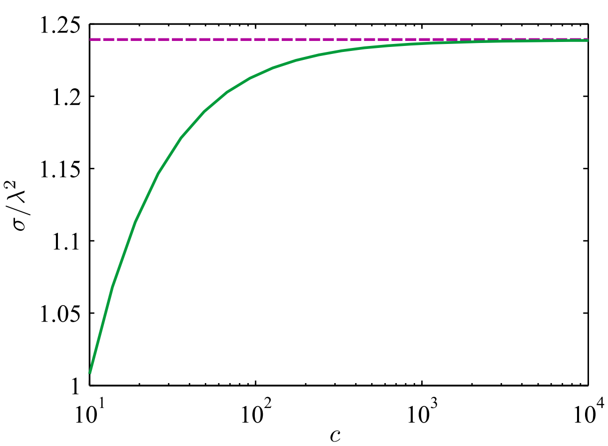

Figure 2: The forward scattering cross section (normalized by the free-space wavelength squared) of a material sphere (left) and cube (right) compared with the corresponding DB scatterer (the wave is normally incident with respect to the face of the cube). The size parameter of the sphere is $kr = 1.5$ and the cube has the same volume as the sphere. The material sphere is uni-impedant $\mu_r/\epsilon_r = 1$ with $\epsilon_r = \mu_r = 1/c$. With decreasing refractive index (increasing c), the response approaches that of the DB-surface scatterer.

3 Discussion

The synthetic approach to the boundary–interface correspondence calls for material realizations for a given boundary condition, which of course can only be approximate. Only in the limit of infinite or zero values for certain components of the material parameters, the effects of the boundary and the interface on electromagnetic fields match. However, the path to arrive at such limits can be taken through various extreme material types, like, for example, in the case of mimicking the DB surface which can be achieved by INZ, IB, or P medium, among others.

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